GATEFLIX

INDUSTRIAL ENGINEERING

For
MECHANICAL ENGINEERING

INDUSTRIAL ENGINEERING

Production Planning and Control: Forecasting models, aggregate production planning, scheduling, materials requirement planning.
Inventory Control: Deterministic and probabilistic models; safety stock inventory control systems
Operations Research: Linear programming, simplex and duplex method, transportation, assignment, network flow models, simple queuing models, PERT and CPM.

ANALYSIS OF GATE PAPERS

Exam Year	1 Mark Ques.	2 Mark Ques.	Total
2003	1	6	13
2004	3	6	15
2005	3	7	17
2006	2	7	16
2007	-	4	8
2008	2	6	14
2009	2	5	12
2010	4	4	12
2011	2	2	6
2012	1	2	5
2013	2	1	4
2014 Set-1	2	1	4
2014 Set-2	2	2	6
2014 Set-3	1	3	7
2014 Set-4	2	3	8
2015 Set-1	1	2	5
2015 Set-2	3	1	5
2015 Set-3	1	2	5
2016 Set-1	-	2	4
2016 Set-2	1	2	5
2016 Set-3	1	1	3
2017 Set-1	0	2	4
2017 Set-2	2	2	6
2018 Set-1	1	2	5
2018 Set-2	2	2	6

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1 FORECASTING

1.1 DEFINITION

Forecasting is defined as an estimate of sales in physical units or monetary value for a specified period.

1.2 TYPE OF FORECASTING

The forecasting may be classified on the basis of time span or range of forecasting. There are three categories as follows:

1.2.1 LONG RANGE FORECASTING

Long range forecasting consists of time period of more than 5 years. Mostly qualitative techniques are used for long range forecasting.

Applications:
i) Power generation
ii) Cement Industry
iii) Sugar Industry
iv) Petroleum Industry

1.2.2 MEDIUM RANGE FORECASTING

The range of medium range forecasting is generally 1 to 5 years. Medium range forecasting needs combination of collective opinion, regression analysis, correlation of different index and inflation etc.

Applications:
i) Inventory planning
ii) Sales planning
iii) Enrolment of students in college etc.

1.2.3 SHORT RANGE FORECASTING

Short range forecasting covers a period typically from one hour to one year. In most cases, it is for one season, a few months or a few weeks. For this exponential smoothing, graphical projections etc. are used.

Application:
i) Monthly forecast of sales

1.3 CATEGORIES OF FORECASTING METHOD

Forecasting can also be classified into two categories:
i) Qualitative Method
ii) Quantitative Method

1.3.1 QUALITATIVE METHOD

Qualitative methods are needed in forecasting when data necessary to use time series are not available .for example , when a new product is to be launched in the market , its past demand data are not available
In such cases human judgment, expert opinion, market research etc. are required for forecasting.

1.3.1.1 TYPE OF QUALITATIVE FORECASTING

1) Delphi Method
2) Market Research
3) Historical Estimates

1.3.2 QUANTITATIVE METHOD

When past data are available then with the help of mathematical equations forecasting can be done. Such method of forecasting is known as quantitative method. It is done for small to medium range forecasting.

1.3.2.1TYPES OF QUANTITATIVE METHOD OF FORECASTING

1) Least square or Regression analysis
2) Time series analysis

1.3.2.2LEAST SQUARE or REGRESSION ANALYSIS

This is the mathematical method of obtaining the line of best fit between the dependent variable & independent variable .In a simple regression analysis, the relationship between the dependent variable y and some independent variable x can be represented by a straight line.
y= a +bx
Where a = y intercept
b = Slope of the line

The values of 'a' & 'b' are determined from two simultaneous equations

$$\sum y = na + b\sum x \ ...\ (i)$$
$$\sum xy = a\sum x + b\sum x^2 \ ...\ (ii)$$

By solving equation (i) & (ii), we get

$$a = \frac{1}{n}\left(\sum y - b\sum x\right)$$
$$b = \frac{\sum x\sum y - n\sum xy}{\left(\sum x\right)^2 - n\sum x^2}$$

Where n = No of observations

1.3.2.3 TIME SERIES ANALYSIS

1) **Simple Moving Average Method:** This method of forecasting uses past data to calculate a rolling average for a period.

$$\text{Moving Average}(y) = \frac{\text{Sum of demands for periods}}{\text{Chosen number of peridos}} = \frac{D_1 + D_2 + D_3 + \ldots + D_n}{n}$$

2) **Weighted Moving Average:** As compared to simple moving average which gives equal effect to each component of the data base, weighted moving average can give any amount of weights for each element, where the sum of all weight should be one.
In simple moving average, equal weightage is given to 1st month, 2nd month and 3rd month in a three month moving average. But the organization wants to attach more weightage to the third month and least to the first month. For example, depending upon the importance it assigns weightage e.g. 0.2 to 1st period 0.3 to second and 0.5 to the third. The sum of these weights should be equal to one.

Example: The past data on the load on the weaving machines is shown below:

Month	Load(hrs)
May-96	
June-96	585
July-96	610
Aug-96	675
Sep-96	750
Oct-96	860
Nov-96	970

a) Compute the load on the weaving machine centre using 5th moving average for the months of December 1996.
b) Compute a weight three months moving average for December, 1996 where the weightage are 0.5 for the latest month, 0.3 and 0.2 for the other months respectively.

Solutions:
a) Five months moving average forecast for Dec.1996

$$= \frac{D\ Nov. + D\ Oct + D\ Sep + D\ Aug + D\ July}{5}$$
$$= \frac{970 + 860 + 750 + 675 + 610}{5}$$
$$= 773\text{hrs}$$

b) A three month weight moving average forecast for Dec. 1996

$$= \frac{(W\ Nov.D\ Nov) + (W\ oct + D\ Oct) + (W\ Sep.D\ Sep)}{W_{Nov} + W_{Oct} + W_{Sep}}$$
$$= \frac{(970 \times 0.5) + (660 \times 0.3) + (750 \times 0.2)}{0.5 + 0.3 + 0.2}$$

=947.8 machine hours.

1) **Exponential Smoothing Method:** The Moving Average Method of forecasting requires maintaining the data for all the previous years. This method overcomes the limitations of moving average and eliminates the requirement of keeping past record. It represents a weighted average for the past observation. The most recent observation is given highest weightage and it decreases in the form of geometric progression when move for older observations. Forecast for the period $t(f_t)$ is given by

$$f_t = f_{t-1} + \alpha\left(D_{t-1} - f_{t-1}\right)$$

Where f_{t-1} =Last period forecasted demand

D_{t-1} =Last period actual demands

α =Smoothing constant which lies between 0-1

Note:

When demand is very stable α = 0.1 to 0.3

When demand is slightly unstable α = 0.4 to 0.6

1.4 FORECASTING ERROR

1.4.1 MEAN ABSOLUTE DEVIATION (MAD)

This is calculated as the average of absolute value of difference between actual and forecasted value.

$$MAD = \frac{\sum_{t=I}^{n}\left|D_t - F_t\right|}{n}$$

Where D_t = Actual demand for period t

F_t = Forecasted demand for period t

n = Number of periods considered for calculating the error

1.4.2 MEAN SUM OF SQUARE ERROR (MSE)

The average of square of all errors in the forecast is termed as MSE. Its interpretation is same as MAD.

$$MSE = \frac{\sum_{t=I}^{n}\left(D_t - F_t\right)^2}{n}$$

1.4.3 BIAS

Bias is measure of overestimation or underestimation

$$Bias = \frac{\sum_{t=I}^{n}\left(D_t - F_t\right)}{n}$$

1.4.4 TRACKING SIGNAL (TS)

It is used to identify those items, which do not keep pace with either positive or negative bias or trend.

$$TS = \frac{\sum_{t=I}^{n}\left(D_t - F_t\right)}{\left(MAD\right)_n}$$

Where

$\left(MAD\right)_n$ = Mean absolute deviation till period n.

$\left(BIAS\right)_n$ = Bias till period n.

2 INVENTORY CONTROL

2.1 INTRODUCTION

The term inventory can be defined as the stocks of raw materials, parts and finished products held by an organization. It is a comprehensive list of movable items which are required for manufacturing the products and to maintain the plant facilities in working conditions.

2.1.1CLASSIFICATION OF INVENTORIES

1) **Direct inventory:** The inventory which play a direct role in the manufacture of a product and becomes an integral part of the finished product are called direct inventories e.g.
   i) Raw material
   ii) In process inventory
   iii) Purchased parts
   iv) Finished goods

2) **Indirect inventory:** The Indirect inventories are those materials which helps the raw materials to get converted into the finished product, but do not becomes an integral part of the finished products
   i) Tools
   ii) Supplies (brooms, cotton, toilet papers, card board boxes, office supplies etc)

2.1.2 REASONS FOR KEEPING INVENTORIES

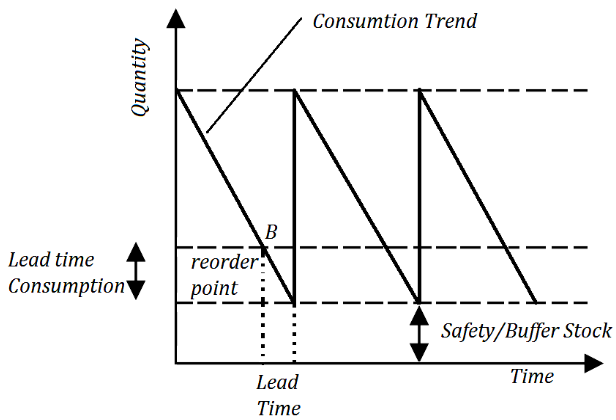
1) To stabilize production
2) To take advantage of price discounts
3) To meet the demands during the replenishment period
4) To prevent loss of sales etc

2.2 INVENTORY CONTROL

Inventory control is a planned approach of determining what to order, when to order and how much to order and how much to stocks that costs associated with buying & storing are optimal without interrupting production and sales. Inventory control basically deals with two problems:
1) When should an order be placed (order level)
2) How much should be ordered (order quantity)

2.2.1 INVENTORY CONTROL TERMINOLOGY

1) **Demand:** It is the number of items required per unit time. The demand may be either deterministic or probabilistic in nature.
2) **Order cycle/ cycle time:** The time period between two successive orders is called order cycle or cycle time.
3) **Lead time:** The length of time between placing an order and receipt of items is called lead time.
4) **Safety stock:** It is also called buffer stock or minimum stock. It is the stock or inventory needed to account for delays in materials supply and to account for sudden increase in demands due to rush orders.



1) **Re-order level (ROL):** It is the point at which the replenishment action is initiated when the stock level reaches ROL, the order is placed for the item.
2) **Re-order quantity:** This is the quantity at re-order level. Normally this quantity equals the economic order quantity.

2.2.2 INVENTORY CONTROL SYSTEMS

1) **Fixed quantity system (Q system):** In fixed quantity (Q system), the order quantity is fixed and ordering time varies according to the fluctuation in demand.

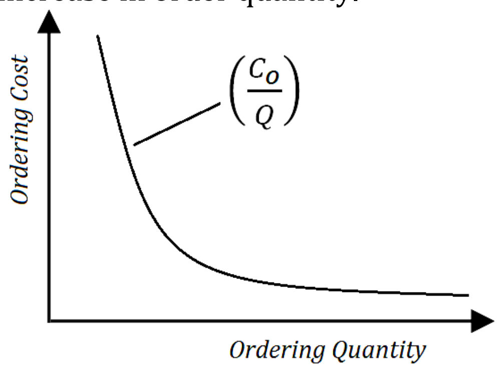
2) **Fixed period system (P System):** In fixed period (P-system), the ordering interval is fixed but the order quantity may vary with changes in demand.

2.3 COSTS ASSOCIATED WITH INVENTORY

2.3.1 ORDERING COST

It is the cost of placing an order from a vendor. This includes all costs incurred from calling for quotations, stationary cost, salaries of staff involved in purchase function etc.

- If $'c_0'$ is the cost of placing an order quantity, then the unit cost of placing an order $= \frac{c_o}{Q}$
- Unit cost of order decreases with increase in order quantity.



2.3.2 CARRYING COST OR HOLDING COST

Holding costs are incurred due to maintain an inventory level in the organization. It is due to following factors:

i) Interest on the held up capital in inventory
ii) Insurance cost
iii) Rent of building
iv) Salaries of staff
v) Obsolescence
vi) Deterioration
vii) Pilferage etc

Generally carrying cost is expressed as a percentage of inventory value.
Assuming that the inventory decreases through use or sale at a constant rate from the order quantity (Q) to zero and then replenished by another order quantity, the average inventory

$$= \frac{Q+0}{2} = \frac{Q}{2}$$

If C_c = Carrying cost per unit per unit time, then average carrying cost is given by

Average carrying cost $= C_c \times$ Average Inventory

$$= C_c \times \frac{Q}{2}$$

2.3.3 SHORTAGE COST

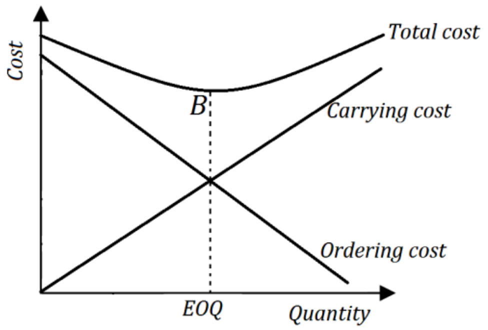
When there is stock-out situation, the customer demand is not satisfied. This causes loss due to emergency purchase, unsatisfied customer, results in loss of goodwill and lost – sale. This is shortage cost.

2.3.4 PRODUCTION OR PURCHASE COST

The value of an item is its purchasing or production cost. This cost becomes significant when availing the price discounts.

2.4 ECONOMIC ORDER QUANTITY

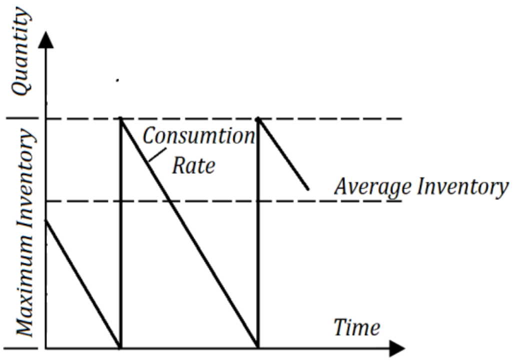
It is to be noted that ordering cost and carrying cost are conflating in nature and in determining the most economical quantity to order; we have to strike a balance between the two. From the point of view of the carrying cost inventory should be as small as possible which is not practical and from the point of view of ordering cost, material should be ordered in large quantities per order, but this will result in high carrying cost.



So for a particular annual consumption there is one order quantity at which the total cost of item (ordering cost + carrying cost) are the lowest and this is called "Economic order quantity"

2.5 INVENTORY MODELS

2.5.1 ECONOMIC ORDER QUANTITY WITH INSTANT ANEOUS STOCK REPLENISHMENT (BASIC INVENTORY MODEL)



Let D = Annual demand (Units per year)
C_o = Ordering costs (Rupees/order)
C_h = Inventory carrying cost (Rupees/unit/untimtime)
Q = Order quantity
Q^* = Economic order Quantity
N = Number of order placed per annum
T_c = Total cost per annum
Annual ordering cost = No. of orders × ordering cost per order

$$= \frac{\text{Annual Demand}}{\text{Order Quantity}} \times \text{ordering cost per order}$$

$$= \frac{D}{Q} \times C_o$$

Annual carrying cost = Average Inventory × Inventory carrying cost

$$= \frac{QC_h}{2}$$

Annual total cost $(T_c) = \frac{DC_o}{Q} + \frac{QC_h}{2}$

To get economic order quantity total, cost is to be differentiated and equated to zero. We get

$$Q^* = EOQ = \sqrt{\frac{2DC_o}{C_h}}$$

The optimal number of order placed per year

$$N^* = \frac{D}{Q^*}$$

Optimal time interval between two orders

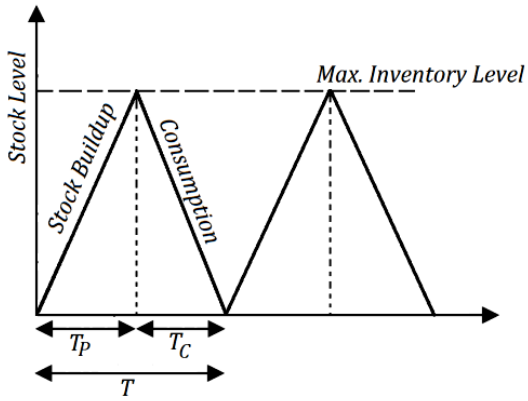
$$T^* = \frac{\text{No.of working days in a year}}{N^*}$$

Minimum total yearly inventory cost

$$T = \sqrt{2DC_o.C_h}$$

2.5.2 ECONOMIC ORDER QUANTITY WHEN STOCK REPLENISHMENT IS NON – INSTANTANEOUS(PRODUCTION MODEL)

This model is applicable when inventory continuously builds up over a period of time at a constant rate



Let
P = Production Rate
D = Demand or consumption rate
T_p=Period during replenishment of inventory takes place
T= Period during which Consumption of inventory takes place (During entire cycle)

Inventory built up rate = p-d
Maximum inventory at the end of production Run= (p-d) T_p

Average Inventory $= \left(\frac{p-d}{2}\right)T_p$

Quantity produced during production Period (Q) = P×T_p

Hence, $T_p = \frac{Q}{P}$

Average Inventory = $\frac{Q}{2}\left(1-\frac{d}{P}\right)$

Annual Inventory carrying cost = Avg. Inventory × Inventory carrying cost

$= \frac{Q}{2}\left(1-\frac{d}{P}\right).C_h$

Annual setup cost = No. of setups × setup cost /setup $= \frac{D}{Q} \times C_o$

Total Annual cost $(T_c) = \frac{DC_o}{Q} + \frac{Q}{2}\left(1-\frac{d}{P}\right)C_h$

To determine the EOQ, different the total cost equation with respect to Q, set the first derivate equal to zero.

$$\frac{DT_c}{dQ} + \frac{1}{2}\left(1-\frac{d}{P}\right)C_h - \frac{DC_o}{Q^2} = 0$$

$$Q^* = EOQ = \sqrt{\frac{2DC_o}{(1-\frac{d}{P})C_h}}$$

Optimal total cost

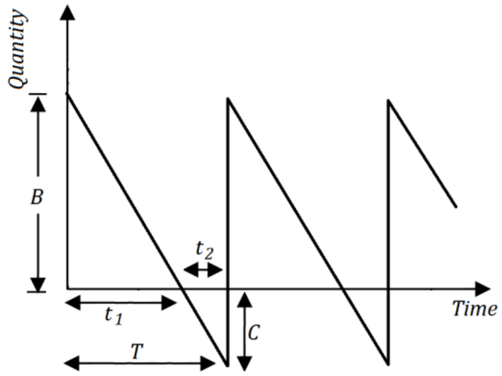
$$T = \sqrt{2DC_oC_h\left(1-\frac{d}{P}\right)}$$

Optimal number of production runs

$$N^* = \frac{\text{Annual Demand}}{EOQ} = \frac{D}{Q^*}$$

2.5.3 INVENTORY MODEL WITH SHORTAGE

There are sine occasions where stock out are economically justified. This situation is observed normally when cost per unit is very high.



Let

C_c = Carryingcost

C_h = shortage cost

C_o = orderingcost

1) $EOQ = \sqrt{\frac{2DC_o}{C_h}\left(\frac{C_s + C_h}{C_s}\right)}$

2) Total optimal inventory cost

$$T = \sqrt{2DC_oC_h\left(\frac{C_s}{C_h + C_s}\right)}$$

2.5.4 PRICE DISCOUNT MODEL

Let D be the annual consumption (Demand)

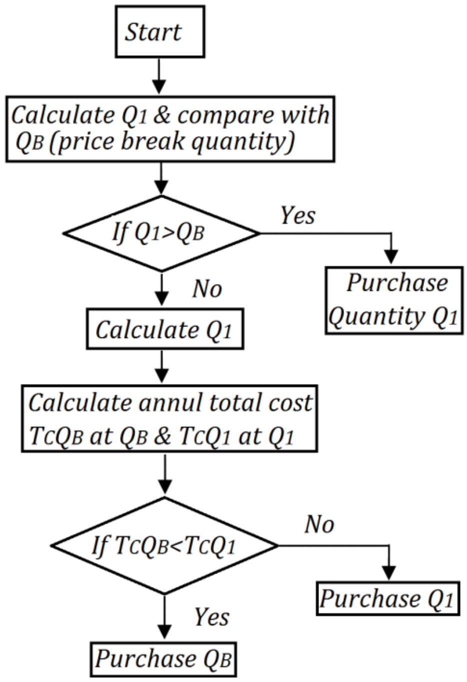
C_1 is the price per unit (Basic price)

C_2 is the discounted price per unit

C_0 is the ordering cost.

I is inventory cost expressed as a percentage of average inventory

Q_B be the price break quantity (Quantity at which the price changes)



Example: A stockiest has to supply 12,000 units of a product per year to his customer. The demand is fixed and known & the shortage cost is assumed is to be infinite. The inventory holding cost is Rs. 0.20 per unit per month and the ordering is Rs. 350. Determine

i) The optimum lot size q_o

ii) Optimum scheduling period t_o

iii) Minimum total variable yearly cost.

Solution: Supply rate,

$$R=\frac{12000}{12}=1,000 \text{ units/month}$$

C_1 = Rs. 0.20 per unit per month, C_3 = Rs. 350 per order

i) $q_o=\sqrt{\frac{2C_3R}{C_1}}=\sqrt{\frac{2\times350\times1000}{0.20}}$

=1870 units/ order

ii) $t_o=\sqrt{\frac{2C_3}{C_1R}}=\sqrt{\frac{2\times350}{0.20\times1000}}=1.87 \text{ months}$

= 8.1 week between order

iii) $C_o=\sqrt{2C_3C_1R}$

$=\sqrt{2\times0.20\times12\times350\times(1000\times12)}$

= Rs. 4490 per year

Example: The demand for a commodity is 100 units per day. Every time an order is placed, a fixed cost of Rs. 400 is incurred. Holding cost is Rs. 0.08 per unit per day. If the lead time is 13 days determine the economic lot size the reorder point.

Solution:

$$q_0=\sqrt{\frac{2C_3R}{C_1}}=\sqrt{\frac{2\times400\times100}{0.08}}=1000\text{units}$$

Length of cycle, $t_0=\frac{1000}{100}=10\text{days}$

As the lead time is 13 days and cycle length is 10 days, re-ordering should occur when the level of inventory is sufficient to satisfy the demand for $13-10=3days$

Recorder point = 100 × 3 = 300 units

It may be noted that the 'effective' lead time is taken equal to 3 days rather than 13 days is because the lead time is longer than t_0.

Example: Find the optimal order quantity for a product for which the price breaks are as follows:

Quantity	**Unit cost (Rs.)**
$0 < q < 500$	Rs. 10
$500 \le q < 750$	Rs. 9.25
$750 \le q$	Rs. 8.75

The monthly demands for the product is 200 units, storage cost is 2%of the unit cost and cost of ordering is Rs. 100

Solution:

EOQ for unit price of Rs. 9.25 $=\sqrt{\frac{2\times100\times200}{9.25\times0.02}}$

= 465 units (infesiable)

Total cost/ month for order quantity of 500 units (non-optimal size)

$=\sqrt{2C_1C_3R}+CR$

$=\sqrt{2\times(10\times0.02)\times100\times200}+10\times200$

$=\text{Rs}(89.45+2,000)=\text{Rs}.2089.45$

Total cost/ month for order quantity of 500 units (non-optimal size)

$=\frac{q}{2}C_1+C_3.\frac{R}{q}+CR$

$=\text{Rs}.\left(\frac{500}{2}\times9.25\times0.02+100\times\frac{200}{500}+9.25\times200\right)$

$=\text{Rs}.(46.25+40+1,850)=\text{Rs}.1,936.25$

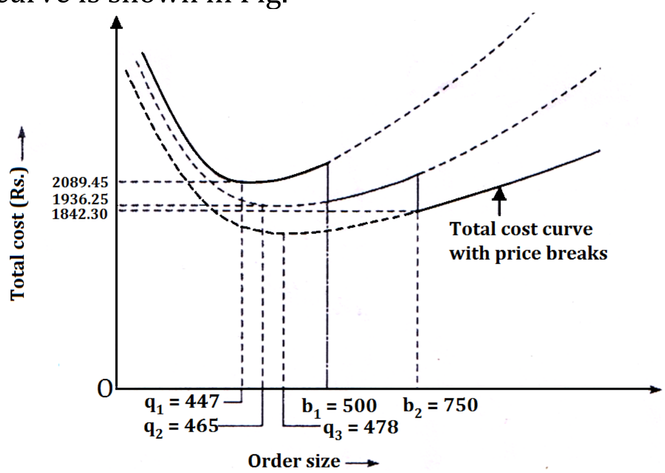
Total cost/month for order quantity of 750 units (non –optional size)

$=\frac{q}{2}C_1+C_3.\frac{R}{q}+CR$

$=\text{Rs}.\left(\frac{750}{2}\times8.75\times0.02+100\times\frac{200}{70}+8.75\times200\right)$

$=\text{Rs}.(65.63+26.67+1,750)=\text{Rs}.1,842.30$

∴ the optimal order quantity is 750 units. The total cost curve, which is a stepped curve is shown in Fig.



3 PROJECT MANAGEMENT(CPM & PERT)

3.1 INTRODUCTION

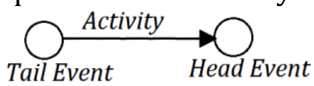
Project is a combination of interrelated activities that must be executed in a defined order for completing the entire task. Project management is the domain that deals with planning organizing, staffing, controlling and directing a project for its effective execution.
CPM (Critical Path Method) & PERT (Program Evaluation & Review Technique) are the techniques used for project management.

3.1.1 DIFFERENCE BETWEEN PERT & CPM

Sr. No.	PERT	CPM
1.	Time estimates are probabilistic	Time estimates are deterministic
2.	It is event oriented technique	It is activity oriented technique
3.	It is used for non-repetitive task	It is used for repetitive task
4.	It is used where the activities are of uncertain time	It is used where the activities are of well known time.

3.2 TERMS RELATED WITH PROJECT MANAGEMENT

1) **Activity:** An activity is a physically identifiable part of the project which consumes time and resources. Each activity has a definite start and end.
2) **Event:** An event represents the start or the completion of an activity.



3) **Activity duration:** It is the physical time required to complete an activity.
4) **Dummy Activity:** An activity that consumes no time but shows precedence among activities. It is useful for proper representation in the network.
5) **Earliest start time:** The earliest time that an activity can start, from the beginning of the project.
6) **Earliest finish time:** The earliest time that an activity can finish from the beginning of the project. EFT = EST + Activity duration
7) **Latest finish time:** It is the latest time that an activity can finish from the beginning of the project, without causing a delay in the completion of the project.
8) **Latest start time:** It is the latest time that an activity can start from the beginning of the project, without causing a delay in the completion of the project.
9) **Float:** Float is a difference between the time available to complete an activity and the actual time necessary to complete the same activity. Floats can further be classified into three types:

   i) **Total float:** It represents the time by which an activity can be delayed or may extended without affecting the total duration for the project. It is also expressed as a difference between latest start time and earliest start time.
   Hence
   Total float =LST-EST=LFT-EFT

   ii) **Free float:** It represents the time by which an activity can be extended or delayed which further does not

delay the start of any succeeding activity.
Free float can be calculated by
Free float = EFT – EST - Activity duration

iii) Independent float: It represents the extra time available with an activity without affecting any earliest and latest time.
Independent float = EFT-LST-Activity duration

10) Critical Path: The sequence or chain of critical activities for the project constitutes critical path. It is the longest duration path which consumes maximum resources.

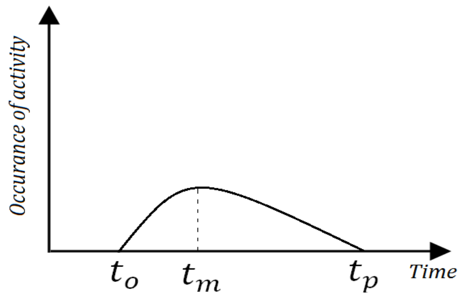
3.3 CRITICAL PATH METHOD

In critical path method the activity times are known with certainty. For each activity earliest start time and latest start times are computed. The path with the longest time sequence is called critical path. The length of the critical path determines the minimum time in which the entire project can be completed. The activities on the critical path are called critical activities.

3.4 PROGRAMME EVALUATION & REVIEW TECHNIQUE

Critical path method has only one time estimate so it does not consider uncertainty in time. In reality the duration of activities may not be deterministic (certain) in all cases. PERT takes into account the uncertainty of activity times. It is a probabilistic model with uncertainty in activity duration. PERT makes use of three estimates of time:
i) optimistic time (t_o)
ii) most likely time (t_m)
iii) pessimistic time (t_p)

Optimistic time (t_o) is the shortest possible time. If everything goes perfectly without any complications .Pessimistic time (t_p) is the longest time taking into consideration all odds. This is the time estimate if everything goes wrong.



Most likely time (t_m) is the best estimate of the activity time.This lies between the optimistic and pessimistic time estimatesthe three time estimates t_o, $t_p$and t_m are combined to develop expected time (t_e) is given by

$$t_e = \frac{t_o + 4t_M t t_p}{6}$$

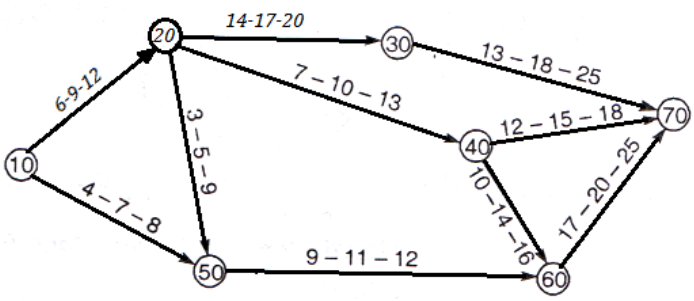
The standard deviation of the time required to complete each activity

$$\sigma = \frac{t_p - t_o}{6}$$

Variance for such activity

$$V = \left(\frac{t_p - t_o}{6}\right)^2 = \sigma^2$$

Example: Consider the network shown in fig. For each activity, the three time estimate $t_0 t_m$ and t_p are given along the arrows in the $t_0 - t_m - t_p$ order. Determine variance & expected time for each activity.



Solutions: We put the events in a tabular form and calculate the variance and expected times. These calculations can be carried on the network also.

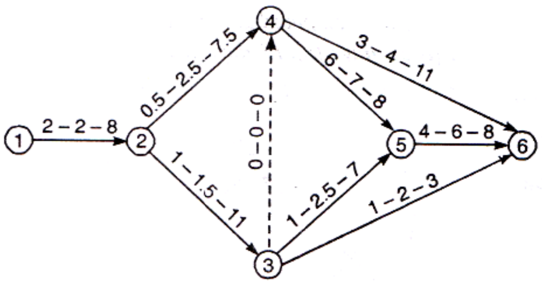
Activity-j		t_0	t_m
Predecessor Event i	**Successor Event j**		
10	20	6	9
10	50	4	7
20	30	14	17
20	40	7	10
20	50	3	5
30	70	13	18
40	60	10	14
40	70	12	15
50	60	9	11
60	70	17	20

t_p	$\sigma = \left(\frac{t_p - t_0}{6}\right)^2$	$t_e = \frac{t_0 + 4t_m + t_p}{6}$
12	1.00	9.00
8	0.44	6.7
20	1.00	17.0
13	1.00	10.0
9	1.00	5.33
25	1.00	18.33
16	4.00	13.67
18	1.00	15.00
12	0.25	10.83
25	1.78	20.33

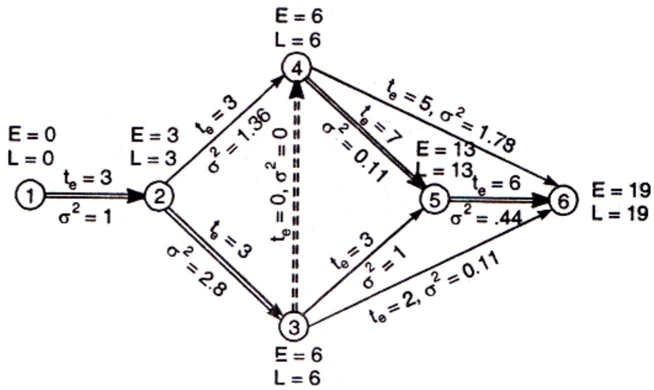
The entry in the tabular form starts with the initial event, by entering first number (10 in this case) in the first row under the column 'predecessor event I'. Then the activities emerging out from the initial event (here 10 – 20 and 10 – 50) are entered in the ascending order. Then we go to the next higher number (here 20) in the predecessor event column and enter all the activities emerging out from this event i.e. 20 – 30 20 – 40 and 20 – 50. This procedure is repeated until all the events are entered. The variance σ^2 and the expected activity time t_e are then computed by employing the relations:

$$\sigma^2 = \left(\frac{t_p - t_0}{6}\right)^2 \text{ And } \quad t_e = \frac{t_0 + 4t_m + t_p}{6}$$

Example: Consider the network shown in fig. The three time estimates for the activities are given along the arrows. Determine the critical path. What is the probability that the project will be completed in 20 days?



Solution: First step is to number to events. In this network the events are already numbered. The calculations can be performed on the network itself or in the tabular form. After calculating the expected times and the variances of the activities they are put along the arrows as shown in fig. By carrying the forward pass and backward pass computations E and L values are determined for all the events. By applying the conditions of critical activities it is determined that 1-2-3-4-5-6 and 1-2-4-5-6 are the two critical paths.



Expected duration of the project t_{cp}=19days
Contractual obligation time, T = 20 days
Standard division of the project

$$\sigma = \sqrt{\sum \sigma_{ij}^2} \text{ for all } i - j \text{ on the critical path}$$

$$\therefore \sigma \text{ for path } 1-2-4-5-6$$

$$= \sqrt{1 + 1.36 + 0.11 + 0.44} = 1.70$$

$$\sigma \text{ for path } 1-2-3-4-5-6$$

$$= \sqrt{1 + 1.28 + 0.11 + 0.44} = 2.08$$

∴ σ = 2.08 is chosen as it is higher of the two values.

$$\therefore \text{Normal deviate, } Z = \frac{T - T_{cp}}{\sigma}$$

$$= \frac{20 - 19}{2.08} = 0.48$$

From standard deviation table,
Probability =68.44%

4 PLANT LAYOUT

4.1 INTRODUCTION

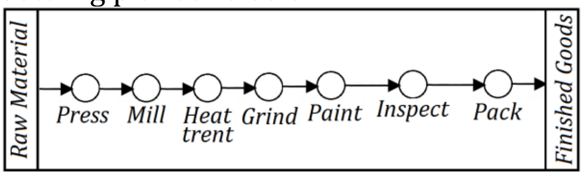
Plant – layout refers to the arrangement of various plant facilities such as equipment, machines, materials, manpower & services of the plant within the area of the plant site.

4.1.1 TYPE OF PLANT LAYOUT

On the basis of type of industry, volume of production and variety of production, we have the following four types of plant layout.

4.1.1.1 PRODUCT LAYOUT

This layout is also called flow – type layout or production line layout. In this layout the machines, equipments and work centers are arranged in a straight or curved line in the order in which they have to be used; that is according to the sequence of operations needed to manufacture a product.
Examples: Automobile assembly lines bottling plant and so on.



Advantages of product layout:

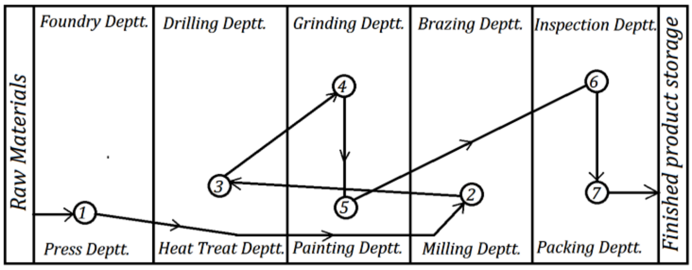
i) Reduced total production time
ii) Minimum of handling & transportation resulting in lower total material handling cost.
iii) Less floor area needed per unit of production
iv) Less work in process inventory

Disadvantages:

i) The layout is fixed, that is, if the product design changes, the whole line will have to be rearranged, that is lower flexibility
ii) The break – down of a single machine in the line leads to shutdown of the whole production line.
iii) High capital investment

4.1.1.2PROCESS LAYOUT

Process layout is also known as functional layout. Here machine performing similar operations are grouped together and are not arranged according to any particular sequence of operations. The work is brought to a machine from a machine on which the previous operation was carried out, this machine may be in another department or even a building. This results in lot of back – tracking or cross movement of the work.



Advantages of process layout:

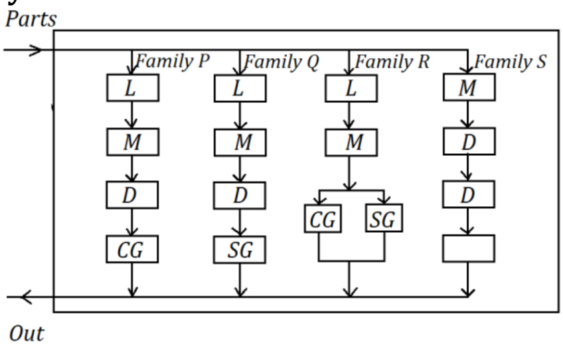
i) Greater flexibility of production. Change in product design can be easily accommodated.
ii) Lower initial investment in machinery because of less duplication of equipment.
iii) Break down of one machine will not shutdown the production as the work of that machine can be transferred to another machine or worker.

Disadvantages of process layout:

i) Generally, more floor space is required.
ii) More handling costs because of back tracking and cross movements of work resulting in chaotic material movement.
iii) Longer production cycles.
iv) High in-process inventory.

4.1.1.3GROUP TECHNOLOGY

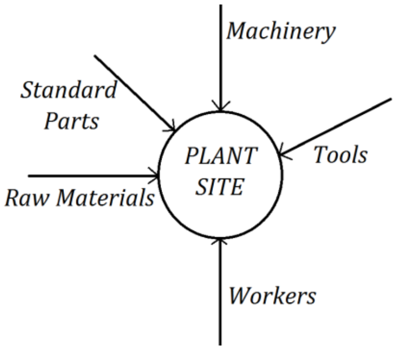
In group technology, similar parts or those requiring similar sequence of operations are grouped together in a family. All the equipments needed to manufacture a family of parts is grouped into a cell, that is why this layout is also called **cellular layout**.



[L: Lathe; M: Milling; D: Drilling CG: Cylindrical Grinding; SG: Surface Grinding]

4.1.1.4 FIXED POSITION LAYOUT

This type of layout is used for products which are very massive (in weight or size or both) it is very inconvenient or is not desirable to move them. Here the wok remains in a fixed position. The machines, materials, equipments and workers needed for production are brought to the site of work. E.g. shipbuilding industry, aircraft production etc.



4.2 PRODUCTION SYSTEM

The production system varies from factory to factory and from product to product however of the most important issues is production volume. The most common type of production system is:

1) **Job Shop Production:** The job shop production system is used when production volume is low. It has following characteristics:
   i) It is commonly used to meet a particular customer need.
   ii) Production lot size is generally small.
   iii) Product variety is generally very high.
   iv) Production equipments are mostly general purpose and flexible.

2) **Batch Production:** Batch production is suited for medium volume lot of same variety. At regular intervals, the production order is repeated. Batch production has the following characteristics:
   i) Commonly used to meet repeated customer orders.
   ii) Production lot size is medium batches.
   iii) Suitable for moderate product variety.

3) **Mass Production:** Mass production is suited for the manufacturing of continues identical parts. Production rate is generally very high. It is characterized by the followings:
   i) Suitable for high demand items.
   ii) Production lot size is very high and production rate is continuous.
   iii) Product variety is very low.
   iv) Special purpose tools & equipment may be needed.

5 LINE BALANCING

5.1 INTRODUCTION

Assembly line balancing is associated with a product layout in which products are processed as they pass through a line of work centers. An assembly line can be considered as a production sequence where parts are assembled together to form an end product.

5.2 SOME IMPORTANT DEFINITIONS

1) **Work station:** A work station is a location on assembly line where given amount of work is performed.

2) **Work elements:** The job is divided into its components so that work may be spread along the line. Work element is a part of the total job content in the line.

3) **Task time:** The standard time to perform element task.

4) **Station time:** Total standard work content of specific workstation.

5) **Cycle time:** It is the amount of time for which a unit that is assembled is available to any operator on the linear. It is the time the product spends at each work station

$$\text{Cycle Time} = \frac{\text{Availble time period}}{\text{out units}}$$

6) **Balance delay:** The percentage of total idle time on the line to total time spent by the product from beginning to end of the line.

7) **Line efficiency:** It is expressed as the ratio of total station time to the cycle time multiply by the number of station. Line efficiency indicates the percentage utilization of facilities.

5.3 IMPORTANT PARAMETERS IN LINE BALANCING

1) Line efficiency $\eta =$

$$\frac{\text{Total station time}}{\text{Cycle time}\times\text{No.of work stations}}\times 100$$

2) $\text{Balance delay} = (1 - \text{Line efficiency})\times 100$

Example: The table gives details of an assembly line

Work stations	**I**	**II**	**III**	**IV**	**V**	**VI**
Total task time at the station (in Min)	7	9	10	9	9	6

Calculate:
1) Balance delay of the assembly line.
2) Line efficiency of the assembly line.

Solution: Cycle Time = Maximum time in work station =10 min
Total work content = 7 + 9 + 7 + 10 + 9 + 6 = 48 min
Work stations = 6

$$\text{Balance Delay} = 1 - \frac{\text{Total work Content}}{\text{workstations}\times\text{cycle time}}$$

$$= 1 - \frac{48}{6\times 10} = 0.2 = 20\%$$

Line Efficiency = 1 – balance delay
= 1 – 0.2
= 0.8 = 80%

6 SEQUENCING MODEL

6.1 INTRODUCTION

When number of jobs are waiting in queue before an operational facility (such as milling) there is need to decide the sequence of processing all the waiting jobs sequencing is basically an order in which the jobs, waiting before an operational facility are processed. For this priority rule, processing time etc are needed.

6.2 GANTT CHART

The Gantt chart is a very useful graphical tool for representing a production schedule. A common production schedule involves a large number of production facilities, such as machine testing etc. Gantt chart contains time on its one axis. The status and scheduling of jobs on a time scale is schematically represented. This gives a clear pictorial representation of relationship among different production related activities of a firm on a time horizon.

6.2.1 PROCESSING OF N JOBS THROUGH ONE MACHINE

The processing of n jobs through one machine can be done by following rules:

1) First Come First Serve Rule (FCFS)
2) Shortest Processing Time Rule (SPT)
3) Earliest Due Date Rule (EDD)
4) Last Come First Service Rule (LCFS)
5) Slack Time Remaining Rule (STR)

These rules described with the help of example given below:

Example: There are five jobs in waiting for getting proceeds on a machine. There sequence of arrival, processing time and due–date are given in the table below. Schedule the jobs using FCFS, SPT, EDD, LCFS and STR rules. Compare the results.

Solution:

i) **First come first serve (FCFS) Rule:** In this, the job, which arrives first, is scheduled first. Then the next arrived job is scheduled, and so on

Job Arrival (in sequence) (i)	Processing Time (Days) (p_i)	Due date (Days from today) (d_i)
J1	4	6
J2	5	7
J3	3	8
J4	7	10
J5	2	3

Job sequence (i)	Processing Time(Days) (p_i)	Due date (Days from today) (d_i)	Flow Time (Days) $F_i=(F_{i-1}+p_1)$	Lateness of job $L_i=(F_i-d_i)$; $F_i>d_i$ Otherwise Zero
J5	2	3	0 + 2 = 2	0
J1	4	6	2 + 4 = 6	0
J2	5	7	6 + 5 = 11	11 – 7 = 4
J3	3	8	11+3= 15	14 – 8 = 6
J4	7	10	14+7= 21	21–10= 11

Flow Time (Days) $F_i = (F_{i-1} + p_i)$	Lateness of job $= (F_i - d_i)$; $F_i > d_i$ Otherwise Zero
0 + 4 = 4	0
4 + 5 = 9	2
9 + 3 = 12	4
12 + 7 = 19	9
19 + 2 = 21	18

Total flow time

=4 + 9 + 12 + 19 + 21 = 65 days

Mean flow time

$$= \frac{\text{time flow time}}{\text{Number of jobs}} = \frac{65}{5} = 13\text{days}$$

Total lateness of job

=0 + 2 + 4 + 9 + 18 = 33 days

Average lateness of job

$$= \frac{33}{5} = 66.6\text{days}$$

ii) **Shortest Processing Time (SPT) Rule or Shortest Operation Time (SOT)**

Rule: This rule gives highest priority to that job, which has shortest processing time. This approach gives following sequence of jobs for the given problem.

Total flow time=2+5+9+14+21=51 days

Mean flow time $=\frac{51}{5}=10.2\text{days}$

Total lateness of job =3+7+11=21 days

Average lateness of job $=\frac{21}{5}=4.2\text{days}$

iii) Earliest Due Date Rule: This rule gives highest priority to the job having earliest due date.

Total flow time =2+6+11+14+21=54 days

Mean flow time $=\frac{54}{5}=10.8\text{days}$

Total lateness of job =0+0+4+6+11=21 days

Average lateness of job $=\frac{21}{5}=4.2\text{days}$

i) Last Come First Serve (LCFS) Rule: This rule gives priority to that job, which has arrived most recently. Most recent job is the last arrived job. The scheduling of jobs on this rule is explained through the earlier example.

Total flow time =2+9+12+17+21=61 days

Mean flow time $=\frac{61}{5}=12.2\text{day}$

Total lateness of job =4+10+15=29 days

Average lateness of job $=\frac{29}{5}=5.8\text{days}$

ii) Slack Time Remaining (STR) Rule: STR is calculated as the difference between the times remaining before the due date minus processing time.

Total flow time =2+6+11+18+20=57 days

Mean flow time $=\frac{57}{5}=11.4\text{days}$

Total lateness of job =4+8+12=24 days

Average lateness of job $=\frac{24}{5}=4.8\text{days}$

Job sequence (i)	Processing Time (Days)	Due- date (Days from hence)	Flow Time (Days)	Lateness of job Otherwise Equal to Zero
J5	2	3	0 + 2 = 2	0
J1	4	6	2 + 4 = 6	0
J2	5	7	6 + 5 =11	11 - 7 = 4
J4	7	10	11 + 7 = 18	18 - 10 = 8
J3	4	8	18 + 2 = 20	20 - 8 = 12

Job sequence(i)	Processing Time (Days) (pi)	Due- date (Days from today) (di)	Flow Time (Days) $F_i=(F_{i}-1+p_i)$	Lateness of $L_i=(F_i-d_i)$; $F_i>d_i$ Otherwise Zero
J5	2	3	0 + 2 = 2	0
J3	3	8	2 + 3 = 5	0
J1	4	6	5 + 4 = 9	9 - 6 = 3
J2	5	7	9 + 5 = 14	14 - 7 = 7
J4	7	10	14 + 7 = 21	21 - 10 = 11

6.2.11 COMPARISON OF SEQUENCING RULES FOR THE GIVEN PROBLEM

Rule	Total Flow Time to Complete Jobs (Days)	Average Time to Complete Jobs (Days)	Average Lateness
FCFS	65	13	6.6
SPT	51	10.2	4.2
D-Date	54	10.8	4.2
LCFS	61	12.2	5.8
STR	57	11.4	4.8

6.2.2 PROCESSING N JOBS THROUGH TWO MACHINES

Johnson rule for solving n jobs through two machines consists of following steps:

Step1: Examine the columns of processing times on machine A and B and find the smaller value [Min $(A_i B_i)$]

Step2: If this value falls in column A, schedule this job first on machine A. If this value falls in column B, schedule this job on machine B (because of the given order AB). If there are equal minimal value (there is tie) one in each column, schedule the one in the first column first on machine A; and the one in the second column, last on machine A. If both equal values are in the first column (A), select the one with lowest entry in column A first. If the equal values are in the second column (B), select the one with lowest entry in column A first.

Step3: Cross out the Job assigned and continue the process (repeat step 1 and 2) placing the jobs next to first or last till all the jobs are scheduled. The resulting sequence will minimize T.

Example: A machine operator has to perform two operations, turning and threading on a number of different jobs. The time required to perform these operations (in minutes) for each job is known. Determine the order in which the jobs should be processed in order to minimize the total time required to turn out all the jobs.

Job	Time for turning (minutes)	Time for threading (minutes)
1	3	8
2	12	10
3	5	9
4	2	6
5	8	3
6	11	1

Also find the total processing time and idle times for turning and threading operations.

Solution: By examining the columns, we find the smallest value. It is threading time of 1 minute for job 6 in second column. This we schedule job 6 last turning (and thereafter for threading) as shown below.

					6

The smallest value is turning item of 2 minutes for job 4 in first column. Thus we schedule job 4 first as shown below:

4					6

There are two equal minimal values: turning time of 3 minutes for job 1 in first column and threading time of 3 minutes for job 5 in second column. According to the rules, job 1 is scheduled next to job to job 4 and 5 next to job 6 as shown below:

4	1			5	6

The smallest value is turning time of 5 minutes for job 3 in first column. Therefore, we scheduled job 3, next to job 1 and we get the optimal sequence as

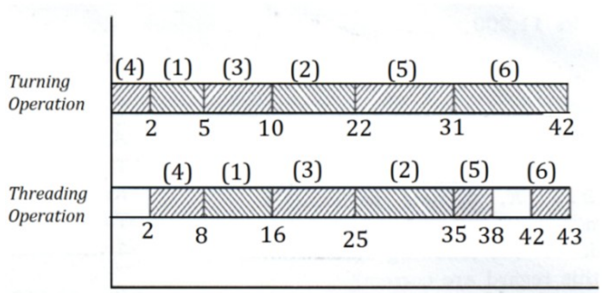
4	1	3	2	5	6

Now we can calculate the elapsed time corresponding to the optimal sequence, using the individual processing times given in the problem.

	Turning operation		Threading operation	
Job	Time in	Time Out	Time in	Time Out
4	0	2	2	8
1	2	5	8	16
3	5	10	16	25
2	10	22	25	35
5	22	31	35	38
6	31	42	42	43

Thus the minimum elapsed time is 43 minutes. Idle time for turning operation is 1 minute (from 42nd minute to 43rd minute) and for threading operation is 12+4=16 minutes (from 0-2 and 38-42 minutes)

Gantt Chart Representation



6.2.3 PROCESSING N JOBS THROUGH THREE MACHINES

For the n jobs three machine problem let A,B, C are the three machine and the method of solving two machine problem is applicable here also but the following two conditions are to be checked.

1) The minimum time on machine A is ≥ maximum time on machine B, and
2) The minimum time on machine C is ≥ Maximum time on machine B.

Now two fictitious machines are denoted by G and H are formed by following way.

$G_i = A_i + B_i$

$H_i = B_i + C_i$

Now this three machine problem is converted into two machine problem and Johnson rule can be applicable here. The problem can be well understood by referring the example given below:

Example: A machine operator has to perform three operations: turning, threading and knurling a number of different jobs. The time required to perform these operations (in minutes) for each job is known. Determine the order in which the jobs should be processed in order to minimize the total time required to turn out all the jobs. Also find the idle times for the three operations.

Job	**Time for turning (minutes)**	**Time for threading (minutes)**	**Time for knurling (minutes)**
1	3	8	13
2	12	6	14
3	5	4	9
4	2	6	12
5	9	3	8
6	11	1	13

Solution: Here min $A_i = 2$, Max $B_i = 8$, and min $C_i = \max B_i$

Hence

Job	**G_i= Turning + Threading (minutes)**	**H_i= Threading +Knurling (minutes)**
1	11	21
2	18	20
3	9	13
4	8	18
5	12	11
6	12	14

Examine the columns G_i and H_i we find that the smallest value is 8 under operation G_i in row 4. Thus we scheduled job 4 first (on machine G_i and thereafter onH_i) as shown below:

4					

The next smallest value is 9 under G_i for job 3. Hence we schedule job 3 as shown below:

4	3				

There are two equal minimal values : processing time of 11 minutes under column G_i for jobs 1 and processing time of 11 minutes under column H_i for job 1 is scheduled next to job 3 and 5 is scheduled last as shown below:

4	3	1			5

Now we may calculate the elapsed time corresponding to the optimal sequence, using the individual processing times given in the problem.

	Turning operation		Threading operation		Knurling operation	
Job	Time in	Time Out	Time in	Time Out	Time in	Time Out
4	0	2	2	8	8	20
3	2	7	8	12	20	29
1	7	10	12	20	29	42
6	10	21	21	22	42	55
2	21	33	33	39	55	69
5	33	42	42	45	69	77

Thus the minimum elapsed is 77minutes. Idle time for turning operation is 77 – 42 = 35 minutes, for threading operation is +1 + 11 + 3 + (77 – 45) = 17 + 32 = 49 minutes and for operation is 8 minutes.

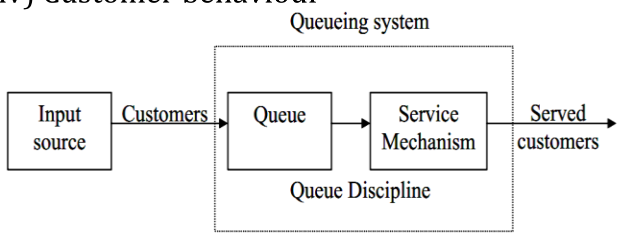
7 QUEUING THEORY

7.1 INTRODUCTION

Items which have been lined up for receiving service is called queue. This happens when the demand for a particular service exceeds the capacity of service facility.

7.1.1 CHARACTERISTICS OF A QUEUING MODEL

A queuing model may be looked for four basic characteristics
i) Arrival characteristics (input source)
ii) Queue or waiting line characteristics
iii) Service facility
iv) Customer behaviour



7.1.1.1 ARRIVAL CHARACTERISTICS / CALLING POPULATION

The calling population has three major features:

1) Size of input source: The size of input source may be considered either finite or infinite. When the arrival to a system at any given time is only a very small fraction, then it is consider as finite otherwise infinite.

2) Arrival pattern: Arrival at service counter may be scheduled or random.

3) Customer attitude: Which may be patient or impatient.

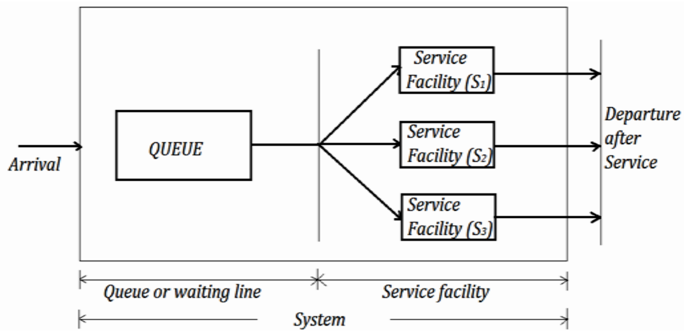
4) Number of waiting lines that are allowed:

7.1.1.2 QUEUE CHARACTERISTICS

Characteristic of a queue is its discipline. Queue discipline is the rule by which customers waiting in queue would receive service. These rules may be:
i) FIFO: First – In – First – Out
ii) LIFO: Last – In – First – Out
iii) SIRO: Service In Random Order etc

7.1.1.3 SERVICE CHARACTERISTICS

Service system may vary depending upon the number of service channels, number of servers etc.



1) Queue or waiting line can be defined as the number of customer waiting to be served excluding the customer being serviced.
2) System can be defined as the combination of queue & service facility.

7.1.1.4 CUSTOMER BEHAVIOR

1) Balking of Queue: Some customers decide not to join the queue due to their observation related to the long length of queue, insufficient waiting space or improper care while customers are in queue. This is balking behavior of customer.

2) Reneging of Queue: Reneging pertains to impatient customers after being in queue for some time, few customers become impatient and may leave the queue. This phenomenon is called as reneging of queue.

3) **Jockeying of Queue:** Jockeying is a phenomenon, in which customers move from one queue to another queue with a hope that they will receive quicker service in the new position.

7.2 KENDALL NOTATIONS

Kendall proposed a set of notations for queuing models. This is widely used in literature. The common pattern of notations of a queuing model is given by:

(a/b/c) : (d/e)

Where,

a: Probability distribution of the inter arrival time

b: Probability distribution of the service time

c: Number of servers in the queuing model

d: Maximum allowed customers in the system

e: Queue discipline.

7.3 QUEUE RELATED OPERATING CHARACTERISTICS

1) **Traffic density (ρ):**

$$\rho = \frac{\lambda}{\mu}$$

Where λ Arrival rate

μ =Service rate

2) **Expected number of units in the system (length of the system,L_s):**

$$L_s = \frac{\lambda}{\mu - \lambda}$$

3) **Expected number of units in queue:**

$$L_q = L_s - \frac{\lambda}{\mu} = \frac{\lambda}{\mu}\left(\frac{\lambda}{\mu - \lambda}\right)$$

4) **Expected waiting time in the system (W_s):**

$$W_s = \frac{L_s}{\lambda} = \frac{1}{\mu - \lambda}$$

5) **Expected waiting time in the queue (W_q):**

$$W_q = W_s - \frac{1}{\mu} = \frac{\lambda}{\mu}\left(\frac{1}{\mu - \lambda}\right)$$

6) **Probability of no unit in the system (i.e. system is idle):**

$$P_O = 1 - \rho$$

7) **Probability of system being occupied or busy:**

$$P = 1 - P_O = \rho$$

8) **Probability of 'n' unit in the system:**

$$P_n = P_O \rho^h$$

Example: A person repairing radios finds that the time spent on the radio sets has exponential distribution with mean 20 minutes. If the radios are repaired in the order in which they come in and their arrival is approximately Passion with an average rate of 15 for 8 hour day. What is there repairman's expected idle time each day? How many jobs are ahead of the average set just brought in?

Solution:

Arrival rate $\lambda = \frac{15}{8 \times 60} = \frac{1}{32}$ units / minute,

Service rate $\mu = \frac{1}{20}$ units / minute,

Number of jobs ahead of the set brought in = Average number of jobs in the system,

$$L_s = \frac{\lambda}{\mu - \lambda} = \frac{1/32}{1/20 - 1/32} = \frac{5}{3}$$

Number of hours for which the repairman remains busy in an 8- hour day

$$= 8\frac{\lambda}{\mu} = 8 \times \frac{1/32}{1/20} = 8 \times \frac{20}{32} = 5\text{hours}$$

$\therefore$ Time for which repairman remains idle in an 8-hour day

=8-5=3 hour

Example: A branch of Punjab National Bank has only one typist. Since the typing work varies in length (number of pages to be typed), the typing rate is randomly distributed approximating a Passion distribution with mean service rate of 8 letters per hour. The letters arrive at a rate of 5 per hour during the entire 8-hour work-day. If the typewriter is valued at Rs. 1.50 per hour determine.

1) Equipment utilization
2) The per cent time that an arriving letter has to wait
3) Average system time.
4) Average cost due to waiting on the part of typewriter i.e. it remaining idle.

Solution:

Arrival rate $\lambda = 5$ perhour

Service rate, $\mu = 8$ perhour.

1) Equipment utilization,

$$\rho = \frac{\lambda}{\mu} = \frac{5}{8} = 0.625$$

2) The per cent time that an arriving letter has to wait
=per cent time the typewriter remains busy
=62.5%

3) Average system time

$$W_s = \frac{1}{\mu - \lambda} = \frac{1}{8-5} = \frac{1}{3}\text{hr.} = 20\text{minuties}$$

4) Average cost due to waiting on the part of typewriter per day
=8×(1-5/8)×Rs.1.50=Rs.4.50

8 LINEAR PROGRAMMING PROBLEM

8.1 INTRODUCTION

Linear programming (LP) is an optimization problem which is used for the following:

i) It attempts to maximize or minimize a linear function of decision variables.

ii) The values of the decision variables are selected in such a way that they satisfy a set of constraints. Every constraint should be a linear function in the form of an equation or a linear inequality.

The function, which is maximized (or minimized), is termed as objective function. The restrictions, which are in the form of equations or inequalities, are termed as constraints.

8.2 SOME DEFINITIONS IN LINEAR PROGRAMMING

8.2.1 DECISION VARIABLES

In a LPP model the decision variable means the variable whose quantitative values are required be found, so as to minimize (or maximize) the objective function.

8.2.2 OBJECTIVE FUNCTION

The decision maker wants to maximize a function such as revenue/profit function, or minimize a function such as cost function under some restrictions. Function, which is maximized or minimized, is termed as objective function.

8.2.3 CONSTRAINT

The restrictions, which are expressed in the form of an equation or inequality (generally assigned with sign ≤or≥, are termed as constrains.

8.2.4 FEASIBLE SOLUTION

A set of values of decision variables, which satisfies the constraints set, contributes to the feasible solution. There may be many feasible solutions for a LP problem & all obviously are not the best solutions.

8.2.5 OPTIMUM SOLUTION

An optimum solution of an LPP is that set of feasible solution, which satisfies the maximization (or minimization) of the objective function. In case of maximization problem, the objective function needs to be maximized. However, in case of minimization problem, objective function is minimized.

8.3 SOLUTION OF LINEAR PROGRAMMING PROBLEM

The linear programming problem may be solved by two methods:

1) Graphical Method

2) Simplex Method.

The graphical method to solve LPP is useful when there are only two decision variables. This is because more than two coordinates are difficult to be represented on a graph paper. Simplex technique can handle any number of variables.

8.3.1 GRAPHICAL METHOD

Following steps are adopted to solve a two variable LPP through graphical method:

Step1: Formulate the problem in standard LPP form. It should have a linear objective function of maximization (or minimization) type. There may be many linear constraints but decision variables should not exceed two.

Step2: Treat each constraint as a line equation by assuming ≥ or ≤ signs as equal to sign. Plot them on a graph paper.

Step3: Based on the original sign (≥ or ≤) of the constraint, mark the feasible region in space.

Step4: Identify the corner points (or intersection of constraints, represented by lines) of the feasible region. Also include the two intersections on two axes by the feasible region. All these points constitute a set of possible solution as optimal solution always lies on the corner points.

Step5: For the objective function, draw straight line, called as is profit/ is o cost line. This may be done by equating the objective function to a very small profit figure or a high cost figure depending upon the nature of the objective function, i.e., maximization or minimization respectively.

Step6: Draw parallel lines to the is o profit line in maximization problem: moving away from the origin. Stop when there is only one point in the feasible region, which is also on the isoprofit line. For the minimization problem, draw parallel lines to the isocost line and move towards the origin stop when there is only one point in the feasible region which is also on the is cost line.

Step7: This point represents optimal solution. From the optimal solution point, draw perpendicular lines on both axes (X and Y axes). The point of intersection on the axis will give the values of two variables which give optimal solution.

8.3.1.1 CHARACTERISTICS OF CORNER POINTS:

The corner points are points where lines representing constraints or axes intersect with each other. In other words lines joining corner points enclose the feasible region. Therefore, step 5 and step 6 may also be undertaken as follows:

Step 5(a):
Calculate the value of objective function at each possible solution (i.e. corner point of feasible region). Find that corner point, which satisfies the objective of maximization or minimization as the case may be. This is the optimum solution point.

Step 6(a):
Draw a line with slope, same as that of objective function, and passing through the optimum solution point.

Go to step 7

8.3.2 SIMPLE PROCEDURE TO SOLVE LPP

1) Formulate the problem as the LPP problem in standard form.
2) Convert the inequalities into equalities by introducing slack or surplus and/or artificial variables as required by the problem.
3) Initial solution–obtain the initial or starting solution by setting n-m variables in the problem equal to zero and m represents the number of constraint equations n represents number of variables. The variables set equal to zero are called non–basic variables. Those (n-m) variables should be equal to zero which give unique solution to the problem.
4) The initial solution is represented in the tabular form & it is called initial simplex table
5) The next step is to improve the solution by the number of iterations till optimal solution is obtained. This step is explained as follows:
   - **i) Determine the entering variable:** The variables with the most negative value in the z equation of the initial table correspond to the entering variable. This is called entering variable as it enters the basis.
   - **ii) Leaving variable:** This is the variable which leaves the basis to give place for new entering variable. To identify the leaving variable – the ratio of entering column co- efficient is calculated for the rows except z (objective) row only +ve values are considered. The variable associated with smallest ratio is called the leaving variable.
   - **iii) The column corresponding to the entering variable:** It is called entering or pivot column. The row corresponding to the leaving variable is called pivot row. The element at intersection of pivot row and pivot column is called the pivot element or key element.
   - **iv) For computing the improvement**:

The pivot element should be 1 and there should be zeros in the pivot column in all other places. This is done using thee Gauss – Jordan elimination method.
The new values are calculated using following equations:

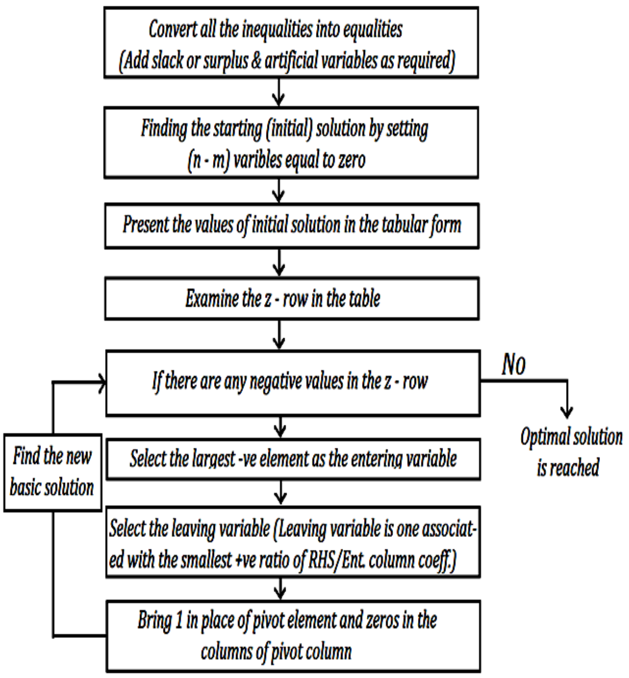
a) New pivot equation $= \dfrac{\text{Old pivot equation}}{\text{pivot element}}$

b) New Z Equation =
$$(\text{old Z equation}) - \begin{pmatrix}\text{its entering column}\\ \text{coefficient}\end{pmatrix} \times \text{New pivot eqation}$$

c) For all other equations,
New Equation=(old equation)–(its entering column coefficien× New pivot eqation

6) Test for optimality: If the values of the non – basic variable in Z row are all +ve, then optimal solution is reached. Get the value of Z and decision variables from the table otherwise. Repeat the step 5 to get improved solution till optimal solution is obtained .Simple procedure for maximization problem is shown in fig.

8.4 FLOWCHART OF SIMPLEX METHOD (MAXIMIZATION PROBLEM)



Example: A manufacture produces two types of products, 1 & 2 at production levels of x_1 and x_2 respectively. The profit given is $2x_1 + 5x_2$. The production constraints are:

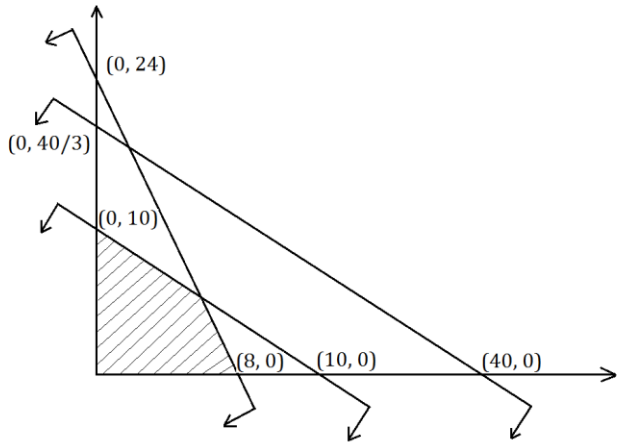
$x_1 + 3x_2 \le 40$
$3x_1 + x_2 \le 24$
$x_1 + x_2 \le 10$
$x_1 > 0$
$x_2 > 0$

Calculate the maximum profit which can meet the constraints

Solution: Objective function: Max(2) = 2 × 1 + 5 × 2

Constraints:
$x_1 + 3x_2 \le 40$
$3x_1 + x_2 \le 24$
$x_1 + x_2 \le 10$
$x_1 > 0, x_2 > 0$

Now, plotting the constrains on x_1x_2 axis



Solving the equation to get- corner points of feasible region

We get $x_1 = 7\ x_2 = 3$

Maximum Profit, Z = (2 × 7) + (5 × 3)
= 2

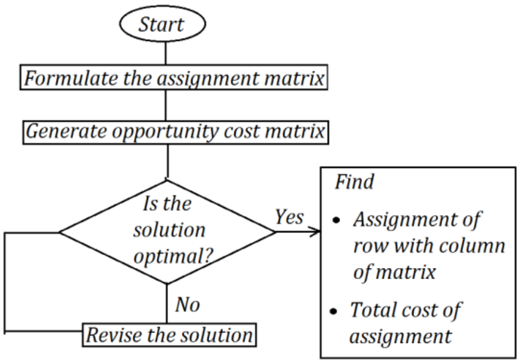
9 ASSIGNMENT MODEL

9.1 INTRODUCTION

Assignment problem pertain to problem of assigning n jobs to n different machines. This model can be effectively used for any other problem in which n items are to be assigned to other n items so that each one of the first group is assigned to one distinct item from the second group.

9.2 SOLUTION METHOD FOR ASSIGNMENT PROBLEM

The assignment problem is solved in the flowing manner (Figure below):



Example: Let us understand it with an example. Let there be four machine and four operators. Operator 1 charges 6, 7, 7 and 8 units on machine I, II III and IV respectively. Operator 2 charges 7, 8, 9 and 7 units, operator 3 charges 8, 6, 7 and 6 units, and operator 4 charges 8, 7, 6 and 9 units respectively. The problem is to assign one operator on one machine so that over – all payment is least.

Model: The assignment model in the form of operator – machine matrix is shown in figure. The entries in the matrix represent unit charge (in Rs.) per hour.

Machine

Operator	I	II	III	IV
1	6	7	7	8
2	7	8	9	7
3	8	6	7	6
4	8	7	6	9

Fig. 1 Representation of an Assignment Model

9.3 ALGORITHM TO SOLVE ASSIGNMENT MODEL

9.3.1 OPPORTUNITY COST APPROACH

Opportunity cost is the cost of possible opportunity which is lost or surrendered. The given problem is related to assigning operators on machine for it least cost objective.
Consider that if operator 2 is assigned on machine I, it will cost Rs. 7. With this no other operators can be assigned machine I as one –to one assignment is required. However, if operator 1 is assigned on machine I it will cost Rs. 6, therefore a potential saving of Rs. 7 – Rs. 6 = Rs. 1 is possible, if instead of operator 2, operator 1 is assigned on machine1. This is nothing but opportunity cost in case we assign operator 2 on machine 1. Similar logic may be put for opportunity cost of not assigning the least cost machine to an operator. So, to form a total opportunity cost matrix, we adopt a very simple two- step method.

Method to find the total opportunity cost matrix:

Step1:

- Select any column & subtract the lowest entry of this column form all the entries of this column and prepare a new column.
- Repeat for all column of the matrix. In this problem it will be the “operator opportunity” matrix.

Step2:

- Select any row of the revised matrix obtained in Step 1 & subtract the lowest entry of this row from all the entries of this row.
- Prepare a fresh row.
- Repeat this for rows of the revised matrix (operator – opportunity matrix).

This would be the total opportunity cost matrix.
For example, in the problem of operator- opportunity matrix as follows (Figure2)
The total opportunity matrix is as follows (Figure3)

		Machine			
		I	II	III	IV
Operator	1	0	1	1	2
	2	1	2	3	1
	3	2	0	1	0
	4	2	1	0	3

Fig. 2 Operator - Opportunity Matrix

		Machine			
		I	II	III	IV
Operator	1	0	1	1	2
	2	0	1	2	0
	3	2	0	1	0
	4	2	1	0	3

Fig. 3 Total Opportunity Cost Matrix

Optimality test of total opportunity cost matrix:

Step1:

- Draw minimum number of possible horizontal and/or vertical lines so that all the zeros of the total opportunity cost matrix are covered.
- If these lines are equal to the number of rows (or columns) then solution is optimal.
- Make assignment as per scheme outlined in Step 3
- If number of vertical and horizontal lines is less than number of rows, go to step 2, as the solution may be non-optimal

Step2:

- From the uncovered entries of Step 1 (i.e. entries which are not struck by lines just drawn) select the lowest entry.
- Subtract this entry from all entries of uncovered position.
- Add this entry at the junction points of line just drawn. By junction point we mean entries where both horizontal and vertical lines meet.
- Check for optimality as per Step 1. If optimal,go to Step 3; otherwise repeat Step 2.

Step 3:
Optimal Assignment of the Matrix:

- Select row (or Column), which has least number of zeros (say, one zero). Note that all rows (or columns) will have at least one zeros.
- Make assignment of this row with corresponding column .Strike- off the already assigned row and column.
- Now, select row and column which have minimum number of zeros. Make next assignment.
- Repeat the process till all rows assigned to one column.

9.3.2 ILLUSTRATION OF OPTIMALITY TEST AND ASSIGNMENT

Refer figure (3) apply step I for the check of optimality. Draw minimum number of possible horizontal/ vertical lines to cover zeros. We can do it in no less than four lines, hence preset assignment is optimal.

		Machine				
		I	II	III	IV	
Operator	1	0	1	1	2	
	2	0	1	2	0	
	3	2	0	1	0	2
	4	2	1	0	3	
		1		4	3	

Fig. 4 Four lines needed to cover all zeros

Therefore the assignment for figure (3) (which is optimal) may be done in this manner. Column II has only one zero. Therefore, assign machine II to operator 3. Remove column II and row 3.

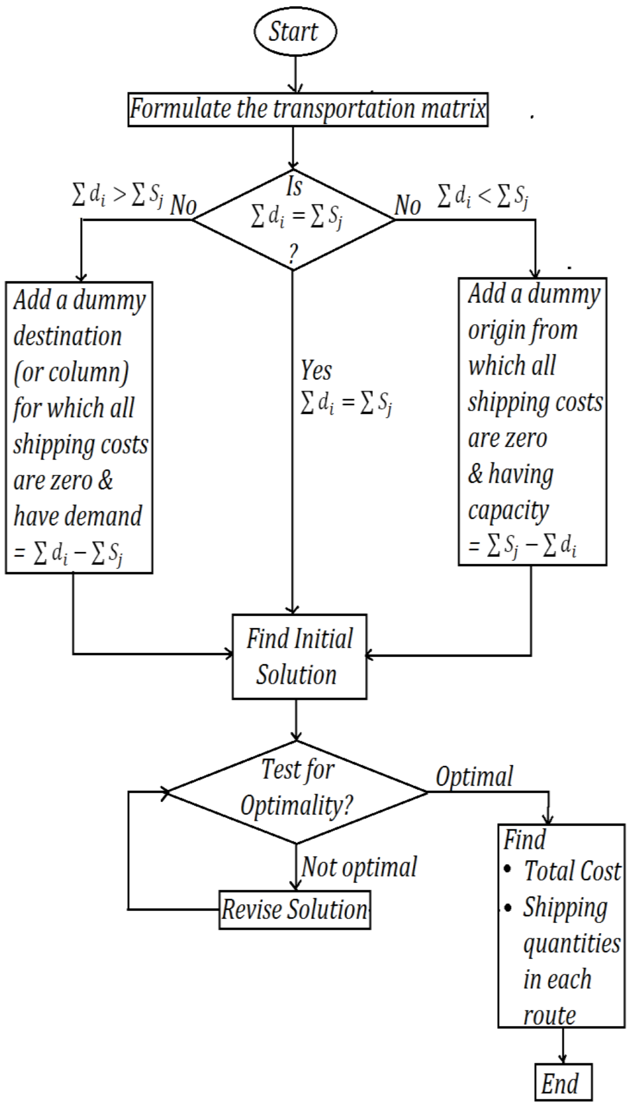
From the remainder matrix, it may be noticed that column III has only one zero. Therefore assign machine III to operator 4. Remove row 4 and column III.
In the remainder matrix, only row 1 and 2 and column I and IV remain. In this, column IV will have one zero at row 2. Therefore, assign machine IV to operator 2. The last assignment is the left over machine I to operator 1.
Thus, the final assignment is:

Operator	*Machine*	*Cost*
1	I	6
2	II	7
3	III	6
4	IV	6
	Total Cost	*Rs. 25*

10 TRANSPORTATION PROBLEM

10.1 INTRODUCTION

Transportation problem is a special variety of classical linear programming problem. In this model, shipments of goods from few locations or origins to others locations or destinations are planned. Due to specific characteristics of transportation problem special approaches are available to solve them.



10.2 HOW TO SOLVE THE TRANSPORTATION PROBLEM (TP)?

A typical TP is represented in standard matrix. From each cell in the main body contains a small cell at "top right" corner. In this cell, we write the unit shipping cost(C_{ij}). For three origins and four destinations TP, the standard matrix representation is as follows:

Origins	Destinations				Capacity of Origin
	D_1	D_2	D_3	D_4	
O_1	C_{11}	C_{12}	C_{13}	C_{14}	S_1
O_2	C_{21}	C_{22}	C_{23}	C_{24}	S_2
O_3	C_{31}	C_{32}	C_{33}	C_{34}	S_3
Demand of Destination	d_1	d_2	d_3	d_4	$\sum_{i=1}^{3} S_i = \sum_{j=1}^{4} d_j$

To solve the TP, an initial feasible solution is obtained that source and supply constrains are satisfied. For this some approaches are:

1) Least – cost method
2) North –West corner method
3) Vogel's approximation method.

The initial solution is further tested for optimality. The total cost is minimized through Modified Distribution (or MODI) method

GATE QUESTIONS

Topics

1. FORECASTING
2. INVENTORY CONTROL
3. LINEAR PROGRAMMING
4. PERT AND CPM
5. PROBABILITY AND STATISTICS
6. PRODUCTS AND PROCESS, PLANNING AND CONTROL
7. QUEUING THEORY & TRANSPORTATION

1 FORECASTING

Q.1 When using a simple moving average to forecast demand, one would

a) give equal weight to all demand data
b) assign more weight to the recent demand data
c) include new demand data in the average without discarding the earlier data
d) include new demand data in the average after discarding some of the earlier demand data

[GATE–2001]

Q.2 The sales of cycles in a shop in four consecutive months are given as 70, 68, 82, 95. Exponentially smoothing average method with a smoothing factor of 0.4 is used in forecasting. The expected number of sales in the next month is

a) 59 b) 72
c) 86 d) 136

[GATE–2003]

Q.3 For a product, the forecast and the actual sales for December 2002 were 25 and 20 respectively. If the exponential smoothing constant (a) is taken as 0.2, then forecast sales for January 2003 would be

a) 21 b) 23
c) 24 d) 27

[GATE–2004]

Q.4 The sales of a product during the last four years were 860, 880, 870 and 890 units. The forecast for the fourth year was 876 units. If the forecast for the fifth year, using simple exponential smoothing, is equal to the forecast using a three period moving average, the value of the exponential smoothing constant α is

a) $\frac{1}{7}$ b) $\frac{1}{5}$
c) $\frac{2}{7}$ d) $\frac{2}{5}$

[GATE–2005]

Q.5 In an MRP system, component demand is

a) forecasted
b) established by the master production schedule
c) calculated by the MRP system from the master production schedule
d) ignored

[GATE–2007]

Q.6 A moving average system is used for forecasting weekly demand $F_1(t)$ and F_2 (t) are sequences of forecasts with parameters m_1 and m_2, respectively, where m_1 and $m_2(m_1 > m_2)$ denote the numbers of weeks over which the moving averages are taken. The actual demand shows a step increase from d_1 to d_2 at a certain time. Subsequently,

a) neither $F_1(t)$ nor $F_2(t)$ will catch up with the value d_2
b) both sequences $F_1(t)$ and $F_2(t)$ will reach d_2 in the same period
c) $F_1(t)$ will attain the value d_2 before $F_2(t)$
d) $F_2(t)$ will attain the value d_2 before $F_1(t)$

[GATE–2008]

Q.7 Which of the following forecasting methods takes a fraction of forecast error into account for the next period forecast?

a) Simple average method
b) Moving average method
c) Weighted moving average method
d) Exponential smoothening method

[GATE–2009]

Q.8 The demand and forecast for February are 12000 and 10275, respectively. Using single exponential smoothening method (smoothening coefficient = 0.25), forecast for the month of March is

a) 431 b) 9587
c) 10706 d) 11000

[GATE–2010]

Q.9 In simple exponential smoothing forecasting, to give higher weight age to recent demand information, the smoothing constant must be close to

a) –1 b) zero
c) 0.5 d) 1.0

[GATE– 2013]

Q.10 In exponential smoothening method, which one of the following is true?

a) $0 \le \alpha \le 1$ and high value of α is used for stable demand
b) $0 \le \alpha \le 1$ and high value of α is used for unstable demand
c) $\alpha \ge 1$ and high value of α is used for stable demand
d) $\alpha \le 0$ and high value of α is used for unstable demand

[GATE-2014(1)]

Q.11 The actual sales of a product in different months of a particular year are given below:

September	October	November	December	January	February
180	280	250	190	240	?

The forecast of the sales, using the 4 month moving average method , for the month of the February is ______

[GATE-2014(3)]

Q.12 For a canteen, the actual demand for disposable cups was 500 units in January and 600 units in February. The forecast for the month of January was 400 units. The forecast for the month of March considering smoothing coefficients as 0.75 is ___

[GATE-2015(1)]

Q.13 Sales data of a product is given in the following table :

Month	January	February	March	April	May
Number of unit sold	10	11	16	19	25

Regarding forecast for the month of June, which one of the following statements is TRUE?

a) Moving average will forecast a higher value compared to regression
b) Higher the value of order N, the greater will be the forecast value by moving average.
c) Exponential smoothing will forecast a higher value compared to regression.
d) Regression will forecast a higher value compared to moving average

[GATE-2015(2)]

Q.14 The demand for a two-wheeler was 900 units and 1030 units in April 2015 and May 2015, respectively. The forecast for the month of April 2015 was 850 units. Considering a smoothing constant of 0.6, the forecast for the month of June 2015 is

a) 850 units b) 927 units
c) 965 units d) 970 units

[GATE-2016(3)]

Q.15 The time series forecasting method that gives equal weightage to each of the *m* most recent observations is

a) Moving average method
b) Exponential smoothing with linear trend
c) Triple Exponential smoothing
d) Kalman Filter

[GATE-2018(1)]

ANSWER KEY:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(d)	(b)	(c)	(c)	(c)	(d)	(d)	(c)	(d)	(b)	239	560.75	(d)	(d)	(a)

EXPLANATIONS

Q.1 (d)

The simple moving average method can be used if the underlying demand pattern is stationary. This method include new demand data in the average after discarding some of the earlier demand data.

Let m_t = moving average at time t

y_t= demand in time t and

n = moving average period

$$m_{t+1} = \frac{y_{t+1} - y_{t-n+1}}{n}$$

Q.2 (b)

We know, from the exponential and smoothing average method, the exponential smoothed average $u_{(t+1)}$ which is the forecast for the next period (t+1) is given by

$u_{(t+1)}=au_t+a(1-a)U_{t-1}+...a(1-a)^nU_{t\,N}+...\infty$

Now, for sales of the fifth month put t = 4 in the above equation,

So,

$u_s=au_4+a(1-a)u_3+a(1-a)^2u_2+a(1a)^2u_1$

Where, u_1, u_2, u_3 and u_4 are 70, 68, 82 and 95 respectively and a = 0.4

Hence

$$u_s = 0.4\times95+0.4(1-0.4)82+0.4(1-0.4)^2 \times68+0.4(1-0.4)3\times70$$

u_s = 38+19.68+9.792+6.048=73.52

Q.3 (c)

Given:

Forecast sales for December u_t = 25

Actual sales for December X_t = 20

Exponential smoothing constant a= 0.2

We know that, Forecast sales for January is given by

$$U_{t+1} = U_t + a[X_t - u_t]$$

= 25 + 0.2 (20 – 25)

= 25 + 0.2×(-5) = 25 – 1 =24

Hence, Forecast sales for January 2003 would be 24.

Q.4 (c)

Gives:

Sales of product during four years were 860, 880, 870 and 890 units.

Forecast for the fourth year u_4= 876

Forecast for the fifth year, using simple exponential smoothing, is equal to the forecast using a three period moving average.

So, $u_s = \frac{1}{3}(880+870+890) \Rightarrow u_s=880$unit

By the exponential smoothing method.

$$u_s = u_4 + a(u_4 - u_4)$$

880 = 876 + a(890 – 876)

4 = a(14)

$$a = \frac{4}{14} = \frac{2}{7}$$

Q.5 (c)

MRP (Material Requirement Planning):

MRP function is a computational technique with the help of which the master schedule for end products is converted into a detailed schedule for raw materials and components used in the end product.

Input to MRP

i) Master production schedule.

ii) The bill of material

iii) Inventory records relating to raw materials.

Q.6 (d)

Here $F_1(t)$ & $F_2(t)$ = Forecasting

m_1 & m_2 = Number of weeks

A higher value of m results in better smoothing. Since here $m_1>m_2$ the weight age of the latest demand would be more in $F_2(t)$.

Hence, $F_2(t)$ will attain the value of d_2 before $F_1(t)$

Q.7 (d)

Exponential smoothing method of forecasting takes a fraction of forecast error into account for the next period forecast.

The exponential smoothed average u_t , which is the forecast for the next period $(t + 1)$ is given by.

$$u_t = \alpha y_t + \alpha(1-\alpha) y_{t-1} + ... \alpha(1-\alpha)^n y_{t-n} + \infty$$

$$= ay_t + (1-a)\left[ay_{t-1} + a(1-a) y_{t-2} + ...\right.$$

$$\left.+a(1-a)^n y_{t-(n-1)} + ..\right]3$$

$$= u_{t-1} + a(y_t - u_{t-1})$$

$$= u_{t-1} + a\, e_t$$

Where $e_t = (y_t - u_{t-1})$ is called error and is the difference between the last Observation y_t and its forecast a period earlier, u_t-1.

The value of α lies between 0 and 1.

Q.8 (c)

Given, forecast for February

F_{t-1} = 10275

Demand for February

D_{t-1} = 12000

Smoothing coefficient

α = 0.25

Which is the forecast for the next period is given by,

$F_1 = \alpha(D_{t-1}) + (1-\alpha) \times F_{t-1}$

=0.25

$(12000) + (1-0.25) \times (10275)$

= 10706.25≃10706

Hence, forecast for the month of March is 10706.

Q.9 (d)

Height weight given to recent demand, the fore $F_t = D_t$

$F_t = F_{t-1} + \alpha(D_t - F_{t-1})$

$\text{or} = F_{t-1}(1-\alpha) + D_t$

Thus from the given condition

$F_{t-1}(1-\alpha) = 0$

Or = 1

The values of smoothing constant (α) lie between 0 and 1.

A low value of α gives more weightage to the past series and less weightage to the recent demand information. Hence, in simple exponential smoothing forecasting, higher value of α , i.e. 1, gives higher weight age to recent demand information and less weight age to the past series.

Q.10 (b)

0≤α≤2

High value of 'α' means more weightage f or immediate forecast.

Less value of 'α' means relatively less weightage for immediate forecast, or almost equal weightage for all previous forecast.

Hence high value of forecast is only chosen when nature of demand is not reliable rather unstable.

Q.11 239 to 241

Number of periods = 4, then the past 4 months average sales is fore cast for next 4 months.

So, $\dfrac{280 + 250 + 190 + 240}{4} = 240.$

Q.12 560.75

Forecast for

$= 400 + \alpha(500 - 400) = 400 + .25 \times 100 = 475$

Forecast march

$= 475 + \alpha(600 - 475) = 560.75$

Q.13 (d)

Q.14 (d)

Month	Demand	Forecast $\left(= \alpha D_{t-1} + (1-\alpha) F_{t-1}\right)$
April	900	850
May	1030	$= 0.6 \times 900 + 0.4 \times 850 = 880$
June		$= 0.6 \times 1030 + 0.4 \times 880 = 970$

$\therefore\ F_{june} = 970$ units

Q.15 (a)

Equal weightage is given to each of most recent observations in moving average method

2 INVENTORY CONTROL

Q.1 Market demand for springs is 8,00,000 per annum. A company purchases these springs in lots and sells them. The cost of making a purchase order is Rs.1200. The cost of storage of springs is Rs.120 per stored piece per annum. The economic order quantity is

a) 400 b) 2,828
c) 4,000 d) 8,000

[GATE–2003]

Q.2 A company has an annual demand of 1000 units, ordering cost of Rs.100/ order and carrying cost of Rs.100/ unit/year. If the stock – out cost are estimated to be nearly Rs. 400 each time the company runs out – of – stock, then safety stock justified by the carrying cost will be

a) 4 b) 20
c) 40 d) 100

[GATE–2004]

Q.3 There are two products P and Q with the following characteristics:

Product	Demand (Units)	Order cost (Rs/order)	Holding Cost(Rs./unit/ year)
P	100	50	4
Q	400	50	1

The economic order quantity (EOQ) of products P and Q will be in the ratio

a) 1 : 1 b) 1 : 2
c) 1 : 4 d) 1 : 8

[GATE–2004]

Q.4 A stockiest wishes to optimize the number of perishable items he needs to stock in any month in his store. The demand distribution for this perishable item is

Demand (in units)	2	3	4	5
Probability	0.10	0.35	0.35	0.20

The stockiest pays Rs. 70 for each item and he sells each at Rs. 90. If the stock is left unsold in any month, he can sell the item at Rs. 50 each. There is no penalty for unfulfilled demand. To maximize the expected profit, the optimal stock level is

a) 5 units b) 4 units
c) 3 units d) 2 units

[GATE–2006]

Q.5 Consider the following data for an item. Annual demand: 2500 units per year, Ordering cost: Rs.100 per order, Inventory holding rate: 25% of unit price quoted by a supplier

Order quantity (units)	Unit price (Rs.)
< 500	10
≥ 500	9

The optimum order quantity (in units) is

a) 447 b) 471
c) 500 d) ≥ 600

[GATE–2006]

Q.6 The maximum level of inventory of an item is 100 and it is achieved with infinite replenishment rate. The inventory becomes zero over one and half month due to consumption at a uniform rate. This cycle continues throughout the year. Ordering cost is Rs.100 per order and inventory carrying cost is Rs.10 per item per month. Annual cost (in Rs.) of the plan, neglecting material cost, is

a) 800 b) 2800
c) 4800 d) 6800

[GATE–2007]

Q.7 In a machine shop, pins of 15 mm diameter are produced at a rate of 1000 per month and the same is consumed at a rate of 500 per month. The production and consumption continue simultaneously till the maximum inventory is reached. Then inventory is allowed to reduce to zero due to consumption. The lot size of production is 1000. If backlog is not allowed, the maximum inventory level is

a) 400 b) 500
c) 600 d) 700

[GATE–2007]

Q.8 The net requirements of an item over 5 consecutive weeks are 50-0-15-20-20. The inventory carrying cost and ordering cost are Rs.1 per item per week and Rs.100 per order respectively. Starting inventory is zero. Use "Least Unit Cost Technique" for developing the plan. The cost of the plan (in Rs.) is

a) 200 b) 250
c) 225 d) 260

[GATE–2007]

Q.9 A company uses 2555 units of an item annually. Delivery lead time is 8 days. The reorder point (in number of units) to achieve optimum inventory is

a) 7 b) 8
c) 56 d) 60

[GATE–2009]

Q.10 Annual demand for window frames is 10000. Each frame cost Rs. 200 and ordering cost is Rs. 300 per order. Inventory holding cost is Rs. 40 per frame per year. The supplier is willing of offer 2% discount if the order quantity is 1000 or more, and 4% if order quantity is 2000 or more. If the total cost is to be minimized, the retailer should

a) order 200 frames every time
b) accept 2% discount
c) accept 4% discount
d) order Economic Order Quantity

[GATE–2010]

Q.11 A local tyre distributor expects to sell approximately 9600 steel belted radial tyres next year. Annual carrying cost is Rs. 16 per tyre and ordering cost is Rs. 75. The economic order quantity of the tyres is

a) 64 b) 212

c) 300 d) 1200

[GATE–2018(2)]

ANSWER KEY:

1	2	3	4	5	6	7	8	9	10	11
(c)	(c)	(c)	(a)	(c)	(d)	(b)	(b)	(c)	(c)	(c)

EXPLANATIONS

Q.1 **(c)**

Given:

D = 800000 per annum

C_v = 1200 Rs.

$C_h = 120$ per piece per annum

We know that,

Economic order quantity (EOQ) = N

$$= \sqrt{\frac{2c_0 D}{c_h}}$$

$$N = \sqrt{\frac{2\times1200\times800000}{1200}} = \sqrt{16\times10^6}$$

$$= 4\times10^3 = 4000$$

Q.2 **(c)**

Given: D = 1000 units,

C_0= 100 / order. C_h=10 unit/ year

C_S= 400 Rs.

We know that, optimum level of stock out will be,

$$\text{S.O.} = \sqrt{\frac{2DC_0}{C_h}} \times \sqrt{\frac{C_S}{C_h + C_S}}$$

$$\text{S.O.} = \sqrt{\frac{2\times1000\times100}{100}} \times \sqrt{\frac{400}{100+400}}$$

= 44.72 × 0.895 = 40

Q.3 **(c)**

For product P:D = 100 units, C_0= 50 Rs. / order. C_h =4 Rs. / unit / year

Economic order quantity (EOQ) for product P.

$$(EOQ)_P = \sqrt{\frac{2C_o D}{C_h}}$$

$$(EOQ)_P = \sqrt{\frac{2\times50\times100}{4}}$$

$$= \sqrt{2500}$$

= 50 .... (i)

For product Q :

D = 400 units,

C_o = 50 Rs. / order.

C_h =1 Rs. unit / year

EOQ for Product Q.

$$(EOQ)_Q = \sqrt{\frac{2C_o D}{C_h}}$$

$$(EOQ)_P = \sqrt{\frac{2\times50\times400}{4}} = \sqrt{40000}$$

= 200 .... (ii)

From equation (i) & (ii),

$$\frac{(EOQ)_P}{(EOQ)_Q} = \frac{50}{200} = \frac{1}{4}$$

$$(EOQ)_P : (EOQ)_Q = 1:4$$

Q.4 **(a)**

Profit per unit sold = 90 - 70 = 20 Rs

Loss per unit unsold item = 70-50 = 20 Rs

Now consider all the options:

Case	Units in stock	Unit sold (Demand)	Profit	Probability	Total Profit
Option (D)	2	2	2 × 20 = 40	0.1	4
Option (C)	3	2	2 × 20 – 1 × 20 = 20	0.1	2
	3	3	3 × 20 = 60	0.35	21
					23
Option (B)	4	2	2 × 20 – 2 × 20 = 0	0	0
	4	3	3 × 20 – 1 × 20 = 40	0.35	14
	4	4	4 × 20 = 80	0.35	28
					42
Option (A)	5	2	2 × 20 – 3 × 20 = 20	0.10	-2
	5	3	3 × 20 – 2 × 20 = 20	0.35	7
	5	4	4 × 20 – 1 × 20 = 60	0.35	21
	5	5	5 × 20 = 100	0.20	20
					46

Thus, For stock level of 5 units, profit is maximum.

Q.5 **(c)**

Given: D = 2500 units per year.

C_o = Rs. 100 per order

C_h = 25 % of unit price

Case (I): When order quantity is less than 500 units.

Then, Unit price = 10 Rs.

and $C_h = 25\%$ of 10 = 2.5 Rs.

$$EOQ = \sqrt{\frac{2C_oD}{C_h}} = \sqrt{\frac{2\times100\times2500}{4}}$$

Q = 447.21 ≃ 447 units

Total cost =

$$D\times\text{units cost} + \frac{Q}{2}\times C_b + \frac{D}{Q}\times C_o$$

$$= 2500\times10 + \frac{447}{2}\times2.5 + \frac{2500}{447}\times100$$

= 25000+ 558.75+559.75=26118 Rs.

Case (II) : when order Quantity is 500 units . Then unit prize = 9.Rs & C_h =25% of 9=2.25 Rs. Q = 500 units

Total Cost

$$= 2500\times9 + \frac{500}{2}\times2.25 + \frac{2500}{500}\times100$$

= 22500 + 562.5 + 500 = 23562.5 Rs.

So, we may conclude from both cases that the optimum order quantity must be equal to 500 units.

Q.6 (d)

Total annual cost = Annual holding t Annual ordering cost Maximum level of inventory N = 100

So , Average investor $= \frac{N}{2} = 50$

Investor carrying cost C_h = Rs. 10 per item per month

= Rs. 10×12 per item per year

= Rs. 120 per item per year

So Annual holding cost $= \frac{N}{2}\times C_b$

$C_{hA} = 50\times120$

Weeks	Quantity			Cost		
	Inventory	Used	Carried forward	Order	Carrying	Total
1	105(ordered)	50	0	100	0	100
2	0	0	0	0	0	0
3	55(ordered)	15	40	100	40	140
4	40	20	20	0	20	20
5	20	20	0	0	0	0

= Rs. 6000 Item per year

And. Ordering cost C_0=100 per order

Number of orders in a year $= \frac{12}{1.5}$

order= 8 order

So, Annual ordering cost C_{0A}

= ordering cost perorder × no. of orders

= 100×8 => Rs. 800 per order

Hence,Total Annual cost=6000+ 800

= Rs. 6800

Q.7 (b)

Given: Number of items produced per month

Weeks	Quantity			Cost		
	Inventory	Used	Carried forward	Order	Carrying	Total
1	65 (ordered)	50	15	100	15	115
2	15	0	15	0	15	15
3	15	15	0	0	0	0
4	40 (ordered)	20	20	100	20	120
5	20	20	0	0	0	0

K = 1000 per month Number of items required per month

R=500 per month Lot size q_0 = 1000

When backlog is not allowed, the maximum inventory level is given by,

$$I_m = \frac{K-R}{K}\times q_0 = \frac{1000-500}{1000}\times1000 = 500$$

Q.8 (b)

Given:

C_h = Rs. 1 per item per week

C_o = Rs. 100 per order

Requirements = 50 - 0 - 15 - 20 – 20

Total cost is the cost of carrying inventory and cost of placing order.

Weeks	Quantity			Cost		
	Inventory	Used	Carried forward	Order	Carrying	Total
1	85 (ordered)	50	35	100	35	135
2	35	0	35	0	35	35
3	35	15	20	0	20	20
4	20	20	0	0	0	0
5	20 (ordered)	20	0	100	0	100

Case (I) Only one order of 105 units is placed at starting.

Weeks	Quantity			Cost		
	Inventory	Used	Carried forward	Order	Carrying	Total
1	105 (ordered)	50	55	100	55	155
2	55	0	55	0	55	55
3	55	15	40	0	40	40
4	40	20	20	0	20	20
5	20	20	0	0	0	0

Total cost of plan = 155 + 55 + 40 + 20 = 270 Rs.

Case (II) Now order is placed two times, 50 units at starting and 55 units after 2nd week.

Total cost of plan=100+140+20=260Rs.

Case (III). The order is placed two times. 65 units at starting and 40 units after 3rd week.

Total cost of plan =115+15+120=250Rs.

Case (IV) Now again order is placed two times, 85 units at starting and 20 units after 4th week.

Total cost of plan =135+35+20+ 100 = 290 Rs.

So, The cost of plan is least in case (III) & it is 250 Rs.

Q.9 (c)

In figure,

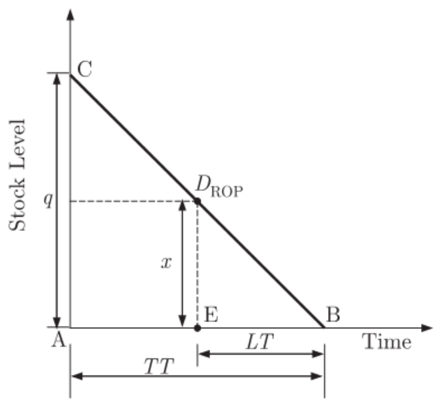
ROP = Reorder point

LT = Lead Time = 8 days

TT = Total Time : 365 days

q = stock level = 2555 units

Let the reorder quantity be x



Now from the similar triangles ΔABC & ΔBDE

$$\frac{q}{TT} = \frac{x}{LT}$$

$$\Rightarrow \frac{2555}{365} = \frac{x}{8}$$

$$x = \frac{2555}{365} \times 8 = 56 \text{ Units.}$$

Alternate Method

Given, Demand in a year D=2555 Units

Lead time T = 8 days

Now Number of orders to be placed in a year

$$N = \frac{\text{Number of days a year}}{\text{Lead Time}} = \frac{365}{8}$$

orders

Now quantity order each time or reorder point.

$$N = \frac{\text{Demand in a years}}{\text{Number of order}} = \frac{255}{\frac{365}{8}} = 56 \text{ units.}$$

Q.10 (c)

Given $D = 10000$

Ordering cost C_0= Rs. 300 per order

Holding C_h= Rs. 40 per frame per year.

Unit cost, C_u = Rs. 200

$$\text{EOQ} = \sqrt{\frac{2C_0D}{C_h}} = \sqrt{\frac{2\times300\times10000}{40}} ;387\text{units}$$

Total cost = Purchase cost + holding cost + ordering cost

For EOQ=387 units

$$\text{Total Cost} = D\times C_u + \frac{Q}{2}\times C_h + \frac{D}{Q}\times C_o$$

Where , Q = EOQ = 387 units

Total cost =

$$100000\times200 + \frac{387}{2}\times40\frac{10000}{387}\times300$$

=2000000+7740+7752=Rs.2015492

Now supplier offers 2% discount if the order quantity is 1000 or more.

For Q = 1000 units.

Total cost =

$$1000\times(200\times0.98) + \frac{1000}{2}\times40\frac{10000}{1000}\times300$$

=1960000 + 20000 + 3000

= Rs. 1983000

Supplier also offer 4% discount if order quantity is 2000 or more.

For Q = 2000 units

Total cost =

$$1000\times(200\times0.96)+\frac{2000}{2}\times40\frac{10000}{2000}\times300$$

= 1920000 + 40000 + 1500

= Rs. 1961500

It is clearly see that the total cost is to be minimized, the retailer should accept 4% discount.

Q.11 (c)

Annual Demand D = 9600

Carrying cost per unit per year C_c = Rs. 16

Ordering cost per order = Rs. 75

$$\therefore \quad EOQ = \sqrt{\frac{2DC_o}{C_C}} = \sqrt{\frac{2\times9600\times75}{16}} = 300$$

3 LINEAR PROGRAMMING

Q.1 A manufacturer produces two types of products, 1 and 2, at production levels of x1 and x2 respectively. The profit is given is 2x1+ 5x2. The production constraints are
$x_1 + 3x_2 \leq 40$
$3x_1 + x_2 \leq 24$
$x_1 + x_2 \leq 10$
$x_1 > 0, x_2 > 0$
The maximum profit which can meet the constraints is
a) 29 b) 38
c) 44 d) 75
[GATE–2003]

Q.2 A company produces two types of toys : P and Q. Production time of Q is twice that of P and the company has a maximum of 2000 time units per day. The supply of raw material is just sufficient to produce 1500 toys (of any type) per day. Toy type Q requires an electric switch which is available @ 600 pieces per day only. The company makes a profit of Rs. 3 and Rs. 5 on type P and Q respectively.
For maximization of profits, the daily production quantities of P and Q toys should respectively be
a) 1000, 500 b) 500, 1000
c) 800, 600 d) 1000, 1000
[GATE–2004]

Q.3 A company has two factories S1, S2, and two warehouses D1, D2. The supplies from S1 and S2 are 50 and 40 units respectively. Warehouse D1 requires a minimum of 20 units and a maximum of 40 units. Warehouse D2 requires a minimum of 20 units and, over and above, it can take as much as can be supplied. A balanced transportation problem is to be formulated for the above situation. The number of supply points, the number of demand points, and the total supply (or total demand) in the balanced transportation problem respectively are
a) 2, 4, 90 b) 2, 4, 110
c) 3, 4, 90 d) 3, 4, 110
[GATE–2005]

Common Data for Q.4 and Q.5

Consider a linear programming problem with two variables and two constraints. The objective function is: Maximize $X_1 + X_2$. The corner points of the feasible region are (0, 0), (0, 2), (2, 0) and (4/3, 4/3)

Q.4 If an additional constraint $X_1 + X_2 \leq 5$ is added, the optimal solution is
a) $\left(\frac{5}{3}, \frac{5}{3}\right)$ b) $\left(\frac{4}{3}, \frac{4}{3}\right)$
c) $\left(\frac{5}{2}, \frac{5}{2}\right)$ d) (5,0)
[GATE–2005]

Q.5 Let Y_1 and Y_2 be the decision variables of the dual and v_1 and v_2 be the slack variables of the dual of the given linear programming problem. The optimum dual variables are
a) Y_1 and Y_2 b) Y_1 and υ_1
c) Y_1 and υ_2 d) υ_1 and υ_2
[GATE–2005]

Q.6 A component can be produced by any of the four processes I, II, III and IV. Process I has a fixed cost of Rs. 20 and variable cost of Rs. 3 per piece. Process II has a fixed cost Rs. 50 and variable cost of Rs.1 per piece. Process III has a fixed cost of Rs. 40 and variable cost of Rs. 2 per piece. Process IV has a fixed cost of Rs.10 and variable cost of Rs. 4 per piece. If the company wishes to produce 100 pieces of the component, form economic point of view it should choose

Work station	I	II	III	IV	V	VI
Total task time at the workstation(in minutes)	7	9	7	10	9	6

a) Process I
b) Process II
c) Process III
d) Process IV

[GATE–2005]

Q.7 A firm is required to procure three items (P, Q, and R). The prices quoted for these items (in Rs.) by suppliers S1, S2 and S3 are given in table. The management policy requires that each item has to be supplied by only one supplier and one supplier supply only one item. The minimum total cost (in Rs.) of procurement to the firm is

Item	Suppliers		
	S1	S2	S3
P	110	120	130
Q	115	140	140
R	125	145	165

a) 350
b) 360
c) 385
d) 395

[GATE–2006]

Common Data For Q.8 and Q.9

Consider the Linear Program (LP)

Max $4x + 6y$ subject to $3x + 2y \leq 6$

$2x + 3y \leq 6$

$x, y \geq 0$

Q.8 After introducing slack variables s and t, the initial basic feasible solution is represented by the table below (basic variables are s = 6 and t = 6, and the objective function value is 0)

	-4	-6	0	0	0
s	3	2	1	0	6
t	2	3	0	1	6
	X	Y	S	t	RHS

After some simplex iteration, the following table is obtained

	0	0	0	2	12
s	5/3	0	1	-1/3	2
y	2/3	1	0	1/3	2
	X	y	s	t	RHS

From this, one can conclude that

a) the LP has a unique optimal solution
b) the LP has an optimal solution that is not unique
c) the LP is infeasible
d) the LP is unbounded

[GATE–2008]

Q.9 The dual for the LP in Q. 9 is

a) Min $6u + 6\upsilon$ subject to
$3u + 2\upsilon \geq 4$
$2u + 3\upsilon \geq 6$ u,
$\upsilon \geq 0$

b) Max $6u + 6\upsilon$ subject to
$3u + 2\upsilon \leq 4$
$2u + 3\upsilon \leq 6$
$u, \upsilon \geq 0$

c) Max $4u + 6\upsilon$ subject to
$3u + 2\upsilon \geq 6$
$2u + 3\upsilon \geq 6u,$
$\upsilon \geq 0$

d) Min $4u + 6\upsilon$ subject to
$3u + 2\upsilon \leq 6$
$2u + 3\upsilon \leq 6u,$
$\upsilon \geq 0$

[GATE–2008]

Q.10 For the standard transportation linear program with m source and n destinations and total supply equalling total demand, an optimal solution (lowest cost) with the smallest number of non-zero x_{ij} values (amounts from source i to destination j) is desired. The best upper bound for this number is

a) mn
b) 2(m + n)
c) m + n
d) m + n – 1

[GATE–2008]

Q.11 Consider the following Linear Programming Problem (LPP):

Maximize $Z = 3x1 + 2x2$

Subject to $x1 \leq 4$

$x2 \leq 6$

$3x1 + 2x2 \geq 18$

$x1 \geq 0$, $x2 \geq 0$

a) The LPP has a unique optimal solution.
b) The LPP is infeasible.
c) The LPP is unbounded.
d) The LPP has multiple optimal solutions.

[GATE–2009]

Q.12 Simplex method of solving linear programming problem uses
a) all the points in the feasible region
b) only the corner points of the feasible region
c) intermediate points within the infeasible region
d) only the interior points in the feasible region

[GATE–2010]

Common Data for Q.13 and Q.14

One unit of product P_1 requires 3 kg of resources R_1 and 1 kg of resources R_2. One unit of product P_2 requires 2 kg of resources R_1 and 2 kg of resources R_2. The profits per unit by selling product P_1 and P_2 are Rs. 2000 and Rs. 3000 respectively. The manufacturer has 90 kg of resources R_1 and 100 kg of resources R_2.

Q.13 The unit worth of resources R2, i.e., dual price of resources R2 in Rs. per kg is
a) 0 b) 1350
c) 1500 d) 2000

[GATE–2011]

Q.14 The manufacturer can make a maximum profit of Rs.
a) 60000 b) 135000
c) 150000 d) 200000

[GATE–2011]

Q.15 A linear programming problem is shown below.

Maximize $3x + 7y$

Subject to $3x + 7y \leq 0$

$4x + 6y \leq 8$

$x, y \geq 0$

It has
a) An unbounded objective function.
b) Exactly one optimal solution.
c) Exactly two optimal solutions.
d) Infinitely many optimal solutions.

[GATE– 2013]

Q.16 Two models, P and Q, of a product earn profits of Rs. 100 and Rs. 80 per piece, respectively. Production times for P and Q are 5 hours and 3 hours, respectively, while the total production time available is 150 hours. For a total batch size of 40, to maximize profit, the number of units of P to be produced is __________.

[GATE– 2017(1)]

Q.17 Maximize $Z = 5x_1 + 3x_2$

Subject to

$x_1 + 2x_1 \leq 10$

$x_1 - x_2 \leq 8$

$x_1, x_2 \geq 0$

In the starting Simplex tableau, x_1 and x_2 are non-basic variables and the value of Z is zero. The value of Z in the next Simplex tableau is ________.

[GATE– 2017(2)]

Q.18 The minimum value of $3x + 5y$ such that

$3x + 5y \leq 15$

$4x + 9y \leq 8$

$13x + 2y \leq 2$

$x \geq 0, y \geq 0$

is _______

[GATE– 2018(1)]

Q.19 The problem of maximizing $z = x_1 - x_2$ subject to constraints $x_1 + x_2 \leq 10$, $x_1 \geq 0$, $x_2 \geq 0$ and $x_2 \leq 5$ has

a) no solution
b) one solution
c) two solutions
d) more than two solutions

[GATE– 2018(2)]

ANSWER KEY:

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
(a)	(a)	(c)	(b)	(d)	(b)	(c)	(b)	(a)	(d)	(d)	(d)	(a)	(b)
15	16	17	18	19									
(b)	15	40	0	(b)									

EXPLANATIONS

Q.1 (a)

Given: Objective function,

$Z=2x_i+5x_2$

and $x_i+3x_2\leq40$

$3x_i+x_2\leq24$

$x_i+x_2\leq10$

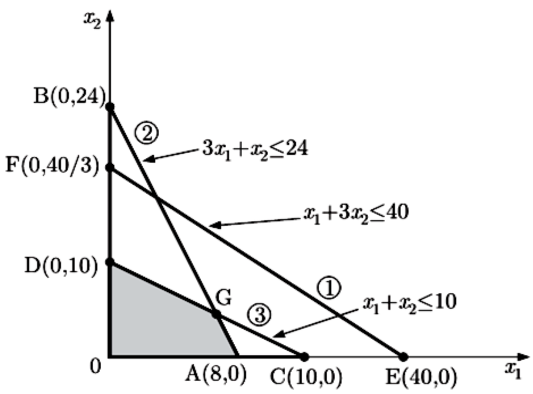
$x_1\leq0$

$x_2\leq0$

Path	Duration (day)
A-D-L	2 + 10 + 3 = 15
A-E-G-L	2 + 5 + 6 + 3 = 18
A-E-H	2 + 5 + 10 = 17
B-H	8 + 1 = 18
C-F-K-M	4 + 9 +3+ 8 = 24
C-F-H	4+ 9 +10 =23
A-E-K-M	2 + 5 + 3 +8 =18
B-G-L	8 + 6 + 3 = 17
B-K-M	8 + 3 + 8 = 19
C-F-G-L	4 + 9 + 6 +3 =22

First we have to make a graph from the given constraints. For draw the graph, substitute alternatively x_1 & x_2 equal to zero in each constraints to find the point on the x_1 & x_2 axis.

Now shaded area shows the common area. Note that the constraint $x_1+3x_2\leq40$ does not affect the solution space and it is the redundant constraint. Finding the coordinates of point G by the equations.



$3x_1+x_2=24$

$x_1+x_2=10$

Subtract these equations.

$(3x_1+x_2)+0=24-10$

$2x_1=14\Rightarrow x_1=7$

$x_2=10-x_1=10-7=3$

So, point G (7,3)

SO, maximum profit which can meet the constraints at G (7,3) is

$Z_{max} = 2 \times 7 + 5 \times 3 = 14 + 15 = 29$

Q.2 (a)

Solve this problem, by the linear programming model.

We have to make the constraints from the given conditions.

For production conditions

$P + 2Q \leq 2000$ ...(i)

For raw material

$P + 2Q \leq 2000$ ...(ii)

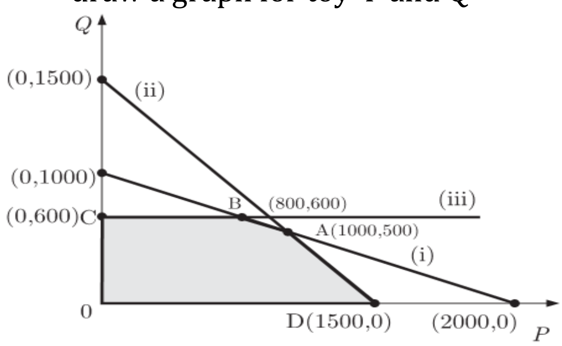
For electric switch

$Q + Q \leq 600$ ...(iii)

For maximization of profit, objective function

$Z = PQ + 5Q$ ...(iv)

From the equations (i), (ii) & (iii), draw a graph for toy P and Q



Line (i) and line (ii) intersects at point A, we have to calculate the intersection point.

$P + 2Q = 2000$

$P + Q = 1500$

After solving their equations, we get, A (1000, 500)

For point B.,

$P + 2Q = 2000$

$Q = 1500, P = 2000 - 1200 = 800,$

So, B. (800, 600)

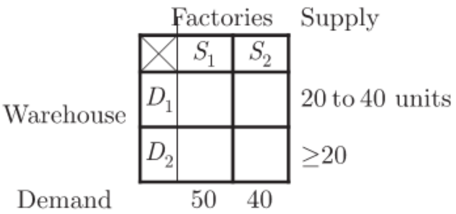
Here shaded are shows the area bounded by the three line equations (common area). This shaded area has five vertices.

	Vertices	Profit Z = 3 P + 5Q
(i)	0 (0,0)	Z = 0
(ii)	A (1000, 500)	Z = 3000 + 2500 = 5500
(iii)	B (800 , 600)	Z = 2400 + 3000 = 5400
(iv)	C (0,600)	Z = 3000
(v)	D (1500,0)	Z = 4500

So, for maximization of profit P = 1000 from point (ii)
Q = 500

Q.3 **(c)**

First we have to make a transportation model from the given details.



We know, Basic condition for transportation model is balanced, if it contains no more than m + n - 1 non-negative allocations, where m is the number of rows and n is the number of columns of the transportation problem.

So, Number of supply point (allocations) = m + n -1 =2+2−1=3

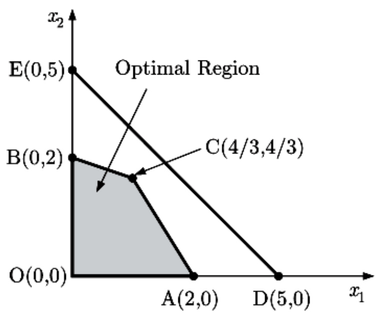
Number of demand point = 4 (No. of blank blocks)

Total supply or demand=50+40 = 90

Q.4 **(b)**

Given: Objective function $Z = X_1+X_2$

From the given corners we have to make a graph for X_1 and X_2



From the graph, the constraint $X_1+X_2 \leq 5$ has no effect on optimal region. Now, checking for optimal solution

	Point	$Z = X_1+X_2$
(i)	O (0,0)	Z =0
(ii)	A (2,0)	Z = 2 + 0 =2
(iii)	B (0,2)	Z = 0 + 2 =2
(iv)	C(4/3, 4/3)	Z = 4/3 + 4/3 = 8/3

The optimal solution occurs at point C (4/3, 4/3)

Q.5 **(d)**

We know, The inequality constraints are changed to equality constraints by adding or subtracting a non-negative variable from the left-hand sides of such constraints.

These variable is called slack variables or simply slacks.

They are added if the constraints are (≤) and subtracted if the constraints are (≥). These variables can remain positive throughout the process of solution and their values in the optimal solution given useful information about the problem. Hence, Optimum dual variables are V_1 and V_2 .

Q.6 **(b)**

For economic point of view, we should calculate the total cost for all the four processes.

Total cost = Fixed cost + Variable cost × Number of piece

For process (I):
Fixed cost = 20 Rs
Variable cost = 3.Rs per piece
Number of pieces = 100
Total cost = 20 + 3×100 = 320 Rs.
For process (II):
Total cost = 50 + 1×100 = 150 Rs.
For process (III):
Total cost = 40 + 2×100 = 240 Rs.
For process (IV):
Total cost = 10 + 4×100 = 410 Rs.

Now, we can see that total cost is minimum for process (II). So process (II) should choose for economic point of view.

Q.7 **(c)**

In figure

	S1	S2	S3
P	110	120	130
Q	115	140	140
R	125	145	165

Step (I) : Reduce the matrix :

In the effectiveness matrix, subtract the minimum element of each row from all the element of that row. The resulting matrix will have at least one zero element in each row.

	S1	S2	S3
P	0	10	20
Q	0	25	25
R	0	20	40

Step (II) : Mark the column that do not have zero element. Now subtract the minimum element of each such column for all the elements of that column.

	S1	S2	S3
P	0	0	0
Q	0	15	5
R	0	10	20

Step (III): Check whether an optimal assignment can be made in the reduced matrix or not.

For this, Examine rows successively until a row with exactly one unmarked zero is obtained. Making square ()4 around it, cross ()# all other zeros in the same column as they will not be considered for making any more assignment in that column.

Proceed in this way until all rows have been examined.

	S1	S2	S3
P	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$
Q	$\cancel{0}$	15	5
R	$\cancel{0}$	10	20

In this there is not one assignment in each row and in each column.

Step (IV):Find the minimum number of lines crossing all zeros. This consists of following sub step

(A) Right marked ( ) the rows that do not have assignment.

(B) Right marked ( ) the column that have zeros in marked column (not al- ready marked).

(C) Draw straight lines through all unmarked rows & marked columns.

	S1	S2	S3	
P	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$	
Q	$\cancel{0}$	15	5	✓
R	$\cancel{0}$	10	20	✓
	✓			

Step (V): Now take smallest element & add, where two lines intersect.

No change, where single line & subtract this where no lines in the block.

	S1	S2	S3	
P	5	$\boxed{0}$	$\cancel{0}$	
Q	$\cancel{0}$	10	$\boxed{0}$	✓
R	$\boxed{0}$	5	15	✓
	✓			

So, minimum cost is = 120 + 140 + 125 = 385

Q.8 **(b)**

The LP has an optimal solution that is not unique, because zero has appeared in the non-basic variable (x and y) column, in optimal solution.

Q.9 **(a)**

The general form of LP is Max Z = CX

Subject to ,$AX \leq B$

And dual f above LP is represented by

M in $Z = B^T Y$

Subject to , $A^T Y \geq C^T$

So, the dual is Min 6u+6v

Subject to $3u + 2v \geq 4$

$2u + 3v \geq 6$

$u + v \geq 0$

Q.10 **(d)**

In a transportation problem with m origins and n destinations, if a basic feasible solution has less than m+n - 1 allocations (occupied cells), the problem is said to be a degenerate transportation problem.

So, the basic condition for the solution to be optimal without degeneracy is.

Number of allocations = m + n – 1

Q.11 **(d)**

Given objective function

$Z_{max} = 3x_i + 2x_2$

and constraints are

$x_i \leq 4$ ...(i)

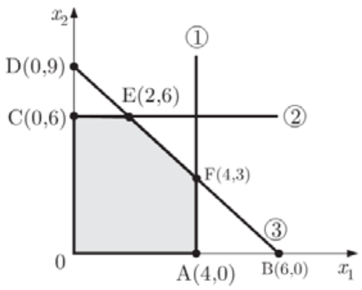
$x_2 \leq 6$ ...(ii)

$3x_1 + 2x_2 \leq 18$ ...(iii)

$x_1 \geq 0$

$x_2 \geq 0$

Plot the graph from the given constraints & find the common area.



Now, we find the point of intersection E & F.

For E. $3x_1 + 2x_1 = 18$

(E is the intersection point of equation (ii)& (iii))

$X_2 = 6$

So $3x_1 + 12 = 18 = 18$

$x_1 = 2$

For F $3x_1 + 2x_2 = 18$

$x_1 = 4$

So $3 \times 4 + 2x_1 = 18$

$x_2 = 3$

Hence, E (2,6) or F(4,3)

At point E (2.6)

= 18

Now at point F (4, 3)

= 18

The objective function and the constraint (represent by equation (iii)) are equal.

Hence, the objective function will have the multiple solutions as at point E & F, the value of objective function $(Z\text{-}3x_i + 2x_2)$ is same.

Q.12 **(d)**

Simplex method provides an algorithm which consists in moving from one point of the region of feasible solutions to another in such a manner that the value of the objective function at the succeeding point is less (or more, as the case may be) than at the preceding point. This procedure of jumping from one point to another is then repeated. Since the number of points is finite, the method leads to an optimal point in a finite number of steps.

Therefore simplex method only uses the interior points in the feasible region.

Q.13 **(a)**

Since, in Z_j Row of final (second) optimum table the value of slack variable S_2 shown the unit worth or dual price of Resource R_2 and the value of S_2 in given below table is zero. Hence the dual Price of Resource R_2 is zero.

Max Z = $2000 P_1 + 3000 P_2$

S.T. $3P_1 + 2P_2 \leq 90 \rightarrow$ R_1 - Resource

$P_1 + 2P_2 \leq 100 \rightarrow$ R_2 - Resource

$P_1, P_2 \geq 0$

Solution

$Z = 2000\, P_1 + 3000\, P_2 + 0.\, S_1 + 0.\, S_2$

S.T. $\quad 3P_1 + 2P_2 + S_1 = 90$

$P_1 + 2P_2 + S_2 = 100$

$P_1, P_2 \geq 0$

$P_1 \geq 0, P_2 \geq 0, S_1 \geq 0. S_2 \geq 0$

First table

		C_j	2000	3000	0	0
C_B	S_B	P_B	P_1	P_2	S_1	S_2
0	S_1	90	3	2 →	1	0
0	S_2	100	1	2	0	1
	z_j		0	0	0	0
	$Z_j - C_j$		-2000	-3000	0	0

Second Table

		C_j	2000	3000	0	0
C_B	S_B	P_B	P_1	P_2	S_1	S_2
3000	P_2	45	3/2	1	½	0
0	S_2	10	-2	0	-1	1
		z_j	4500	3000	1500	0→ Unit Worth Of R_2
		$Z_j - C_j$	2500	0	1500	0

Q.14 **(b)**

Since all $Z_j - C_j \geq 0$, an optimal basic feasible solution has been attained. Thus, The optimum solution to the given LPP is

Max Z = 2000 ×0 + 3000×45

= Rs.135000 with P_1=0 and P_2= 45

Q.15 **(b)**

We have

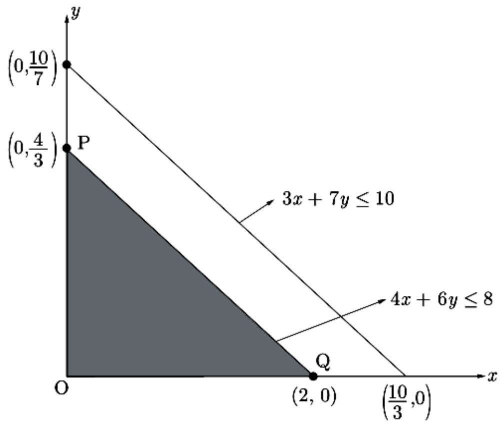
Maximize $\quad 3x + 7y$`

Subject to $\quad 3x + 7y \leq 10$

$4x + 6y \leq 8$

$x, y \geq 0$

From these equation



The solution of the given problem must lie in the shaded area. One of the points O,P. and Q of shaded area must given the optimum solution of problem So

At P $\left(0, \frac{4}{3}\right)$

$$Z = 30 + 7 \times \frac{4}{3} = \frac{28}{3} = 9.33$$

and at Q (2.0)

Z = 3 × 2 + 7× 0 = 6

Hence, there is only a single optimal solution of the problem which is at point P $\left(0, \frac{4}{3}\right)$

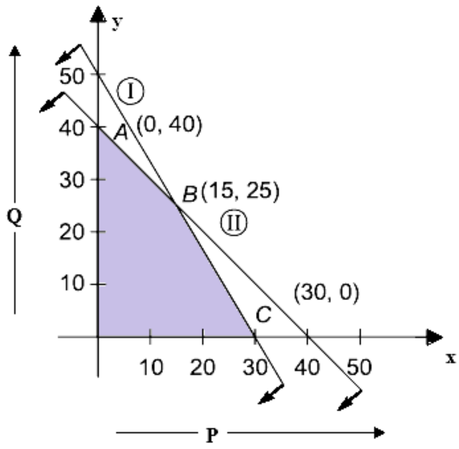
Q.16 Max z = 100P+80Q

$5P + 3Q \leq 150$

$P + Q \leq 40$

$$\frac{P}{30} + \frac{Q}{50} = 1$$

$$\frac{P}{40} + \frac{Q}{40} = 1$$



For A (0, 40)

Z_A=3200

For B(15, 25)

Z_B=3500

For c (30, 0)

Z_c=3000

Maximum of B,   P=15, Q= 25

Q.17   (40)

$\text{Max} z = 5x_1 + 3x_2 + 0 \times s_1 + 0 \times s_2$

Constraint

$x_1 + 2x_1 + s_1 = 10$

$x_1 - x_2 + s_2 = 8$

First iteration

e_i	Basis	x_1	x_2	S_1	S_2	b_i	Replacement ratio $(\theta) = \frac{b_i}{a_{ij}}$
0	S_1	1	2	1	0	10	10
0	S_2	(1)	-1	0	1	8	8 ←key row
C_j		5	3	0	0	-	-
$Z_j = \sum e_i a_{ij}$		0	0	0	0	-	-
$\Delta_j = C_j - Z_j$		5	3	0	0	-	-

↑

Key column

2nd iteration

e_i	Basis	x_1	x_2	S_1	S_2	b_i	Replacement ratio $(\theta) = \frac{b_i}{a_{ij}}$
0	S_1	0	3	1	-1	2	
5	x_1	1	-1	0	1	8	
C_j		5	3	0	0	-	-
Z_j		5	-5	0	5	-	-
Δ_j		0	8	0	-5	-	-

$x_1 = 8$ & $s_1 = 2$

Max z =5×8+0×2=40

Q.18   (0)

Minimize the objective function,

Z = 3x + 5y

Subject to constraints

$3x + 5y \le 15$

$4x + 9y \le 8$

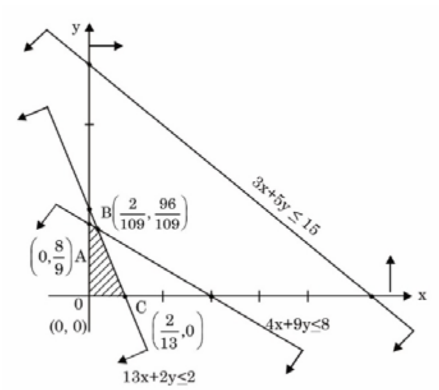
$13x + 2y \le 2$

$x \ge 0,\ y \ge 0$

$3x + 5y = 15\ [(x = 0,\ y = 3)\ \text{and}\ (y = 0,\ x = 5)]$

$4x + 9y = 8\left[\left(x = 0,\ y = \frac{8}{9}\right)\ \text{and}\ (y = 0,\ x = 2)\right]$

$13x + 2y = 2\left[(x = 0,\ y = 1)\ \text{and}\ \left(y = 0,\ x = \frac{2}{13}\right)\right]$



$\therefore$ The value of objective function at points

$Z_0 = 3 \times 0 + 5 \times 0 = 0$ (min)

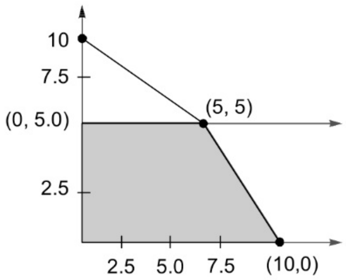
$Z_A = 3 \times 0 + \frac{8}{9} \times 5 = 4.44$

$Z_B = 3 \times \frac{2}{109} + 5 \times \frac{96}{109} = 0.46$

$Z_C = 3 \times \frac{2}{13} + 0 \times 5 = 4.45$

$\therefore Z_{min} = 0$

Q.19 (b)



Max., $Z = x_1 - x_2$

Constratins : $x_1 + x_2 \leq 10$;

$x_1 \geq 0, x_2 \geq 0$ *and* $x_2 \leq 5$

$Z(0, 5) = 0 - 5 = -5$

$Z(5, 5) = 5 - 5 = 0$

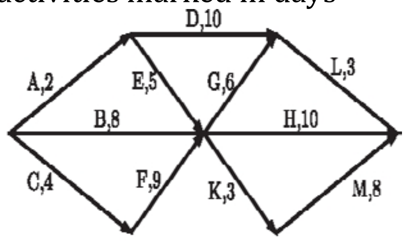
$Z(10, 0) = 10 - 0 = 10$

$Z_{max} = 10$ at $(10, 0)$

$\therefore$ The problem has one solution

4 PERT & CPM

Q.1 A project consists of activities A to M shown in the net in the following figure with the duration of the activities marked in days



The project can be completed
a) between 18, 19 days
b) between 20, 22 days
c) between 24, 26 days
d) between 60, 70 days
[GATE–2003]

Q.2 The symbol used for Transport in work study is
a) ⟹ b) T
c) ☐ d) ∇
[GATE–2004]

Q.3 In PERT analysis a critical activity has
a) maximum float b) zero float
c) maximum cost d) minimum cost
[GATE–2004]

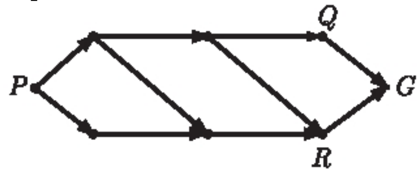
Q.4 A project has six activities (A to F) with respective activity duration 7, 5, 6, 6, 8, 4 days. The network has three paths A-B, C -D and E – F. All the activities can be crashed with the same crash cost per day. The number of activities that need to be crashed to reduce the project duration by 1 day is
a) 1 b) 2
c) 3 d) 6
[GATE–2005]

Q.5 An assembly activity is represented on an Operation Process Chart by the symbol
a) ☐ b) A
c) D d) O
[GATE–2005]

Q.6 The expected completion time of the project is
a) 238 days b) 224 days
c) 171 days d) 155 days
[GATE–2006]

Q.7 The standard deviation of the critical path of the project is
a) $\sqrt{151}$ days b) $\sqrt{155}$ days
c) $\sqrt{200}$ days d) $\sqrt{238}$ days
[GATE–2006]

Q.8 For the network below, the objective is to find the length of the shortest path from node P to node G. Let d_{ij} be the length of directed arc from node i to node j. Let S_j be the length of the shortest path from P to node j. Which of the following equations can be used to find S_G ?



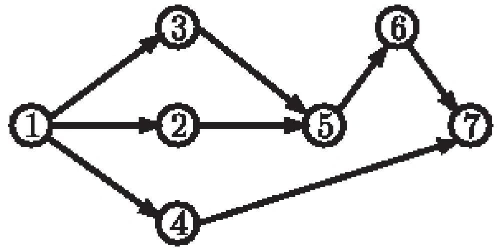
a) $S_G = \text{Min}\{S_Q, S_R\}$
b) $S_G = \text{Min}\{S_Q - d_{QG}, S_R - d_{RG}\}$
c) $S_G = \text{Min}\{S_G + d_{QG}, S_R + d_{RG}\}$
d) $S_G = \text{Min}\{d_{QG}, d_{RG}\}$
[GATE–2008]

Q.9 A set of 5 jobs is to be processed on a single machine. The processing time (in days) is given in the table below. The holding cost for each job is Rs. K per day.

Job	Processing time
P	5
Q	2
R	3
S	2
T	1

A schedule that minimizes the total inventory cost is
a) T -S -Q.R-P b) P-R-S -Q.T
c) T -R-S-Q.P d) P-Q.R-S –T
[GATE–2008]

Common Data For Q.10 and Q.11
Consider the following PERT network:
The optimistic time, most likely time and pessimistic time of all the activities are given in the table below:



Activity	Optimistic time (days)	Most likely time (days)	Pessimistic time (days)
1 - 2	1	2	3
1 - 3	5	6	7
1 - 4	3	5	7
2 - 5	5	7	9
3 - 5	2	4	6
5 - 6	4	5	6
4 - 7	4	6	8
6 - 7	2	3	4

Q.10 The critical path duration of the network (in days) is
a) 11 b) 14
c) 17 d) 18
[GATE–2008]

Q.11 The standard deviation of the critical path is
a) 0.33 b) 0.5
c) 0.77 d) 1.66
[GATE–2008]

Q.12 Six jobs arrived in a sequence as given below:

Jobs	Processing Time (days)
I	4
II	9
III	5
IV	10
V	6
VI	8

Average flow time (in days) for the above jobs using Shortest Processing time rule is
a) 20.83 b) 23.16
c) 125.00 d) 139.00
[GATE–2009]

Q.13 The expected time (te) of a PERT activity in terms of optimistic time t0, pessimistic time (tp) and most likely time (t*l*) is given by
a) $t_e = \frac{t_o + 4t_1 + t_p}{6}$ b) $t_e = \frac{t_o + 4t_p + t_1}{6}$
c) $t_e = \frac{t_o + 4t_1 + t_p}{3}$ d) $t_e = \frac{t_o + 4t_p + t_1}{6}$
[GATE–2009]

Common Data For Q.14 and Q.15

Four jobs are to be processed on a machine as per data listed in the table.

Job	Processing time (in days)	Due date
1	4	6
2	7	9
3	2	19
4	8	17

Q.14 If the Earliest Due Date (EDD) rule is used to sequence the jobs, the number of jobs delayed is
a) 1 b) 2
c) 3 d) 4
[GATE–2010]

Q.15 Using the Shortest Processing Time (SPT) rule, total tardiness is
a) 0 b) 2
c) 6 d) 8
[GATE–2010]

Q.16 The project activities, precedence relationships and durations are described in the table. The critical path of the project is

Activity	Precedence	Duration (in days)
P	-	3
Q	-	4
R	P	5
S	Q	5
T	R, S	7
U	R, S	5
V	T	2
W	U	10

a) P-R-T -V b) Q.S -T –V
c) P-R-U -W d) Q.S -U –W
[GATE–2010]

Common Data for Q.17and Q.18
For a particular project, eight activities are to be carried out. Their relationships with other activities and expected durations are mentioned in the table below.

Activity	Predecessors	Durations(days)
A	-	3
B	A	4
C	A	5
D	A	4
E	B	2
F	D	9
g	c, e	6
h	f, g	2

Q.17 The critical path for the project is
a) a - b - e - g – h b) a - c - g – h
c) a - d - f - h d) a- b - c - f – h
[GATE–2012]

Q.18 If the duration of activity f alone is changed from 9 to 10 days, then the
a) critical path remains the same & the total duration to complete the project changes to 19 days.
b) critical path and the total duration to complete the project remains the same.
c) critical path changes but the total duration to complete the project remains the same.
d) critical path changes and the total duration to complete the project changes to 17 days.
[GATE–2012]

Q.19. A project starts with activity A and ends with activity F. The precedence relation and durations of the activities are as per the following table:

Activity	Immediate predecessor	Duration (days)
A	–	4
B	A	3
C	A	7
D	B	14
E	C	4
F	D, E	9

The minimum project completion time (in days) is ______
[GATE–2017(2)]

Q.20 The arc lengths of a directed graph of a project are as shown in the figure. The shortest path length from node 1 to node 6 is _______.
[GATE–2018(2)]

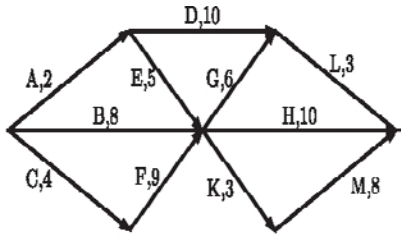
ANSWER KEY:

1	2	3	4	5	6	7	8	9	10	11	12	13	14
(c)	(a)	(b)	(c)	(d)	(d)	(a)	(c)	(a)	(d)	(c)	(a)	(a)	(c)
15	**16**	**17**	**18**	**19**	**20**								
(d)	(d)	(c)	(a)	30	7								

EXPLANATIONS

Q1 **(c)**

The various path and their duration are :-



Here maximum time along the path CFKM . So, it is a critical path and project can be completed in 4+9+3+8 =24 days.

Q.2 **(a)**

Q.3 **(b)**

PERT (Programme Evaluation and Review Technique) uses even oriented network in which successive events are joined by arrows. Float is the difference between the maximum time available to perform the activity and the activity duration. In PERT analysis a critical activity has zero float.

Q.4 **(c)**

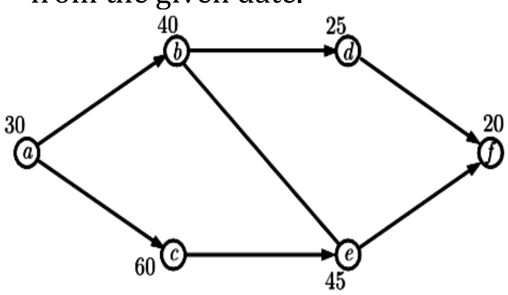
The 3 activity need to be crashed to reduce the project duration by 1 day.

Q.5 **(d)**

In operation process chart an assembly activity is represented by the symbol O

Q.6 **(d)**

We have to make a network diagram from the given date.



For simple projects, the critical path can be determined quite quickly by enumerating all paths and evaluating the time required to complete each. There are three paths between and af. The total time along each path is

(i) For path a -b -d - f

T_{abdf} =30+ 40+ 25+ 20 = 115 days

(ii) For path a -c -e – f

T_{acef} =30 + 60 + 45 + 20 = 155 days

(iii) For path a -b -e – f

T_{abef} = 30+ 40 + 45 + 20 = 135 days

Now, path a -c -e - f be the critical path time or maximum excepted completion time T = 155 days

Q.7 **(a)**

The critical path of the network is a -c -e - f .

Now, for variance.

Task	Variance(days^2)
a	25
c	81
e	36
f	9

Total variance for the critical path = 25 + 81 + 36 + 9 => 151 days^2

We know the standard deviation of critical path is

$$\sigma = \sqrt{V_{critical}} = \sqrt{151}\text{days}$$

Q.8 **(c)**

There are two paths to reach from node P to node G.

(i) Path P -Q -G (ii) Path P - R -G

For Path P -Q -G ,

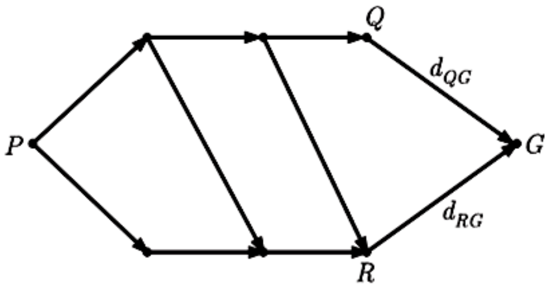
Length of the path $S_G = S_Q + d_{QG}$

For Path P -R -G ,

Length of the path $S_G = S_R + d_{RG}$

So, shortest path

$$S_G = \text{Min}\left\{S_Q + d_{QG}, S_R + d_{RG}\right\}$$

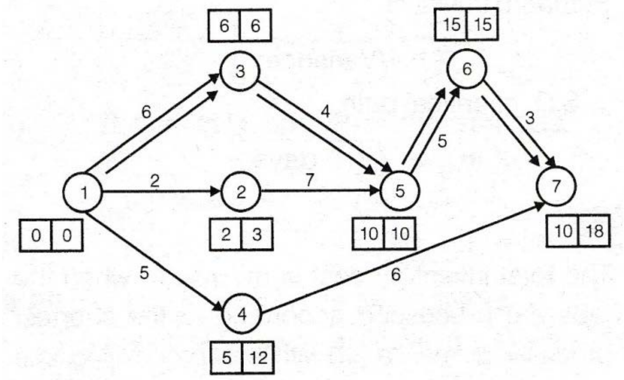


Q.9 (a)

Total inventory cost will be minimum, when the holding cost is minimum. Now, from the Johnson's algorithm, holding cost will be minimum, when we process the least time consuming job first. From this next job can be started as soon as possible. Now, arrange the jobs in the manner of least processing time.

T - S -Q - R - P or T -Q - S - R - P (because job Q and S have same processing time).

Q.10 (d)



Activities	Duration $t_e = \frac{t_0 + 4t_m + t_p}{6}$	EST	LST	EFT	LFT	LST−EST = Float	Variance σ^2 $\left(\frac{t_p - t_0}{6}\right)^2$
1−2	$\frac{1+4\times2+3}{6}=2$	0	1	2	3	1	$\left(\frac{3-1}{6}\right)^2=\frac{4}{36}$
1−3	$\frac{5+6\times4+7}{6}=6$	0	0	6	6	0	$\left(\frac{7-5}{6}\right)^2=\frac{4}{36}$
1−4	$\frac{3+4\times5+7}{6}=5$	0	7	5	12	7	$\left(\frac{7-3}{6}\right)^2=\frac{16}{36}$
2−5	$\frac{5+4\times7+9}{6}=7$	2	3	9	10	1	$\left(\frac{9-5}{6}\right)^2=\frac{16}{36}$
3−5	$\frac{2+4\times4+6}{6}=4$	6	6	10	10	0	$\left(\frac{6-2}{6}\right)^2=\frac{16}{36}$
5−6	$\frac{4+4\times5+6}{6}=5$	10	10	14	15	0	$\left(\frac{6-4}{6}\right)^2=\frac{4}{36}$
4−7	$\frac{4+4\times6+8}{6}=6$	5	11	11	18	6	$\left(\frac{8-4}{6}\right)^2=\frac{16}{36}$
6−7	$\frac{2+4\times3+4}{6}=3$	15	15	17	18	0	$\left(\frac{4-2}{6}\right)^2=\frac{4}{36}$

Critical path = 1→ 3→ 5→6→7 (Marked by double lines) Therefore, critical path duration of the network is = 6 + 4 + 5 + 3 = 18 days

Q.11 (c)

The critical path is 1 – 3 – 5 – 6 – 7

Variance along this critical path is,

$$\sigma^2 = \sigma_{1-3}^2 + \sigma_{3-5}^2 + \sigma_{5-6}^2 + \sigma_{6-7}^2$$

$$= \frac{1}{9} + \frac{4}{9} + \frac{1}{9} + \frac{1}{9} = \frac{7}{9}$$

We know,

Standard deviation $= \sqrt{\text{variance}\left(\sigma^2\right)}$

$$= = \sqrt{\frac{7}{9}} = 0.88$$

The most appropriate answer is 0.77

Q.12 (a)

In shortest processing time rule, we have to arrange the jobs in the increasing order of their processing time and find total flow time.

So, job sequencing are I - III - V - VI - II – IV

Jobs	Processing Time (days)	Flow time (days)
I	4	4
III	5	4 + 5 = 9
V	6	9 + 6 = 15
VI	8	15 + 8 =23
II	9	23 + 9 = 32
IV	10	32 + 10 = 42

Now Total Flow Time

t=4+9+15+23+32+42=125

$$\text{Average flow time} = \frac{\text{Total flow time}}{\text{Number of jobs}}$$

$$T_{\text{average}} = \frac{125}{6} = 2083\text{day}$$

Q.13 (a)

Under the conditions of uncertainty, the estimated time for each activity for PERT network is represented by a probability distribution. This probability distribution of activity time is based upon three different time estimates made for each activity. These are as follows.

t_0= the optimistic time, is the shortest possible time to complete the activity if all goes well.

t_p = the pessimistic time, is the longest time that an activity could take if everything goes wrong

t_l = the most likely time, is the estimate of normal time an activity would take.

The expected time (t_e) of the activity duration can be approximated as the arithmetic mean of $(t_0\text{-}t_p)/2$ and $2t_1$ Thus,

$$(t_e)=\frac{1}{3}\left[2t_l+\frac{(t_o+t_p)}{2}\right]=\frac{t_o+4t_l+t_p}{6}.$$

Q.14 (c)

In the Earliest due date (EDD) rule, the jobs will be in sequence according to their earliest due dates. Table shown below:

Job	Processing time (In days)	Due date	Operation Start	Operation end
1	4	6	0	0 + 4 =4
2	7	9	4	4 + 7 = 11
4	8	17	11	11 + 8 =19
6	2	19	19	19 + 2 = 21

We see easily from the table that, job 2, 4 & 3 are delayed.

Number of jobs delayed is 3.

Q.15 (d)

by using the shortest processing time (SPT) rule & make the table

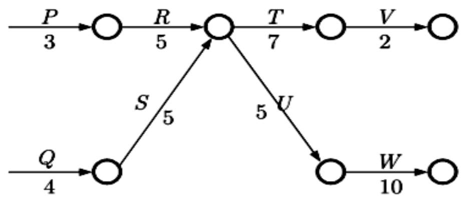
Job	Processing time (In days)	Flow time		Due date	Tardiness
		Start	End		
3	2	0	2	19	0
1	4	2	2 + 4 = 6	6	0
2	7	6	6 + 7 = 13	9	4
4	8	13	13 + 8=21	17	4

So, from the table

Total Tardiness = 4 + 4 =8

Q.16 (d)

We have to draw a arrow diagram from the given date.



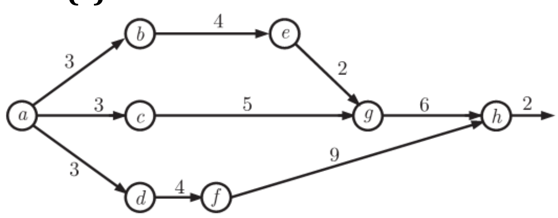
Here Four possible ways to complete the work.

	Path	Total duration (days)
(i)	P – R – T – V	T = 3 + 5 + 7 + 2 = 17
(ii)	Q – S – T – V	T = 4 + 5 + 7 + 2 = 18
(iii)	Q – S – U – W	T = 4 + 5 + 5 + 10 = 24
(iv)	P – R – U – W	T = 3 + 5 + 5 + 10 = 23

The critical path is the chain of activities with the longest time durations

So, Critical path = Q – S – U – W

Q.17 (c)



For path	Duration
a-b-e-g-h	= 3 + 4 + 2 + 6 + 2= 17 days
a-c-g-h	= 3 + 5 + 6 + 2 = 16 days
a-d-f-h	= 3 + 4 + 9 + 2 = 18 days

The critical path is one that takes longest path.

Hence, path a-d-f-h=18 days is critical path

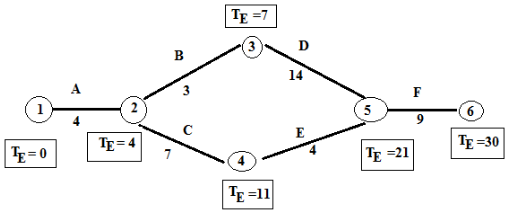
Q.18 (a)

From previous question

For critical path a-d-f-h = 18 days, the duration of activity f alone is changed from 9 to 10 days, then a-d-f-h=3+4+10+2=19days

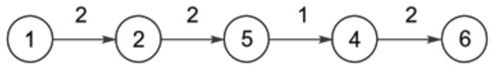
Hence critical path remains same and the total duration to complete the project changes to 19 days.

Q.19 : 30 day



Q.20 (7)

Shortest path is



Shortest length = 7

It is the problem of shortest path which will be 7.

In this question do not confuse with critical path. Examiner ask shortest path. Critical path is the longest path which is not asked in this question.

5 PROBABILITY & STATISTICS

Q.1 A residential school stipulates the study hours as 8.00 pm to 10.30 pm. Warden makes random checks on a certain student 11 occasions a day during the study hours over a period of 10 days and observes that he is studying on 71 occasions. Using 95% confidence interval, the estimated minimum hours of his study during that 10 day period is

a) 8.5 hours b) 13.9 hours
c) 16.1 hours d) 18.4 hours

[GATE–2003]

Q.2 The distribution of lead time demand for an item is as follows:

Lead time demand	**Probability**
80	0.20
100	0.25
120	0.30
140	0.25

The reorder level is 1.25 times the expected value of the lead time demand. The service level is

a) 25% b) 50%
c) 75% d) 100%

[GATE–2005]

Q.3 The jobs arrive at a facility, for service, in a random manner. The probability distribution of number of arrivals of jobs in a fixed time interval is

a) Normal b) Poisson
c) Erlang d) Beta

[GATE-2014]

Q.4 Demand during lead time with associated probabilities is shown below:

Demand	50	70	75	80	50
Probability	0.15	0.14	0.21	0.20	0.30

Expected demand during lead time is ___

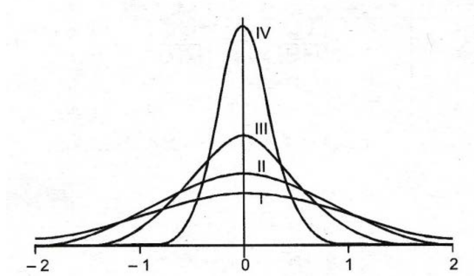
[GATE-2014]

Q.5 Jobs arrive at a facility at an average rate of 5 ir an 8 hour shift. The arrival of the jobs follows Poisson distribution. The average service time of a job on the facility is 40 minutes. The service time follows exponential distribution. Idle time (in hours) at the facility per shift will be

a) $\frac{5}{7}$ b) $\frac{14}{3}$
c) $\frac{7}{3}$ d) $\frac{10}{3}$

[GATE-2014]

Q.6 Among the four normal distributions with probability density functions as shown below which one has the lowest variance?



a) I b) II
c) III d) IV

[GATE-2015]

ANSWER KEY:

1	2	3	4	5	6
(c)	(d)	(b)	75.55	(b)	(d)

EXPLANATIONS

Q.1 (c)

Warden checks the student 11 occasions a day during the study hours over a period of 10 days.

So, Total number of observations in 10 days.

= 11×10 = 110 observations

Study hours as 8.00 pm to 10.30 pm.

So. total study hours in 10 days

= 2.5×10 = 25 hours.

Number of occasions when student studying = 71

So, Probability of studying

$$P = \frac{\text{No.of obsrnations when student studing}}{\text{Total observations}}$$

$$= \frac{71}{110} = 0.645$$

Hence,

Minimum hours of his study during 10 day period is

T = P × Total study hours in 10 days

= 0.645 × 25 = 16.1 hours

Q.2 (d)

The expected value of the lead time demand= 80×0.20+100×0.25+120×0.30+140 ×0.2 =12

Reorder level is 1.25 time the lead time demand.

So, record value = 1.25×112 = 140

Hence both the maximum demand or the reorder value are equal.

Hence , service level = 100%

Q.3 (b)

Q.4 (74.55)

Expected Demand = 50 × 0.15 + 70 × 0.14 + 75×0.21 + 80 × 0.20 + 85×0.3 = 74.55

Q.5 (b)

$$\lambda = \frac{5}{8}$$

$$\mu = \frac{60}{40}.\frac{3}{2}$$

Probability of idleness = $= 1 - \frac{\lambda}{\mu}$

$$= 1 - \frac{5}{8} \times \frac{2}{3} = \frac{7}{12}$$

Idle time (in hrs) per shift $= \frac{7}{12} \times 12$

$$= \frac{14}{3}$$

6 PRODUCTS AND PROCESS, PLANNING AND CONTROL

Q.1 Production flow analysis (PFA) is a method of identifying part families that uses data from
a) Engineering drawings
b) Production schedule
c) Bill of materials
d) Route sheets
[GATE–2001]

Q.2 Two machines of the same production rate are available for use. On machine 1, the fixed cost is Rs.100 and the variable cost is Rs. 2 per piece produced. The corresponding numbers for the machine 2 are Rs.200 and Re. 1 respectively. For certain strategic reasons both the machines are to be used concurrently. The sales price of the first 800 units is Rs. 3.50 per unit and subsequently it is only Rs. 3.00. The breakeven production rate for each machine is
a) 75 b) 100
c) 150 d) 600
[GATE–2003]

Q.3 A standard machine tool and an automatic machine tool are being compared for the production of a component. Following data refers to the two machines.

	Standard Machine Tool	Automatic Machine Tool
Setup time	30 min	2 hours
Machining time per piece	22 min	5 min
Machine rate	Rs. 200 per hour	Rs. 800 per hour

The break even production batch size above which the automatic machine tool will be economical to use, will be
a) 4 b) 5
c) 24 d) 225
[GATE–2004]

Q.4 A soldering operation was work – sampled over two days (16 hours) during which an employee soldered 108 joints. Actual working time was 90% of the total time and the performance rating was estimated to be 120 per cent. If the contract provides allowance of 20 percent of the time available, the standard time for the operation would be
a) 8 min b) 8.9 min
c) 10 min d) 12 min
[GATE–2004]

Q.5 An electronic equipment manufacturer has decided to add a component subassembly operation that can produce 80 units during a regular 8–hours shift. This operation consist of three activities as below

Activity	Standard time (min)
M. Mechanical assembly	12
E. Electric wiring	16
T. Test	3

For line balancing the number of work stations required for the activities M, E & T would respectively be
a) 2, 3, 1 b) 3, 2, 1
c) 2, 4, 2 d) 2, 1, 3
[GATE–2004]

Q.6 A welding operation is time-studied during which an operator was pace-rated as 120%. The operator took, on an average, 8 minutes for producing the weld–joint. If a total of 10% allowances are allowed for this operation. The expected standard production rate of the weld –joint (in units per 8 hour day) is
a) 45 b) 50
c) 55 d) 60
[GATE–2005]

Q.7 The table gives details of an assembly line. What is the line efficiency of the assembly line?

a) 70% b) 75%
c) 80% d) 85%

[GATE–2006]

Q.8 An manufacturing shop processes sheet metal jobs, wherein each job must pass through two machines (M1 and M2, in that order). The processing time (in hours) for these jobs is

Machine	Jobs					
	P	Q	R	S	T	U
M1	15	32	8	27	11	16
M2	6	19	13	20	14	7

The optimal make – span (in hours) of the shop is

a) 120 b) 115
c) 109 d) 79

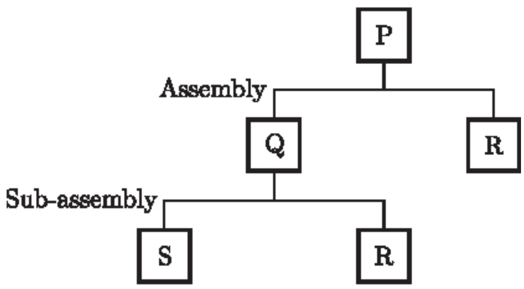
[GATE–2006]

Q.9 Capacities of production of an item over 3 consecutive months in regular time are 100, 100 and 80 and in overtime are 20, 20 and 40. The demands over those 3 months are 90, 130 and 110. The cost of production in regular time and overtime are respectively Rs. 20 per item and Rs. 24 per item. Inventory carrying cost is Rs. 2 per item per month. The levels of starting and final inventory are nil. Backorder is not permitted. For minimum cost of plan, the level of planned production in overtime in the third month is

a) 40 b) 30
c) 20 d) 0

[GATE–2007]

Q.10 The product structure of an assembly P is shown in the figure.



Estimated demand for end product P is as follows:

Week	1	2	3	4	5	6
Demand	1000	100	1000	1000	1200	1200

Ignore lead times for assembly and sub-assembly. Production capacity (per week) for component R is the bottleneck operation. Starting with zero inventory, the smallest capacity that will ensure a feasible production plan up to week 6 is

a) 1000 b) 1200
c) 2200 d) 2400

[GATE–2008]

Q.11 Vehicle manufacturing assembly line is an example of

a) product layout b) process layout
c) manual layout d) fixed layout

[GATE–2010]

Q.12 The word 'kanban' is most appropriately associated with

a) Economic order quantity
b) Just-in-time production
c) Capacity planning
d) Product design

[GATE–2011]

Q.13 Which one of the following is NOT a decision taken during the aggregate production planning stage?

a) Scheduling of machines
b) Amount of labour to be committed
c) Rate at which production should happen
d) Inventory to be carried forward

[GATE–2012]

Q.14 During the development of a product, an entirely new process plan is made based on design logic, examination of geometry and tolerance information. This type of process planning is known as

a) retrieval
b) generative
c) variant
d) group technology based

[GATE-2015]

Q.15 Following data refers to the jobs (P, Q, R, S) which have arrived at a machine for scheduling. The shortest possible average flow time is _________ days.

Job	Processing Time (days)
P	15
Q	9
R	22
S	12

[GATE-2017(1)]

Q.16 Processing times (including setup times) and due dates for six jobs waiting to be processed at a work centre are given in the table. The average tardiness (in days) using shortest processing time rule is ____________ (correct to two decimal places).

Job	Processing time (days)	Due date (days)
A	3	8
B	7	16
C	4	4
D	9	18
E	5	17
F	13	19

[GATE-2017(2)]

ANSWER KEY:

1	2	3	4	5	6	7	8	9	10	11	12	13	14
(d)	(a)	(d)	(d)	(a)	(a)	(c)	(b)	(b)	(c)	(a)	(b)	(a)	(d)

15	16
31	6.33

EXPLANATIONS

Q.1 (d)

Production flow analysis (PFA) is a comprehensive method for material analysis, Part family formation, design of manufacturing cells and facility layout design.

These information are taken from the route sheet.

Q.2 (a)

Given : For machine M1 :

Fixed cost 100 = Rs.

Variable cost 2 = Rs. per piece

For machine M2 :

Fixed cost = 200 Rs.

Variable cost = 1 Rs. per piece

Let, n number of units are produced per machine, when both the machines are to be used concurrently.

We know that,

Total cost = Fixed cost + Variable cost × Number of units

For M1 Total cost of production

= 100 + 2× n

For M2 Total cost of production

= 200 + n

Hence,

Total cost of production on machine M1 & M2 is

= 100 +2 n + 200 + n

= 300 + 3 n

We know, Breakeven point is the point, where total cost of production is equal to the total sales price

Assuming that Number of units produced are less than 800 units and selling price is Rs. 3.50 per unit.

So at breakeven point,

300 + 3 n = 3.50 (n + n)

300 + 3 n = 3.50 × 2 n

300 = 4 n

$$n = \frac{300}{4} = 75 \text{ units}$$

Q.3 (d)

Let The standard machine tool,

Total cost = Fixed cost + Variable cost×Number. of components

$$(TC)_{SMT} = \left[\frac{30}{60} + \frac{22}{60} \times x_1\right] \times 200 =$$

$$\frac{30}{60} \times 200 + \frac{22}{60} \times x_1 \times 200 = 100 + \frac{220}{3} x_1$$

...(i)

If automatic machine tool produce x_2 Number of components then the total cost for automatic machine tool is

$$(TC)_{AMT} = \left(2 + \frac{5}{60} x_2\right) = 800$$

$$= 1600 + \frac{200}{3} x_2 \qquad \text{...(ii)}$$

Let, at the breakeven production batch size is x and at breakeven point.

$$(TC)_{SMT} = (TC)_{AMT}$$

$$= 100 + \frac{200x}{3} = 1600 + \frac{200x}{3} \text{...(ii)}$$

$$\frac{200x}{3} - \frac{200x}{3} - 1600 - 100$$

$$\frac{20x}{3} = 1500$$

$$x = \frac{1500 \times 3}{20} = 225$$

So, breakeven production batch size is 225.

Q.4 **(d)**

Total time T=16 hours =16×60 =960 min

Actual working time was 90 % total time

So, Actual time $T_{actual} = 90\%$ of 960

$$= \frac{90}{100} \times 960. \ T_{actual} = 960 \, \text{min}$$

Performance rating was 120 percent.

So Normal time $T_{actual} = 120\%$ of 864

$$= \frac{120}{100} \times 960. \ T_{actual} = 1036.8 \text{ min}$$

Allowance is 20% of the total available time.

So total standard time

$$T_{standard} = \frac{T_{normal}}{\left(1 - \frac{20}{100}\right)} = \frac{1036.8}{1 - 0.2} = \frac{1036.8}{0.8}$$

= 1296 min

Number of joints soldered, N = 108

Hence, Standard time for operation

$$= \frac{1296}{108} = 12 \text{ min}$$

Q.5 **(a)**

Number of units produced in a day = 80 units.

Working hours in a day = 8 hours

Now, Time taken to produce on unit is, $T = \frac{8}{80} \times 60 = 6 \, \text{min}$

Activity	**Standard time(min)**	**No. of work stations (S.T. / T)**
M. Mechanical assembly	12	12/6 = 2
E Electric wiring	16	16/6=2.666= 3
T. Test	3	3/6 = 0.5 = 1

Number of work stations are the whole numbers, not the fractions. So, number of work stations required for the activities M, E and T would be 2, 3 and 1, respectively.

Q.6 **(a)**

Given: Rating factor = 120%

Actual time T_{actual} = 8 min

Normal time T_{normal} =actual time × Rating factor

$$T_{normal} = 8 \times \frac{120}{100} = 9.6 \, \text{min}$$

10% allowance is allowed for this operation.

So, Standard time

$$T_{normal} = \frac{T_{normal}}{1 - \frac{10}{100}}$$

$$= \frac{9.6}{0.9} = 10.67 \, \text{min}$$

Hence, standard production rate of the weld joint

$$= \frac{8 \times 60}{10.67} = 45 \ \ 45 \text{ Units.}$$

Q.7 **(c)**

Total time used = 7+9+7+10+9+6 = 48 min

Number of work station = 6

Maximum time per work station (cycle time) = 10 min

We know

$$\text{Line efficience } \eta_L = \frac{\text{Total time used}}{\text{Number of work stations} \times \text{cycle time}}$$

$$\eta_L = \frac{48}{6 \times 10} = 0.8 = 80\%$$

Q.8 **(b)**

First finding the sequence of jobs, which are entering in the machine. The solution procedureis described below :

By examining the rows, the smallest machining time of 6 hours on machine M2 . Then scheduled Job P last for machine M2

					P

After entering this value, the next smallest time of 7 hours for job U on machine M2 . Thus we schedule job U second last for machine M2 as shown below

				U	P

After entering this value, the next smallest time of 8 hours for job R on machine M1 . Thus we schedule job R first as shown below.

R				U	P

After entering this value the next smallest time of 11 hours for job T on machine M1 . Thus we schedule job T after the job R .

R	T			U	P

After this the next smallest time of 19 hours for job Q on machine M2. Thus schedule job Q left to the U and remaining job in the blank block. Now the optimal sequence as :

R	T	S	Q	U	P

Then calculating the elapsed time corresponding to the optimal sequence, using the individual processing time given in the problem. The detailed are shown in table.

Jobs	MI		M2	
	In	Out	In	Out
R	0	8	8	8 + 13 = 21
T	8	8 + 11 = 19	21	21+14 = 35
S	19	19+ 27 = 46	46	46+20 = 66
Q	46	46+ 32 = 78	78	78+19 = 97
U	78	78+ 16 = 94	97	97+7= 104
P	94	94+ 15 = 109	109	109+6= 115

We can see from the table that all the operations (on machine 1st and machine 2nd) complete in 115 hours. So the optimal make-span of the shop is 115 hours.

Q.9 (b)

We have to make a table from the given data.

Month	Production (Pieces)		Demand	Excess or short form (Pieces)	
	In regular time	In over time		Regular	Total
1	100	20	90	10	10 + 20 = 30
2	100	20	130	-30	-30 + 20 = -10
3	80	40	110	-30	-30 + 40 = 10

From the table,

For 1st month there is no need to overtime, because demand is 90 units and regular time production is 100 units , therefore 10 units are excess in amount. For 2nd month the demand is 130 unit and production capacity with overtime is 100 +20 = 120 units, therefore 10 units (130 – 120 =10) are short in amount, which is fulfilled by 10 units excess of 1st month. So at the end of 2nd month there is no inventory.

Now for the 3rd month demand is 110 units and regular time production is 80 units. So remaining 110 – 80 = 30 units are produced in overtime to fulfil the demand for minimum cost of plan.

Q.10 (c)

From the product structure we see that 2 piece of R is required in production of 1 piece P .

So, demand of R is double of P.

week	Demand (P)	Demand (R)	Inventory level I =Production Demand
1	1000	2000	R - 2000
2	1000	2000	2R - 4000
3	1000	2000	3R - 6000
4	1000	2000	4R - 8000
5	1000	2400	5R - 10400
6	1000	2400	6R - 12800

We know that for a production system with bottleneck the inventory level should be more than zero.

So, $6R = 12800 \geq 0$

For minimum inventory

$6R-12800=0$

$6R =12800$

$R = 2133$

$\simeq 2200$

Hence, the smallest capacity that will ensure a feasible production plan up to week 6 is 2200.

Q.11 (a)

Vehicle manufacturing assembly line is an example of product layout. A product-oriented layout is appropriate for producing one standardized product, usually in large volume. Each unit of output requires the same sequence of operations from beginning to end.

Q.12 (b)

Kanban Literally, a "Visual record"; a method of controlling materials flow through a Just-in-time manufacturing system by using cards to authorize a work station to transfer or produce materials.

Q.13 (a)

Costs relevant to aggregate production planning is as given below.

(i) Basic production cost : Material costs, direct labour costs, and overhead cost.

(ii) Costs associated with changes in production rate : Costs involving in hiring, training and laying off personnel, as well as, overtime compensation.

(iii) Inventory related costs.

Hence, from above option (A) is not related to these costs. Therefore option (A) is not a decision taken during the APP.

Q.14 (31)

sequence of the job by shortest processing rule

Q S P R

Job	Time	Job Flow Time
Q	9	0+9=9
S	12	9+12=21
P	15	21+15=36
R	22	36+22=58
		Total=124

Avg Job Flow Time

$$= \frac{\text{Total Job Flow Time}}{\text{no. of Jobs}}$$

$$= \frac{124}{4} = 31\text{days}$$

Q.15 (6.33)

Jobs arranged according to shortest processing time

Job	Processing time (days)	Due Date	Job flow time	Tardiness
A	3	8	3	0
C	4	4	$3+4=7$	3
E	5	17	$7+5=12$	0
B	7	16	$12+7=19$	30
D	9	18	$19+9=28$	10
F	13	19	$28+13=41$	22
			Total	**38**

$$\therefore \quad \text{Average Tardiness} = \frac{38}{6} = 6.33$$

7 QUEUING THEORY & TRANSPORTATION

Q.1 Arrivals at a telephone booth are considered to be Poisson, with an average time of 10 minutes between successive arrivals. The length of a phone call is distributed exponentially with mean 3 minutes. The probability that an arrival does not have to wait before service is

a) 0.3 b) 0.5
c) 0.7 d) 0.9

[GATE–2002]

Q.2 The supplies at three sources are 50, 40 and 60 unit respectively while the demands at the four destinations are 20, 30, 10 and 50 unit. In solving this transportation problem

a) a dummy source of capacity 40 unit is needed.
b) a dummy destination of capacity 40 unit is needed.
c) no solution exists as the problem is infeasible.
d) no solution exists as the problem is degenerate.

[GATE–2002]

Q.3 A maintenance service facility has Poisson arrival rates, negative exponential service time and operates on a 'first come first served' queue discipline. Breakdowns occur on an average of 3 per day with a range of zero to eight. The maintenance crew can service an average of 6 machines per day with a range of zero to seven. The mean waiting time for an item to be serviced would be

a) $\frac{1}{6}$ day b) $\frac{1}{3}$ day
c) 1 day d) 3 day

[GATE–2004]

Q.4 Consider a single server queuing model with Poisson arrivals (λ = 4/hour) and exponential service (μ = 4/hour). The number in the system is restricted to a maximum of 10. The probability that a person who comes in leaves without joining the queue is

a) $\frac{1}{11}$ b) $\frac{1}{10}$
c) $\frac{1}{9}$ d) $\frac{1}{2}$

[GATE–2005]

Q.5 The number of customers arriving at a railway reservation counter is Poisson distributed with an arrival rate of eight customers per hour. The reservation clerk at this counter takes six minutes per customer on an average with an exponentially distributed service time. The average number of the customers in the queue will be

a) 3 b) 3.2
c) 4 d) 4.2

[GATE–2007]

Q.6 In an M/M/1 queuing system, the number of arrivals in an interval of length T is a Poisson random variable (i.e. the probability of there being arrivals in an interval of length T is $\frac{e^{-\lambda T}(\lambda T)^n}{n!}$). The probability density function f (t) of the inter – arrival time is

a) $\lambda^2(e^{-\lambda^2 t})$ b) $\frac{e^{-\lambda^2 t}}{\lambda^2}$
c) $\lambda e^{-\lambda t}$ d) $\frac{e^{-\lambda t}}{\lambda}$

[GATE–2008]

Q.7 Little's law is a relationship between

a) stock level and lead time in an inventory system

b) waiting time and length of the queue in a queuing system
c) number of machines and job due dates in a scheduling problem
d) uncertainty in the activity time and project completion time

[GATE–2010]

Q.8 Cars arrive at a service station according to Poisson's distribution with a mean rate of 5 per hour. The service time per car is exponential with a mean of 10 minutes. At steady state, the average waiting time in the queue is

a) 10 minutes b) 20 minutes
c) 25 minutes d) 50 minutes

[GATE–2011]

Q.9 Customer arrive at a ticket counter at a rate of 50 per hr and tickets are issued in the order of their arrival. The average time taken for issuing a ticket is 1 min. Assuming that customer arrivals from a Poisson process and service times and exponentially distributed, the average waiting time is queue in min is

a) 3 b) 4
c) 5 d) 6

[GATE– 2013]

Q.10 If there are m sources and n destinations in a transportation matrix, the total number of basic variables in a basic feasible solution is

a) m+ n b) m+ n+ 1
c) m + n- 1 d) m

[GATE-2014]

Q.11 The total number of decision variables in the objective function of an assignment problem of size n x n (n jobs and n machines) is

a) n^2 b) 2n
c) 2n -1 d) n

[GATE-2014]

Q.12 At a work station, 5 jobs arrive every minute. The mean time spent on each job in the work station is 1/8 minute. The mean steady state number of jobs in the system is ___

[GATE-2014]

Q.13 In the notation (a/b/c) : (dI el f) for summarizing the characteristics of queueing situation, the letters b and d stand respectively for

a) service time distribution & queue discipline
b) number of servers & size of calling source
c) number of servers & queue discipline
d) service time distribution & maximum number allowed in system

[GATE-2015]

Q.14 In a single-channel queuing model, the customer arrival rate is 12 per hour and the serving rate is 24 per hour. The expected time that a customer is in queue is ___minutes.

[GATE-2016]

Q.15 For a single server with Poisson arrival and exponential service time, the arrival rate is 12 per hour. Which one of the following service rates will provide a steady state finite queue length?

a) 6 per hour b) 10 per hour

c) 12 per hour d) 24 per hour

[GATE-2017(2)]

Q.16 A product made in two factories p and Q, is transported to two destinations, R and S. The per unit costs of transportation (in Rupees) from factories to destinations are as per the following matrix:

Factory \ Destination	R	S
P	10	7
Q	3	4

Factory P produces 7 units and factory Q produces 9 units of the product. Each destination requires 8 units. If the north-west corner method provides the total transportation cost as X (in Rupees) and the optimized (the minimum) total transportation cost Y (in Rupees), then (X-Y), in Rupees, is

a) 0 b) 15

c) 35 d) 105

[GATE-2017(2)]

Q.17 The arrival of customers over fixed time intervals in a bank follow a Poisson distribution with an average of 30 customers/hour. The probability that the time between successive customer arrivals is between 1 and 3 minutes is ______ (correct to two decimal places).

[GATE-2018(2)]

ANSWER KEY:

1	2	3	4	5	6	7	8	9	10	11	12	13	14
(c)	(b)	(a)	(a)	(b)	(c)	(b)	(d)	(c)	(c)	(a)	1.67	(a)	2.5

15	16	17
(d)	(*)	0.383

EXPLANATIONS

Q.1 (c)

Given:

Average time between arrivals min = 10 MIN

Mean arrival rate (Number of arrivals per unit time) λ = 6 per hour

Average time between call = 3 min

Mean service rate

$$\mu = \frac{60}{3} = 20 \text{ per hour}$$

SO, the probability that an arrival does not have to wait before service is,

$$P_0 = 1 - \frac{\lambda}{\mu} = 1 - \frac{6}{20} = 1 - 0.3 = 0.7$$

Q.2 (b)

Total supply 50 +40+60 = 150 units

Total demand=20+30+10+50 = 110 units

In this question, the total availability (supply) may not be equal to the total demand, i.e., $\sum_{i-1}^{m} a_i \neq \sum_{j-1}^{n} b_j$

Such problems are called unbalanced transportation problems.

Here total availability is more than the demand. So we add a dummy destination to take up the excess capacity and the costs of shipping to this destination are set equal to zero. So, a dummy destination of capacity 40 unit is needed.

Q.3 (a)

Given:

Mean arrival rate λ =3 per day

Mean service rate μ= 6 per day

We know that, for first come first serve queue.

Mean waiting time of an arrival,

$$t = \frac{\lambda}{\mu(\mu - \lambda)} = \frac{3}{6(6-3)} = \frac{1}{6} \text{ day}$$

Q.4 (a)

Given λ = 4 / hour μ= 4/ hour

The sum of probability

$$\sum_{n=0}^{n-10} P_n = n = 10$$

$$P_0 + P_1 + P_2 ..... + P_{10} = 1$$

In the term of traffic intensity

$$\rho = \frac{\lambda}{\mu} \Rightarrow \rho = \frac{4}{4} = 1$$

SO,

$$P_b + \rho P_0 + \rho^2 + ...\rho^{10} P_0 = 1$$

$$P_1 = \rho P_0, P_2 = \rho^2$$

P_0 and so on

P_0 (1 + 1 + 1 +....)= 1

$P_0 \times 1$

$$P_0 = \frac{1}{11}$$

Hence, the probability that a person who comes in leaves without joining the queue is,

$$P_{11} = \rho^{11}. \times P_0$$

$$P_{11} = \rho^{11} \times \frac{1}{11} = \frac{1}{11}$$

Q.5 (b)

Given λ= 8 per hour

μ= 6 min per customer

$$= \frac{60}{5} \text{ customer / hours}$$

= 10 customer / hour

We know, for exponentially distributed service time.

Average number of customers in the queue.

$$L_q = \frac{\lambda}{\mu} \times \frac{\lambda}{(\mu\text{-}\lambda)} = \frac{8}{10} \times \frac{8}{(10\text{-}8)} = 3.2$$

Q.6 (c)

The most common distribution found in queuing problems is

Poisson distribution. This is used in single-channel queuing problems for random arrivals where the service time is exponentially distributed. Probability of n arrivals in time t

$$P = \frac{(xt)^n . e^{-\lambda T}}{nl}$$

where n = 0,1,2...

So, Probability density function of inter arrival time (time interval between two consecutive arrivals.

$f(t)=\lambda.e^{-\lambda t}$

Q.7 (b)

Little's law is a relationship between average waiting time and average length of the queue in a queuing system. The little law establish a relation between Queue length (L_q) , Queue waiting time (W_q) and the Mean arrival rate λ.

So, $L_q = \lambda W_q$

Q.8 (d)

Given: λ = 5 per hour, $\mu = \frac{1}{10} \times 60$ per hour = 6 per hour

Average waiting time of an arrival

$$W_q = \frac{\lambda}{\mu(\mu - \lambda)} = \frac{5}{6(6-5)}$$

$= \frac{5}{6}$ hours = 50 min

Q.9 (c)

Average waiting time of a customer (in a queue) is given by

$$E(w) = \frac{\lambda}{\mu(\mu - \lambda)}$$

Where λ = 50 customer per hour or 0.834 customer /min

μ = 1 per min

Therefore $E(w) = \frac{0.834}{1 \times (1 - 0.834)}$

= 5 min

Q.10 (c)

Q.11 (a)

Q.12 (1.67)

λ (arrival rate) = 5 jobs/minute

μ(service rate) = 8 jobs/minute

$$Ls = \frac{\lambda}{\mu - \lambda} = \frac{5}{8-5} = \frac{5}{3} = 1.67$$

Q.13 (a)

Q.14 (2.5)

$\lambda = 12 hr^{-1}$

$\mu = 24\ hr^{-1}$

$$W_s = \frac{\lambda}{\mu(u - \lambda)} = \frac{12}{24 \times 12} \times 60$$

= 2.5 seconds

Q.15 (d)

Q.16: given options are incorrect

Q.17 (0.383)

Given, arrival rate, λ = 30/hour

$\lambda = \frac{1}{2}$ / min.

P = prob. $= 1 - e^{-\lambda t}$

$P(1) = 1 - e^{-\frac{1}{2} \times 1} = 0.393$

$P(3) = 1 - e^{-\lambda t} = 1 - e^{-\frac{1}{2} \times 3}$

$= 1 - e^{-1.5} = 0.7768$

$P(1 \le T \le 3 \text{ min}) = 0.7768 - 0.393 = 0.383$

ASSIGNMENT QUESTIONS

Q.1 Which one of the following forecasting techniques is not suited for making forecasts for planning production schedules in the short range?
a) Moving average
b) Exponential moving average
c) Regression analysis
d) Delphi

Q.2 Which one of the following methods can be used for forecasting the sales potential of a new product?
a) Time series analysis
b) Jury of Executive Opinion method
c) Sales Force Composite method
d) Direct Survey method

Q.3 Which one of the following methods can be used for forecasting when a demand pattern is consistently increasing or decreasing?
a) Regression analysis
b) Moving average
c) Variance analysis
d) Weighted moving average

Q.4 Which one of the following forecasting techniques is most suitable for making long range forecasts?
a) Time series analysis
b) Regression analysis
c) Exponential smoothing
d) Market surveys

Q.5 Which one of the following is not a purpose of Long-term forecasting?
a) To plan for the new unit of production
b) To plan the long-term financial requirement
c) To make the proper arrangement for training the personnel
d) To decide the purchase programme

Q.6 Which one of the following is not a technique of Long Range Forecasting?
a) Market Research & Market Surve
b) Delphi
c) Collective Opinion
d) Correlation and Regression

Q.7 Which of the following is the measure of forecast error?
a) Mean absolute deviation
b) Trend value
c) Moving average
d) Price fluctuation

Q.8 For sales forecasting, pooling of expert opinions made use of in
a) Statistical correlation
b) Delphi technique
c) Moving average method
d) Exponential smoothing

Q.9 Which of the following is not the characteristic of exponential smoothing? method of forecasting?
a) This represents a weighted average of the past observations.
b) All observations are assigned equal weightage.
c) If smoothening coefficient is 1 then the latest forecast would be equal to previous period actual demand.
d) The technique is not simple as compared to moving average method.

Q.10 The interchange ability can be achieved by
a) standardization
b) better process planning
c) bonus plan
d) better product planning

Q.11 Standardization of products is done to

a) eliminate unnecessary varieties in design
b) simplify manufacturing varieties in design
c) make interchangeable manufacture possible
d) reduce material cost

Q.12 The standard time of an operation while conducting a time study is
a) mean observed time + allowances
b) normal time - allowances
c) mean observed time x rating factor +allowances
d) normal time x rating factor + allowances

Q.13 In time study, the rating factor is applied to determine
a) standard time of a job
b) merit rating of the worker
c) fixation of incentive rate
d) normal time of job

Q.14 Work study is mainly aimed at
a) determining the most efficient method of performing a job
b) establishing the minimum time of completion of job
c) developing the standard method and standard time of a job
d) economizing the motions involved on the part of worker while performing a job

Q.15 Which one of the following is not a technique of PMTS?
a) Synthetic data
b) Stopwatch time study
c) Work-factor
d) MTM

Q.16 In performing a task, motion economy refers to the manner in which
a) human energy can be conserved
b) electric energy can be conserved
c) machine movements can be reduced
d) material movements can be reduced

Q.17 In the Kendall's notation for representing queuing models the first position represents
a) Probability law for the arrival
b) Probability law for the service
c) Number of channels
d) Capacity of the system

Q.18 In a single server queuing system with arrival rate of λ and mean service time of 'μ' the expected number of customers in the system is $\frac{\lambda}{\mu-\rho}$ What is the expected waiting time per customer in the system?
a) $\frac{\lambda^2}{\mu-\lambda}$
b) $\frac{\lambda}{\mu-\lambda}$
c) $\frac{1}{\mu-\lambda}$
d) $\frac{(\mu-\lambda)}{\lambda}$

Q.19 Flexible manufacturing systems are generally applied in
a) high variety and low volume production
b) medium volume and medium variety production
c) low v3.riety and low volume production
d) high variety and high volume production

Q.20 Vehicle manufacturing assembly line is an example of
a) product layout b) process layout
c) manual layout d) fixed layout

Q.21 Transfer lines are mostly used
a) Aero industries
b) Ship building
c) Automobile industries
d) Machine tool manufacturing

Q.22 Which one of the following types of layout is used for the manufacture of huge Aircrafts?
a) Product layout
b) Process layout
c) Fixed position layout
d) Combination layout

Q.23 Which one of the following is military type organization?
a) Line organization
b) Functional organization
c) Line and staff organization
d) Line and functional organization

Q.24 Which type of layout is preferred in order to avoid excessive multiplication of facilities?
a) Process layout
b) Product layout
c) Fixed position layout
d) Cellular manufacturing

Q.25 In-process inventory will be maximum case of a plant with
a) Process layout
b) Product layout
c) Mixed process and Product layout
d) Machine cell layout

Q.26 The number of pieces inspected per lot is generally greatest in the following sampling plan
a) Single b) Double
c) Multiple d) None of these

Q.27 Which one of the following is not the characteristic of acceptance sampling?
a) This is widely suitable in mass production
b) It causes less fatigue to inspectors
c) This is much economical
d) It gives definite assurance for the conformation of the specifications for all the pieces

Q.28 Which one of the following is true in case of simplex method of linear programming?
a) The constants of constraints equation may be positive or negative
b) Inequalities are not converted into equations
c) It cannot be used for two-variable problems
d) The simplex algorithm is an iterative procedure

Q.29 The solution in a transportation model (of dimension m x n) is said to be degenerate if it has
a) exactly (m + n - 1) allocations
b) fewer than (m + n- 1)
c) more than (m + n- 1) allocations
d) (m x n) allocations

Q.30 In a transportation problem, the materials are transported from 3 plants to 5 warehouses. The basic feasible solution must contain exactly, which one of the following allocated cells?
a) 3 b) 5
c) 7 d) 8

Q.31 Simplex method of solving linear programming problem uses
a) all the points in the feasible region
b) only the corner points of the feasible region
c) intermediate points within the infeasible region
d) only the interior points in the feasible region

Q.32 In linear programming a basic feasible solution
a) satisfies constraints only
b) satisfies constraints and non-negativity restrictions
c) satisfies non-negativity
d) optimizes the objective function

Q.33 A tie for leaving variables in simplex procedure implies
a) Optimality b) Cycling
c) No solution d) Degeneracy

Q.34 Two groups of costs in inventory control are
a) carrying costs and ordering costs
b) relevant costs and ordering costs
c) carrying costs and total costs
d) relevant costs and total costs

Q.35 In basic economic order quantity model for the optimal order quantity,
a) holding cost is more than ordering cost
b) holding cost is less than ordering cost
c) holding cost is equal to ordering cost
d) holding cost is two times the ordering cost

Q.36 The inventory carrying cost includes
a) expenditure incurred for payment of bills
b) placing an order
c) receiving and inspecting
d) obsolescence and depreciation

Q.37 In the basic EOQ model, if demand is Rs.60 per month, ordering cost is Rs. 12 per order, holding cost is Rs. 10 per unit per month, what is the EOQ?
a) 12 b) 144
c) 24 d) 28

Q.38 ABC analysis deals with
a) analysis of process chart
b) flow of materials
c) ordering schedule of job
d) controlling inventory cost

Q.39 In ABC analysis of inventory control 'A' items have
a) very high cost
b) intermediate cost
c) low cost
d) very low cost

Q.40 In ABC analysis the following is true for 'C' items:
a) Low consumption value
b) Low control
c) Bulk ordering
d) Accurate forecasting for material planning

Q.41 The following is the general policy for A class items in ABC analysis:
1. Very strict control
2. Frequent review of their consumption
3. Safety stock kept
Which of the above statements is/are correct?
a) 1 only b) 1 and 2 only
c) 2 only d) 1, 2 and 3

Q.42 The term value in value engineering refers to
a) total cost
b) utility of the product
c) selling price
d) depreciated value

Q.43 In value engineering important consideration is given to
a) cost reduction
b) profit maximization
c) function concept
d) customer satisfaction

Q.44 Aluminum tie pin and gold tie pin, both serve the purpose of keeping the tie in position, but the gold pin has significance due to:
a) Exchange value b) Use value
c) Esteem value d) Cost value

Q.45 Consider the following basic steps involved in value analysis:
1. Create 2. Blast
3. Refine
The correct sequence of these steps is
a) 1, 2, 3 b) 3, 1, 2
c) 1, 3, 2 d) 2, 1, 3

Q.46 In CPM network critical path denotes the
a) path where maximum resources are used
b) path where minimum resources are used
c) path where delay of one activity prolongs the duration of completion of project
d) path that gets monitored automatically

Q.47 In PERT, the distribution of activity times is assumed to be
a) Normal b) Gamma
c) Beta d) Exponential

Q.48 The critical path of a network is the path that
a) Takes the shortest time
b) Takes the longest time
c) Has the minimum variance
d) Has the maximum variance

Q.49 The variance of the completion time for a project is the sum of variance of
a) All activity times
b) Non-critical activity times
c) Critical activity times
d) Activity times of first and last activities of the project

Q.50 Bin cards are used for
a) machine loading
b) stores
c) accounts
d) inventory control

Q.51 Value Engineering aims at finding out
a) depreciation value of a product
b) resale value of a product
c) major functions of the item and accomplishing the same at the least cost without change in quality
d) break-even point when machine requires change

Q.52 MRP-11 means
a) material requirement planning
b) manufacturing resource planning
c) man requirement planning
d) money requirement planning

Q.53 Which one of the following is not a function of production control?
a) Forecasting b) Routing
c) Scheduling d) Dispatching

Q.54 Which of the following is not an input to the MRP system?
a) MPS (Master Production Schedule)
b) Bill of materials
c) Cost of materials
d) Inventory status

Q.55 Under the Emerson Efficiency Plan, the worker gets a normal wage at the efficiency of
a) 50% b) 66.66%
c) 75% d) 85%

Q.56 An equipment costs P and its service in number of years N. If the annual P-L .depreciation charge is N, then L is the
a) maintenance cost
b) salvage value
c) production cost
d) ideal cost

Q.57 In an assembly line the cycle time is equal to the
a) maximum time of all station times
b) average time of all station times
c) sum of all station times
d) time for total work content

Q.58 In manufacturing management, the term "Dispatching" is used to describe
a) dispatch of sales order
b) dispatch of factory mail
c) dispatch of finished product to the user
d) dispatch of work orders through shop floor

Q.59 A production line is said to be balanced when
a) There are equal number of machines a teach work station
b) There are equal number of operators a teach work station
c) The waiting time for service at each station is the same
d) The operation time at each station is the same

Q.60 In production, planning and control, the document which authorizes the start of an operation on the shop floor is the:
a) Dispatch order b) Route plan
c) Loading chart d) Schedule

Q.61 Sequencing is a subset of
a) routing b) scheduling
c) expediting d) none of the above

Q.62 PERT stands tor
a) project evaluation and review technique
b) process evaluation and review technique
c) programme evaluation and review technique
d) planning estimatimation and review technique

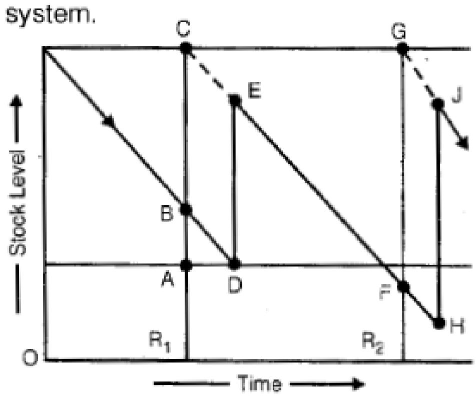
Q.63 Which one of the following statements is not correct tor the TQC?
a) It is about the product quality & quality of all business processes.
b) It is restricted to product quality.
c) It utilizes quality circles.
d) It utilized zero defect programmes.

Q.64 Which one of the following is not the purpose of method study?
a) Save time
b) Improve manpower planning
c) Improve product design
d) Reduce worker fatigue

Q.65 What is the act of authorizing the work-man actually to perform the work according to the method outlined using prescribed tools at a predetermined standard & schedule?
a) Dispatching b) Loading
c) Planning d) Scheduling

Q.66 Mass production is characterized by
a) low volume items with maximum flexibility in their design
b) high volume items with maximum flexibility in their design
c) high volume items with minimum flexibility in their design
d) low volume items with minimum flexibility in their design

Q.67 The given figure shows the details of stock level in the periodic review inventory control system.



Match the List-1 (Characteristics) withList-11 (Line) and select the correct answer using the code given below the lists:

List-I	**List-II**
A. Lead time	1 . DE
B. Ordered quantity	2. FH
C. Safety stock	3. CG
D. Review period	4. R1A
	5. AD

Codes:

	A	B	C	D
a)	3	4	2	5
b)	5	1	4	3
c)	3	1	4	5
d)	5	4	2	3

Q.68 A machine tool manufacturing industry requires to purchase 2000 helical gears/year from a gear manufacturing industry. The ordering cost is Rs. 100/- per order and the carrying cost per unit per year is Rs. 10/-. The economic order quantity is
a) 2000 b) 1000
c) 200 d) 100

Q.69 If item cost, inventory carrying cost, ordering cost and demand get doubled, what is the ratio of modified economic order quantity (EOQ) and the present EOQ?

a) $\sqrt{2}$ b) 2
c) 4 d) 8

Q.70 If orders are placed once a month to meet an annual demand of 6,000 units, then the average inventory would be

a) 200 b) 250
c) 300 d) 500

Q.71 The expected demand of a product is 150 per day. The lead time is 10 days. An order is placed when the inventory falls to 4000units. The safety stock is

a) 1500 units b) 2500 units
c) 3000 units d) 4000 units

Q.72 A dealer sells a radio set at Rs. 900 and makes 80% profit on his investment. If he
can sell it at Rs. 200 more, his profit as percentage of investment will be:

a) 160 b) 180
c) 100 d) 120

Q.73 A purchasing assistant has calculated the annual carrying cost for an item to be Rs. 4/annum. EOQ worked out is 500 units. What is the annual ordering cost for the item?

a) Rs. 125 b) Rs. 500
c) Rs. 1000 d) Rs. 2000

Q.74 The probability distribution for the sale of a product is as follows:

Sale quantity	Probability
1	0.3
2	0.4
3	0.3

On the sale of an item the profit is Rs. 500. The expected profit is

a) 1000 b) 500
c) 250 d) 200

Q.75 Patients arrive at a doctor's clinic according to the Poisson distribution. Check uptime by the doctor follows an exponential distribution. If on an average, 9 patients/hr arrive at the clinic and the doctor takes on average 5 minutes to check a patient, the number of patients in the queue will be

a) 1 b) 1.25
c) 2.25 d) 3.25

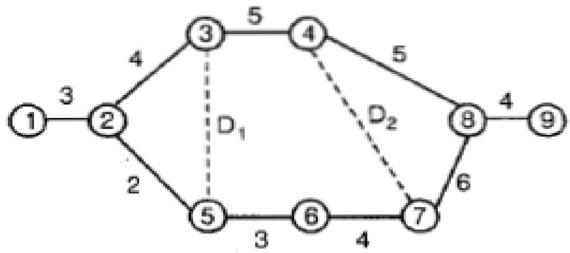
Q.76 For a M/M/1:oo/FCFS queue, the mean arrival rate is equal to 10 per hour and the mean service rate is 15 per hour. The expected queue length is:

a) 1.33 b) 1.53
c) 2.75 d) 3.20

Q.77 The pessimistic, most likely and optimistic times for an activity are 5 days, 3 days and 1 day respectively. Assuming Beta distribution, the expected duration of the activity is

a) 3 days b) 3.5 days
c) 4 days d) 5 days

Q.78 In the network shown below the critical path is along:



a) 1-2-3-4-8-9
b) 1-2-3-5-6-7-8-9
c) 1-2-3-4-7-8-9
d) 1-2-5-6-7-8-9

Q.79 In a PERT network, expected project duration is found to be 36 days from the start of the project. The variance is four days. The probability that the project will be completed in 36 days is:

a) zero b) 34%
c) 50% d) 84%

Q.80 Time estimates of an activity in a PERT network are: Optimistic time t_0 = 9 days; pessimistic time t_P= 21 days and most likely time t_m= 15 days. The approximates probability of completion of this activity in 13 days is:

a) 16% b) 34%
c) 50% d) 84%

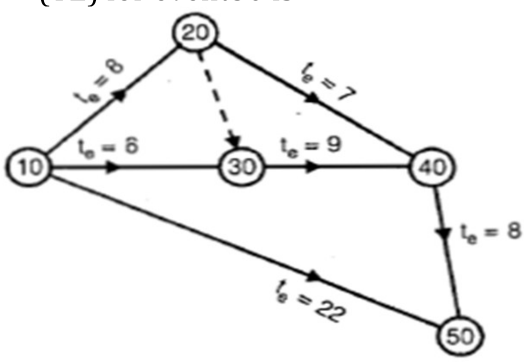
Q.81 If the earliest starting time for an activity is8 weeks, the latest finish time is 37 weeks· and the duration time of the activity is 11 weeks, then the total float is equal to:

a) 18 weeks b) 14 weeks
c) 56 weeks d) 40 weeks

Q.82 Weekly production requirements of a product are 1000 items. The cycle time of producing one product on a machine is10 minutes. The factory works on two shift basis in which total available time is16 hours. Out of the available time about 25% is expected to be wasted on breakdowns, material unavailability and quality related problems. The factory works for5 days in a week. How many machines are required to fulfill the production requirements?

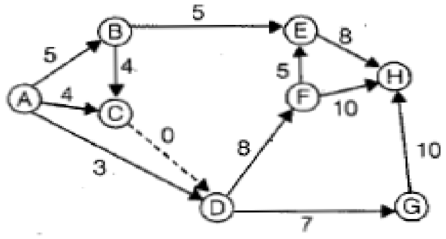
a) 2 b) 3
c) 4 d) 6

Q.83 Consider the network. Activity times are given in number of day. The earliest expected occurrence time (TE) for event50 is



a) 22 b) 23
c) 24 d) 25

Q.84 For the network shown in the given figure, the earliest expected completion time of the project is: 0.36



a) 26 days b) 27 days
c) 30 days d) Indeterminable

Q.85 The fixed costs for a year is Rs. 8 lakes, variable cost per unit is Rs. 40/- and the selling price of each unit is Rs. 200/-. If the annual estimated sales is Rs. 20,00,000/-, then the break-even volume is

a) 2000 b) 3000
c) 3333 d) 5000

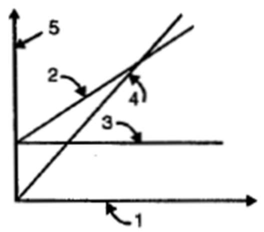
Q.86) The lead time consumption is 500 units. The annual consumption is 8000 units. The company has a policy of EOO ordering and maintenance of 200 units as safety stock. The reorder point (ROP) is

a) 500 units b) 700 units
c) 200 units d) None of these

Q.87 A company sells 14,000 units of its product. It has a variable cost of Rs. 15 per unit. Fixed cost is Rs. 47,000 & total required profit is Rs. 23,000. Per unit product price (in Rs.) Will be

a) 60 b) 40
c) 30 d) 20

Q.88 Match List-1 (Cost revenue parameter) with List-II (Break-even chart's parameter) &select the correct answer using the code given below the lists:

<table>
<tr><th>List-1</th><th>List-2</th></tr>
<tr><td>A. Facility cost</td><td rowspan="4">
</td></tr>
<tr><td>B. Total cost</td></tr>
<tr><td>C. Sales revenue</td></tr>
<tr><td>D. Production quantity</td></tr>
</table>

Codes:

	A	B	C	D
a)	3	2	5	1
b)	1	4	5	3
c)	3	2	4	1
d)	1	2	4	3

Q.89 Consider the following statements in respect of break-even point:
1. Revenue is equal to total cost.
2. Revenue is equal to variable cost.
3. Profit/Loss is equal to zero.

Which of these statements is/are correct?
a) 1, 2 and 3 b) 1 and 2 only
c) 2 and 3 only d) 1 and 3 only

Q.90 If a company's total sales is Rs. 50,000 and (PN) ratio is 50% and margin of safety percentage is 40%, then break-even point sale is
a) 20,000 b) 25,000
c) 30,000 d) 40,000

Q.91 Following table shows the data about a set of single operation jobs to be machined in a milling machine.

Job	1	2	3	4	5
Processing time	6	17	5	7	11

The optimum sequence that minimizes the mean flow time is
a) 1-2-3-4-5 b) 5-4-3-2-1
c) 3-1-4-5-2 d) 2-5-4-1-3

Q.92 There are two machines M1 and M2 which process jobs A, B, C, D, E and F. The processing sequence for these jobs is M1followed by M2. Consider the following detain this regard: Process time required in minutes

Jobs	A	B	C	D	E	F
M_1	4	7	3	12	11	9
M_2	11	7	10	8	10	13

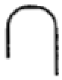
The processing sequence of jobs that would minimize the make span is:
a)C-A-B-F-E-0 b) C-A-B-0-E-F
c) C-A-0-B-F-E d) E-F-0-B-A-C

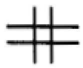
Q.93 In a 'C' chart the value of C' is 36. The upper control limit of the chart is equal to
a) 114 b) 126
c) 90 d) 54

Q.94 Match List-1 (Thermbling symbol) with Ust-11(Description) and select the correct answer using the code given below the lists:

List-I

A. 

B. 

C. 

D. 

List-II
1. Prolonged grasp of the object
2. Locate an article
3. Releasing the object at the desired place
4. Taking hold of the object
5. Putting parts together

Codes:

	A	B	C	D
a)	3	1	4	2
b)	2	4	5	3
c)	3	4	1	2
d)	1	5	4	3

Q.95 Consider the following statements:
1. To prepare estimates of labor and costs.
2. To prepare realistic work schedule.
3. To establish better methods of work.
4. To compare times required for alternative methods.

Which of these are the objectives of work measurement?
a) 1 only b) 2 and 3
c) 1 and 2 d) 2, 3 and 4

Q.96 The following data is available from time study on a job Observed time =

0.75 min., rating = 110%, relaxation allowance = 3%, delay allowance = 2%. All allowances expressed in % of normal time. What is the approximate standard time for this job?
a) 0.95 minutes b) 0.625 minutes
c) 0.825 minutes d) 0.74 minutes

Q.97 Time study of an operator with a performance rating of 120% yields a time of 2 minutes. If the allowances of 10% of the total available time are to be given, then what is the standard time of operation?
a) 2.00 minutes b) 2.40 minutes
c) 2.64 minutes d) 2.67 minutes

Q.98 Match List-I with List-II and select the correct answer using the code given below the lists:
List-I
A. Loading
B. Dispatching
C. Follow-up
D. Scheduling
List-II
1. Reviewing the status of orders as they progress through the system.
2. Authorizing actual performance of the
3. Matching the capacity requirements of orders received (or expected) to the existing capacity.
4. Process of fitting a production order into the available time of machines.

Codes:

	A	B	C	D
a)	4	1	2	3
b)	3	1	2	4
c)	4	2	1	3
d)	3	2	1	4

Q.99 Consider the following statements:
1. Volume of purchases
2. Discounts offered
3. Disposal of scrap
Which of these factors will provide economy in materials management?
a) 1 and 3 b) 2 and 3
c) 1 and 2 d) 3 only

Q.100 Which of the following can be considered to the advantages of product layout?
1. Reduced material handling
2. Greater flexibility
3. Lower capital investment
4. Better utilization of men and machines
a) 1 and 3 b) 2 and 3
c) 1 and 4 d) 2 and 4

ANSWER KEY:

1	2	3	4	5	6	7	8	9	10	11	12	13	14
(d)	(d)	(a)	(d)	(c)	(d)	(a)	(b)	(b)	(a)	(c)	(c)	(d)	(c)
15	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**
(a)	(a)	(a)	(c)	(b)	(a)	(c)	(c)	(a)	(a)	(a)	(a)	(d)	(d)
29	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**
(b)	(c)	(b)	(b)	(d)	(a)	(c)	(c)	(a)	(d)	(a)	(b)	(d)	(b)
43	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**
(a)	(c)	(d)	(b)	(c)	(c)	(c)	(b)	(c)	(b)	(a)	(c)	(b)	(b)
57	**58**	**59**	**60**	**61**	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**
(a)	(d)	(d)	(a)	(b)	(c)	(a)	(c)	(a)	(c)	(b)	(c)	(a)	(d)
71	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**	**81**	**82**	**83**	**84**
(b)	(d)	(c)	(a)	(c)	(a)	(a)	(b)	(c)	(a)	(a)	(b)	(d)	(c)
85	**86**	**87**	**88**	**89**	**90**	**91**	**92**	**93**	**94**	**95**	**96**	**97**	**98**
(d)	(b)	(d)	(c)	(d)	(c)	(d)	(a)	(d)	(b)	(d)	(a)	(c)	(d)
99	**100**												
(c)	(c)								`				

EXPLANATIONS

Q.1 (d)

Q.2 (d)
Opinion survey method is relatively simple and practical method for forecasting demands and especially for new products.

Q.3 (a)
Linear Regression Analysis: This method is very useful forecasting techniques if the past data appear to fall about a straight line. The general equation for the regression line is given by: y = a + bx, where 'a and b are constants.

Q.4 (d)
Time series analysis (Regression Analysis, Exponential smoothing) are quantitative analysis of forecasting and suitable for short range while market survey is a long range forecasting.

Q.5 (c)

Q.6 (d)
Correlation and Regression method is used for short and medium range forecasting.

Q.7 (a)

Q.8 (b)

Q.9 (b)

Q.10 (a)

Q.11 (c)
Standardization is done to make manufacturing simplicity ie., the subassemblies can be made somewhere, where it can be made easily and assembling in done somewhere else.

Q.12 (c)

Q.13 (d)
A rating factor is a factor by which the observed time is multiplied in order.
To adjust for differences in operator's performance.
Normal time = observed time performance rating factor.

Q.14 (c)
Work study investigates work done in an organization and it aims at finding the best and most efficient way of using available resources.

Q.15 (a)
Techniques of PMTS are:
1. Method Time Measurement (MTM)
2. Work Factor System (WFS)
3. Basic Motion Time Study (BMTS)

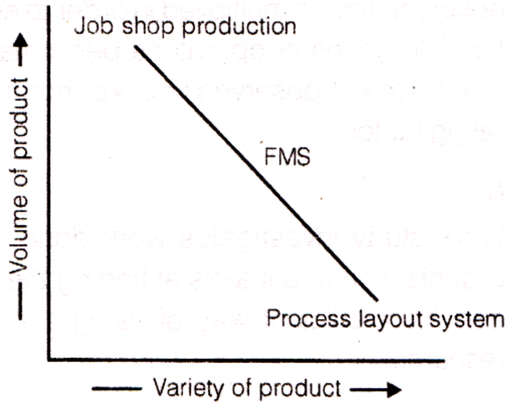
Q.16 (a)
Principle of motion economy: A worker while performing a task makes use of number of motions or movements.The careful examination of these motions may reveal that some of these motions
- are unnecessary and can be eliminated
- can be simplified by effective changes in the work place layout
- can be performed more efficiently by other members of the body.

Q.17 (a)

Q.18 (c)
Waiting time in system.

$$= \frac{\lambda}{(\mu-\lambda)} \times \frac{1}{\lambda} = \frac{1}{\mu-\lambda}$$

Q.19 (b)



Job shop production is used for mass production i.e., high volume low variety, process less out is used for low volume high variety of product while flexible manufacturing system is used for mid volume mid variety product.

Q.20 (a)

Q.21 (c)

Q.22 (c)

Process layout: It is characterized by keeping similar machines or similar operation at one location.

Process layout: Implies that various operations on raw material are performed in a sequence and the machine are placed along the product flow line, i.e. , machines are arranged in the sequence in which raw material will be operated upon.

Fixed position layout: In this case men and equipment are moved to the material, which remain at one place and product in completed at that place where material lies. It is used in ship building, aircraft manufactured big pressure vessel fabrication etc.

Q.23 (a)

Q.24 (a)

Q.25 (a)

Q.26 (a)

The number of pieces inspected per lot is generally more in single sampling than double sampling.

Q.27 (b)

Q.28 (d)

Simplex method of linear programming is an iterative procedure.

Q.29 (b)

In a Transportation Problem, if the number of non-negative independent allocations is less than m + n - 1 there exists a degeneracy. where
m = no. of rows
n = no. of columns

Q.30 (c)

Degeneracy arises when number of allocation are less than 3÷5 -1 = 7

Q.31 (b)

Q.32 (b)

Q.33 (d)

Q.34 (a)

Two costs are involved in inventory control viz. carrying cost & ordering cost.

Q.35 (c)

For economic order quantity
Holding cost = ordering cost

Q.36 (c)

Q.37 (a)

$$\text{EOQ} = \sqrt{\frac{2RC_0}{C_C}}$$

$$= \sqrt{\frac{2\times(60\times12)\times12}{(10\times12)}} = 12$$

Q.38 (d)
ABC analysis is technique of inventory control.

Q.39 (a)
In ABC analysis, item A' are high usage, valued item and extra care is required for these items.

Q.40 (b)

Q.41 (d)

Q.42 (b)
The term value refers to the ratio of utility of product and cost.

$$\text{Value} = \frac{\text{utility}}{\text{cost}}$$

Q.43 (a)
Value engineering is the application of the concept of value analysis at the design or pre-manufacture stage of the component parts with a view to cut down the unnecessary cost, without impairing the function or utility of the product.

Q.44 (c)
Esteem Value: The properties, features or attractiveness of an object makes its ownership desirable.
Use Value: The properties or qualities which accomplish a use, work or service.
Cost Value: The sum of labour material and other cost required to produce the object (also called as Economic value).
Exchange Value: The properties or qualities of an object that make it possible to exchange it for something else that one wants.

Q.45 (d)
Value analysis procedure:
(a) Blast:
- Identify the product
- Collect the relevant information
- Define the different function

(b) Create:
- Create different alternatives
- Critically evaluate the alternative

(c) Refine:
- Develop the best alternatives
- Implement the alternative

Q.46 (b)

Q.47 (c)
Critical path is the path having zero delay and if any delay occurs in the activity of critical path whole cycle get affected.

Q.48 (c)
In PERT the activity times follow Beta distribution.

Q.49 (c)

Q.50 (b)
Bin cards are used in stores to identify various inventory.

Q.51 (c)
Value engineering is a critically investigation & analysis of different aspects of material, design and production of each & every component of product.
Value engineering targeted to improve the value of a product and it can be improved by increasing the utility of product by maintaining the same cost or by minting the same utility by decreasing the cost.

Q.52 (b)

Q.53 (a)
Routing, scheduling and dispatching are the function of production control.

Q.54 (c)

Q.55 (b)

Q.56 (b)

Q.57 (a)

Q.58 (d)
Dispatching basically is the physical delivery of orders and instructions to all the persons who are involved in actual projection and other supporting activities.

Q.59 (d)

Q.60 (a)

Q.61 (b)

Q.62 (c)

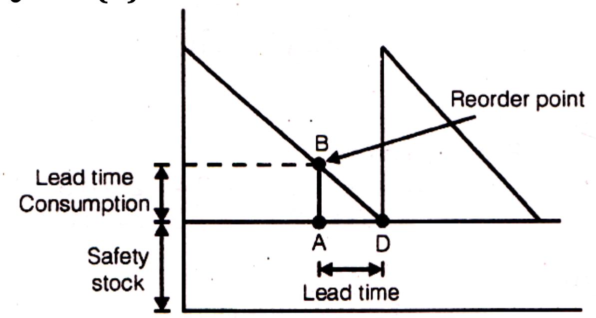
Q.63 (a)

Q.64 (c)

Q.65 (a)

Q.66 (c)

Q.67 (b)



Q.68 (c)

$$\text{EOQ} = \sqrt{\frac{2RC_0}{C_C}}$$

$$= \sqrt{\frac{2 \times 2000 \times 100}{100}}$$

= 200 units

$$\text{EOQ} = \sqrt{\frac{2RC_0}{C_C}}$$

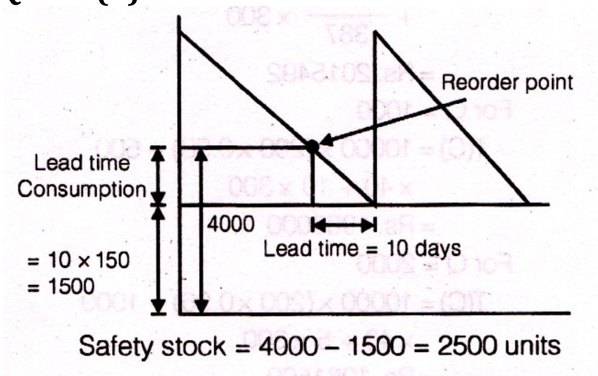
$$\text{EOQ} = \sqrt{\frac{RC_0}{C_C}}$$

Q.69 (a)

Q.70 (d)

$$Q_{avg} = \frac{6000}{12} = 500$$

Q.71 (b)



Q.72 (d)

$$\text{C.P.} = \frac{900 \times (100)}{(100+80)} = 500$$

S.P. if he self at 200 more = 900 + 200 = 1100

$$\therefore \text{Proft (in \%)} = \frac{100-500}{500} \times 100 = 120\%$$

Q.73 (c)

Q.74 (a)
Expected profit
= 500(0.3 x 1 + 0.4 x 2 + 0.3 x 3)
= 500 (0.3 + 0.8 + 0.9) = 1000

Q.75 (c)
λ= 9 Patients / hr
μ= 12 Patients/hr

$$\rho = \frac{\lambda}{\mu} = \frac{9}{12} = 0.75$$

length of queue

$$L_n = \frac{p^2}{p-1}$$

$$L_n = \frac{(0.75)^2}{(1-0.75)}$$

L_n =2.25

Q.76 (a)

$\lambda = 10$ Per / hr

$\mu = 15$ Par/hr

$$\rho = \frac{10}{15} = \frac{2}{3}$$

length of queue $L_n = \frac{p^2}{p-1}$

$$= \frac{\left(\frac{2}{3}\right)^2}{1-\frac{2}{3}} = \frac{\frac{4}{9}}{\frac{1}{3}} = 1.33$$

Q.77 (a)

$t_p = 5$ days

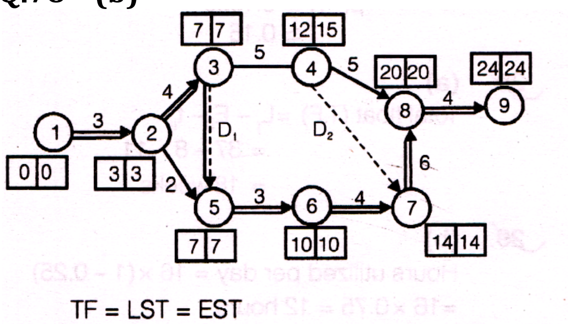
$t_m = 3$ days

$t_o = 1$ days

Expected time $= \frac{t_p + 4t_m + t_0}{6}$

$= \frac{5+12+1}{6}$ =3 days

Q.78 (b)



Activities	Duration	EST	LST	LET	Total Flood
1—2	3	0	0	3	0
2—3	4	3	3	7	0
3—4	5	7	10	15	3
2—5	2	3	5	7	2
4—8	5	12	15	20	3
5—6	3	7	7	10	0
6—7	4	10	10	14	0
7—8	6	14	14	20	0
8—9	4	20	20	24	0
4—7	0	12	14	14	2
3—5	0	7	7	7	0

Therefore critical path

1—2—3—5—6—7—8—9

and total project duration

= 3 + 4 + 3 + 4 + 6 + 4 = 24

Q.79 (c)

Standard deviation (a):

$$\sqrt{\text{Variance}} = \sqrt{4} = 2$$

$$Z = \frac{\overline{X} - \overline{X}}{\sigma} = \frac{36-36}{2} = 0$$

p(0) = 0.5

$\therefore$ the probability that the project will be completed in 36 days is 50%

Q.80 (a)

$$t_{expected} = \frac{t_o + 4t_m + t_p}{6}$$

$$= \frac{9 + (4\times15) + 12}{6} = 15$$

$$\text{S.D.} = \frac{(t_p - t_o)}{6} = \frac{21-9}{6} = 2$$

$$Z = \frac{\overline{X} - \overline{X}}{\sigma} = \frac{13-15}{2} = -1$$

p(-1) = 0.1586 $\simeq$0.16

Q.81 (a)

Total float (T.F.) = $L_1 - E_1 - T_I$

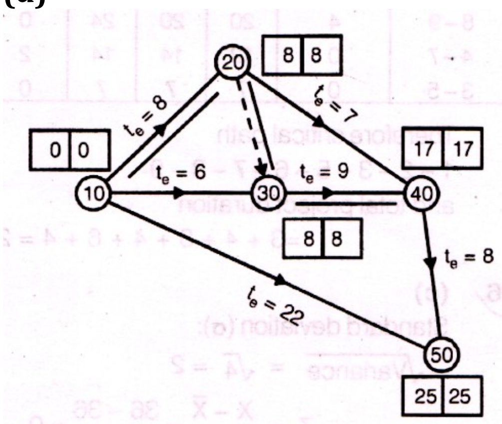
= 37 – 8 -11= 18 WEEKS

Q.82 (b)

Hours utilized per day = 16 x(1 — 0.25) =16 x0.75 = 12 hours hours utilized in a week = 12 x5 = 60 hours Number of items produced on a machine $= \frac{60\times60}{10} = 360$

therefore number of machines required $= \frac{1000}{360} = 277 \simeq 3$

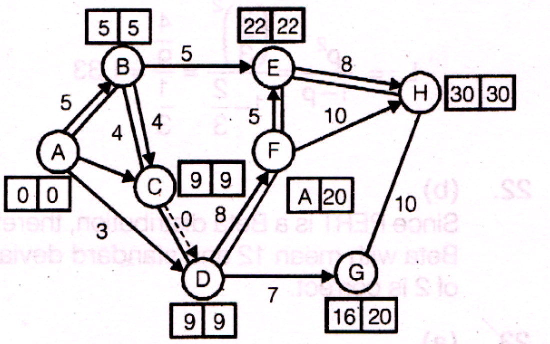
Q.83 (d)



Critical path is given by 10-20-30-40-50

$\therefore$ The earliest expected occurrence time (TE) for the event is 25.

Q.84 (c)



∴ project duration is 30 days

Q.85 (d)

$$BSP = \frac{\text{Fixed cost}}{\text{Selling price} - \text{Variance ocst}}$$

$$= \frac{800000}{200-40} = 5000\text{units}$$

Q.86 (b)

ROP=Safety cost + lead time consumption
= 200 + 500 = 700 units

Q.87 (d)

Total number of product sold by a company = 14000
Variable cost per unit = Rs. 15
Fixed cost = Rs. 47,000
Total profit = As. 23 000

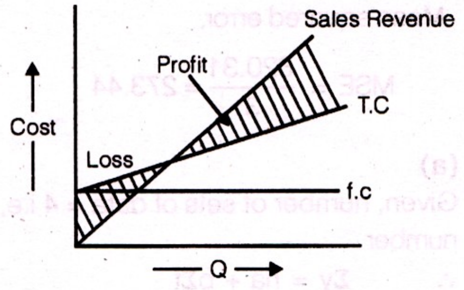
Fixed cost per unit $= \frac{47000}{14000} = \frac{47}{14}$

profit per unit $= \frac{23000}{14000} = \frac{23}{14}$

Unit product price = Fixed cost per unit + variable cost per unit + profit per unit

$$= 15 + \frac{47}{14} + \frac{23}{14} = 15 + \frac{70}{14} = 20$$

Q.88 (c)



Q.89 (d)

Q.90 (c)

$$MOS\% = \frac{\text{Sales} - \text{BEP sales}}{\text{Sales}}$$

$$\Rightarrow 0.4 = \frac{50000 - \text{BEP Sales}}{50000}$$

$$\Rightarrow \text{BEP Sales} = 30000$$

Q.91 (d)

Q.92 (a)

Examine the column for processing time on machine M1 &M2 & find the smallest value [mm (M1, M2)j and allocate on extreme side if M2 is smallest otherwise on extreme left side in sequence

M_1 M_2

C	A	B	F	E	D

Q.93 (d)

$$UCL = 36 + 3\sqrt{36} = 36 + 18 = 54$$

Q.94 (b)

Q.95 (d)

Q.96 (a)

Observed time = 0.75 mm
Rating = 110%
Total allowance = 3 + 2 = 5%
Normal time =0.75×1.10=0.825 min

$$\text{Standard time} = 0.825 + \frac{0.825 \times 5}{100}$$

= 0.866 min

Q.97 (c)

Performance rating = 120%
Observed time = 2 mm
Allowance = 10%
Normal time=2×1.2 = 2.4 mm

$$\text{Observed time} = 2.4 + \frac{2.5 \times 10}{100}$$

= 2.64 minutes

Q.98 (d)

Q.99 (c)

Q.100 (c)