GATEFLIX

STRUCTURAL ANALYSIS

For

CIVIL ENGINEERING

STRUCTURAL ANALYSIS

SYLLABUS

Structural Analysis: Statically determinate and indeterminate structures by force/ energy methods; Method of superposition; Analysis of trusses, arches, beams, cables and frames; Displacement methods: Slope deflection and moment distribution methods; Influence lines; Stiffness and flexibility methods of structural analysis.

ANALYSIS OF GATE PAPERS

Exam Year	1 Mark Ques.	2 Mark Ques.	Total
2003	2	-	12
2004	3	-	9
2005	2	-	10
2006	-	3	6
2007	1	4	9
2008	-	3	6
2009	-	1	2
2010	1	1	3
2011	-	-	-
2012	-	-	-
2013	1	3	7
2014 Set-1	2	2	6
2014 Set-2	1	1	3
2015 Set-1	1	1	3
2015 Set-2	1	1	3
2016 Set-1	1	1	3
2016 Set-2	1	1	3
2017 Set-1	1	3	7
2017 Set-2	1	1	3
2018 Set-1	1	3	7
2018 Set-2	1	1	3

CONTENTS

7. METHODS OF STRUCTURE ANALYSIS

8. MATRIX METHOD OF ANALYSIS

![GATEFLIX]

1 STATIC INDETERMINACY & KINEMATIC INDETERMINACY

1.1 Structure Analysis: It is the application of solid mechanics to predict the response (in terms of force and displacements) of given structure subjected to specified load.

1.2 Structure: It may be defined as an assemblage of load bearing elements in a construction.

1.3 Scope of structural Analysis: Determination of response quantities in terms of support reactions, internal forces (SF, BM, torsion and axial force) and displacements (deflections and rotations).

1.4 Idealization of structure:

(A) Line Elements (Skeletal): When one dimension of member is very large compare to other two dimensions, it is idealized as line element. Ex. Truss elements, beams, columns.

(B) Surface Elements (Area element): When two dimensions of member are very large compare to other two dimensions, it id idealized as line element. Ex. Slab, Shear wall, Shell.

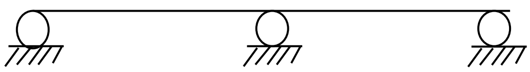
(C) Solid Elements: When all three dimensions of member are large, it is known as Solid elements. Ex. Massive foundation.

1.5 Structural stability: If structure deforms elastically and immediate elastic restrained is developed under the action of external load, structure is called stable.

NOTE: Structure is unstable externally if all unknown reactions are PARALLEL or CONCURRENT.

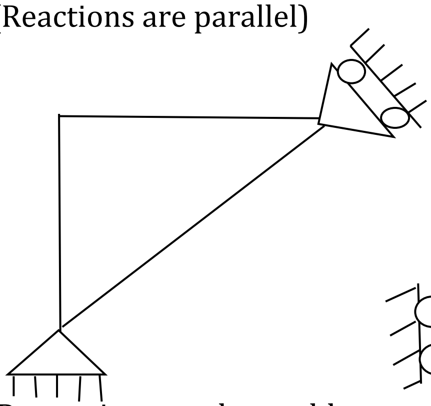
Determinate and unstable

(Reactions are parallel)



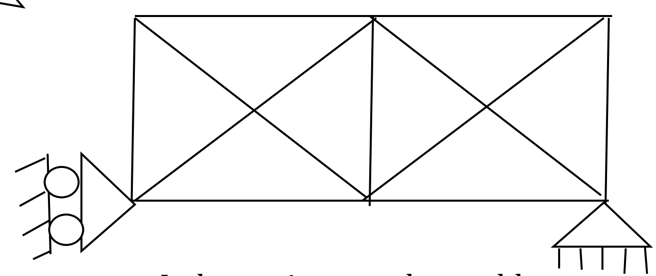
Indeterminate and unstable

(Reactions are parallel)



Determinate and unstable

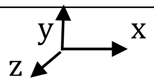
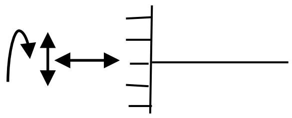
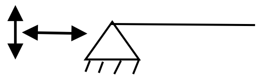
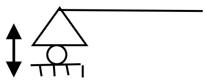
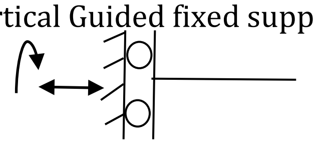
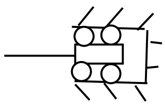
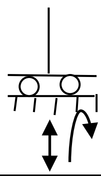
(Reactions are concurrent)



Indeterminate and unstable

(Reactions are concurrent)

1.6 Support Reactions:

Types of support 	**Reactions**	
	Plane structure	**Space structure**
Fixed support 	F_x, F_y, M_z	F_x, F_y, F_z, M_x, M_y, M_z
Hinged or Pinned support 	F_x, F_y	F_x, F_y, F_z
Roller support 	F_y	F_y
Vertical Guided fixed support 	F_x, M_z	F_x, M_x, M_y, M_z
Horizontal guided fixed support **OR** Horizontal shear release support  	F_y, M_z	F_y, M_x, M_y, M_z

1.7 Equation of Static Equilibrium

- In a 2-D structure or planer structure (in which all members and forces are in plane only) the equations of equilibrium are

$$\left.\begin{aligned}\sum F_X &= 0\\ \sum F_Y &= 0\\ \sum M &= 0\end{aligned}\right\} \text{ 3 nos.}$$

- In a. 3-D structures or space structures (in which members and forces are in 3rD). The equations of equilibrium are

$$\left.\begin{aligned}\sum F_X = 0 \quad \sum M_X = 0\\ \sum F_Y = 0 \quad \sum M_Y = 0\\ \sum F_Z = 0 \quad \sum M_Z = 0\end{aligned}\right\} \text{ 6 nos.}$$

1.8 Degree of Static Indeterminacy (D_S): If all reactions of structure cannot be determine by using static equilibrium equations alone, it is called statically indeterminate structure.

> **Note:** In this case additional equations needed which are obtained by relating the applied loads & reactions to the displacements or slopes known at different points on the structure. These equations are called **Compatibility equations.**

$_s$ = D_{se} + D_{si}

Where, D_{se} = External indeterminacy
D_{si} = internal indeterminacy

<u>Rigid jointed plane Frame</u>	**<u>Rigid jointed space Frame</u>**
$D_{se} = R - r - r'$ $D_{si} = 3C$ Where, R = No of unknown reactions r = No of equilibrium equations = 3 C = No of closed loops r' = No of extra equilibrium equations because of internal hinge or link **<u>OR</u>** **$D_s = 3m + R - 3j$** Where, m = No of members j = No of joints	$D_{se} = R - r$ $D_{si} = 6C$ Where, R = No of unknown reactions r = No of equilibrium equations = 6 C = No of closed loops **<u>OR</u>** **$D_s = 6m + R - 6j$** Where, m = No of members j = No of joints
<u>Pinned jointed plane Frame (Plane Truss)</u> $D_{se} = R - r$ $D_{si} = m + r - 2j$ Where, R = No of unknown reactions r = No of equilibrium equations = 3 **$D_s = m + R - 2j$**	**<u>Pinned jointed space Frame (Space Truss)</u>** $D_{se} = R - r$ $D_{si} = m + r - 3j$ Where, R = No of unknown reactions r = No of equilibrium equations = 6 **$D_s = m + R - 3j$**

<u>NOTE:</u>

<u>Internal hinge:</u> If internal hinge is provided anywhere in the structure, one part of structure cannot transmit moment from one part to other part which provides extra conditional equation, i.e, $\sum M = 0$.

r' = no of member connecting at internal hinge – 1

structure cannot transmit moment & horizontal force from one part to other part which

Rigid jointed plane Frame	Rigid jointed space Frame	Pinned jointed plane Frame	Pinned jointed space Frame	Stability & Indeterminancy
R – r < 0	R – r < 0	m + R < 2j	m + R < 2j	Unstable & determinate
R – r = 0	R – r = 0	m + R = 2j	m + R = 2j	Stable & determinate
R – r > 0	R – r > 0	m + R > 2j	m + R > 2j	Stable & indeterminate

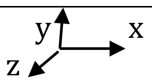
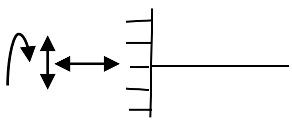
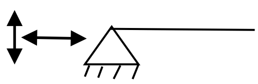
1.9 Kinematic Indeterminacy (Degree of freedom): The degree of kinematic indeterminacy may be defined as the total number of degree of freedom at various joints in skeletal structure.

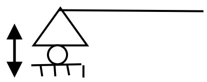
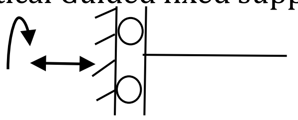
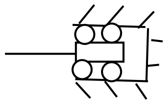
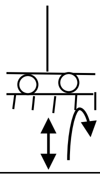
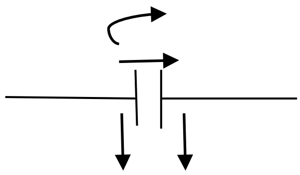
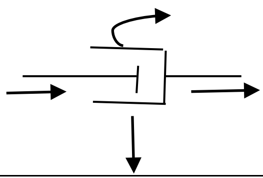
> **Note:** To determine no of unknown DOF, one need same no of compatibility equations.

Rigid jointed plane Frame	**Rigid jointed space Frame**
$D_k = 3j - R + r'$ Where, R = No of unknown reactions m = No of members j = No of joints r' = No of DOF because of internal hinge	**$D_k = 6j - R$** Where, R = No of unknown reactions m = No of members j = No of joints r' = No of DOF because of internal hinge
Pinned jointed plane Frame (Plane Truss) **$D_k = 2j - R$**	**Pinned jointed space Frame (Space Truss)** **$D_k = 3j - R$**

> **D_{knad} = KI neglecting axial deformation**
>
> **$= D_k - m$**

1.10 Degree of freedom of various joints:

Types of support 	DOF	Description
Fixed support 	0	0
Hinged or Pinned support 	1	**θ_1**

Roller support 	2	**δ_x, θ_1**
Vertical Guided fixed support 	1	**δ_y**
Horizontal guided fixed support **OR** Horizontal shear release support  	1	**δ_x**
	4	**δ_{y1}, δ_{y2}, δ_x, θ_1**
	4	**δ_{x1}, δ_{x2}, δ_y, θ_1**

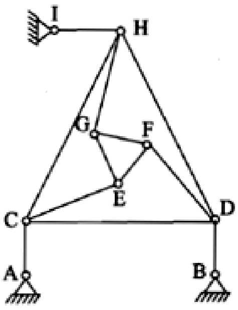
NOTE:

Internal hinge: If internal hinge is provided anywhere in the structure,

DOF = No of member connected at internal hinge + 2

GATE QUESTIONS

Q.1 The following two statements are made with reference to the planar truss shown below:



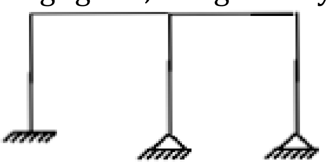
I. The truss is statically determinate
II. The truss is kinematically determinate

With reference to the above statements, which of the following applies?

a) Both statements are true
b) Both statements are false
c) II is true but I false
d) I is true but II is false

[GATE- 2000]

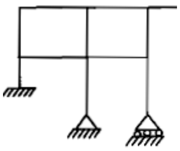
Q.2 The degree of static indeterminacy, Ns and the degree of kinematic indeterminacy, N_k for the plane frame shown below, assuming axial deformations to be negligible, are given by:



a) N_s = 6 and Nk = 11
b) N_s = 6 and Nk = 6
c) N_s = 4 and Nk = 6
d) N_s = 4 and Nk = 4

[GATE-2001]

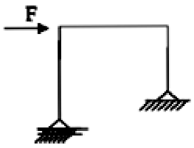
Q.3 For the plane frame with an overhang as shown below, assuming negligible axial deformation, the degree of static indeterminacy, d, and the degree of kinematic indeterminacy, k, are



a) d = 3 and k = 10
b) d = 3 and k = 13
c) d = 9 and k = 10
d) d = 9 and k =13

[GATE – 2004]

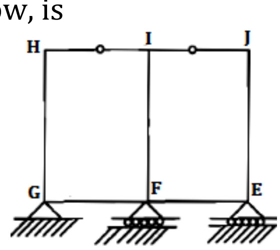
Q.4 Considering beam as axially rigid, the degree of freedom of a plane frame shown below is



a) 9 b) 8
c) 7 d) 6

[GATE – 2005]

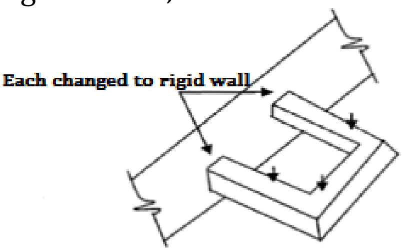
Q.5 The degree of static indeterminacy of the rigid frame having two internal hinges as shown in the figure below, is



a) 8 b) 7
c) 6 d) 5

[GATE-2008]

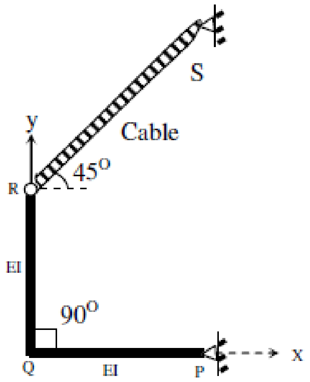
Q.6 The degree of static indeterminacy of a rigidly jointed frame in a horizontal plane and subjected to vertical load only, as shown in figure below, is



a) 6 b)4
c) 3 d) 1

[GATE-2009]

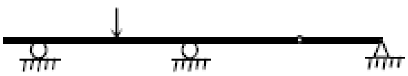
Q.7 The degree of static indeterminacy of a rigid jointed frame PQR supported as shown in the figure is



a) Zero b) One
c) Two d) Unstable

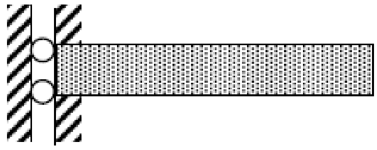
[GATE-2014]

Q.8 The static indeterminacy of the two-span continuous beam with an internal hinge, shown below, is ____



[GATE-2014]

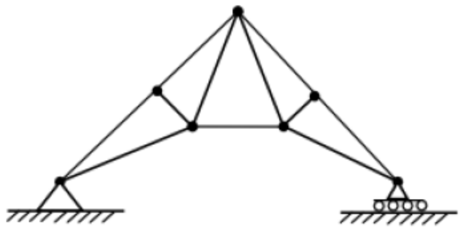
Q.9 A guided support as shown in the figure below is represented by three springs (horizontal, vertical and rotational) with stiffness k_x, k_y and k_θ respectively. The limiting values of k_x, k_y and k_θ are



a) $\infty, 0, \infty$ b) ∞, ∞, ∞
c) $0, \infty, \infty$ d) $\infty, \infty, 0$

[GATE-2015]

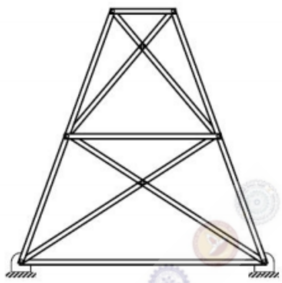
Q.10 The kinematic indeterminacy of the plane truss shown in the figure is



a) 11 b) 8
c) 3 d) 0

[GATE-2016]

Q.11 A planar tower shown in figure.



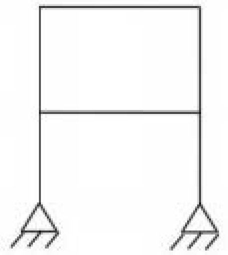
Consider the following statements for external and internal determinacies of truss.

(P) Externally determinate
(Q) External SI = 1
(R) External SI = 2
(S) Internally determinate.
(T) Internal SI = 1
(U) Internal SI = 2

Which one of following options is correct?

	P	Q	R	S	T	U
A	False	True	False	False	False	True
B	False	True	False	False	True	False
C	False	False	True	False	False	True
D	True	True	False	True	False	True

Q.12 Consider frame shown in figure. If axial and shear deformation in different members are assumed to be neglected, the reduction in kinematic indeterminacy would be



a) 5 b) 6
c) 7 d) 8

ANSWER KEY:

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
(d)	(c)	(d)	(b)	(d)	(a)	(a)	0	(a)
10	**11**	**11**	12					
(a)	(a)	(a)	(d)					

EXPLANATIONS

Q.1 (d)

$D_s = m + R - 2j$
$= 12 + 6 - 2(9)$
$= 0$
$D_k = 2j - R$
$= 2(9) - 6$
$= 12$

Hence the given truss is statically determinate and kinematically indeterminate.

Q.2 (c)

Ns = Ds = Dse + Dsi
Dse = 7 – 3 = 4
Dsi = 0
∴ Ns = 4
$D_k = 3j - R$
$= 3(6) - 7 = 11$
$Nk = D_{knad} = D_k - m = 11 - 5 = 6$

Q.3 (d)

$D_{se} = R - r = 6 - 3 = 3$
$D_{si} = 3C = 3(2) = 6$
$D_s = 3 + 6 = 9$
$D_k = 3j - R$
$= 3(10) - 6 = 24$
$Nk = D_{knad} = D_k - m = 24 - 11 = 13$

Q.4 (b)

$D_k = 3j - R$
$= 3(4) - 3 = 9$
$D_{knad} = D_k - m = 9 - 1 = 8$
(beam is only axially rigid)

Q.5 (d)

$D_{se} = R - r - r' = 4 - 3 - 2 = -1$
$D_{si} = 3C = 3(2) = 6$
$D_s = -1 + 6 = 5$

Q.6 (a)

Space frame:
$D_{se} = R - r = 12 - 6 = 6$
$D_{si} = 3C = 0$
$D_s = 6$

Q.7 (a)

$D_{se} = R - r - r' = 4 - 3 - 1 = 0$
$D_{si} = 3C = 0$
$D_s = 0$

Q.8.

$D_{se} = R - r - r' = 4 - 3 - 1 = 0$
$D_{si} = 3C = 0$
$D_s = 0$

Q.9 (a)

As rotation and horizontal deflection in zero as per given figure. Therefore its stiffness is '∞' as deflection = 0.

$$\text{Stiffness} = \frac{\text{Force}}{\text{deflection}}$$

and stiffness is zero in y direction

Q.10 (a)

$D_k = 2j - R$
$= 2(7) - 3$
$= 11$

Q.11 (a)

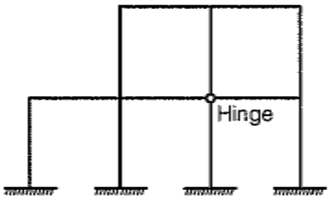
$D_{se} = R - r = 4 - 3 = 1$
$D_{si} = m + r - 2j = 15 + 3 - 2(8) = 2$

Q.12 (d)

$D_k = 3j - R$
$= 3(6) - 4 = 14$
$D_{knad} = D_k - m = 14 - 6 = 8$

ASSIGNMENT

Q.1 Total degree of indeterminacy (both internal and external) of the plane frame shown in figure is



a) 10 b) 12
c) 12 d) 15

Q.2 If there are m unknown member forces, r unknown reaction components and j number of joins, then the degree of static indeterminacy of a pin-joined plane frame is given by
a) m + r + 2j b) m – r + 2j
c) m + r – 2j d) m + r – 3j

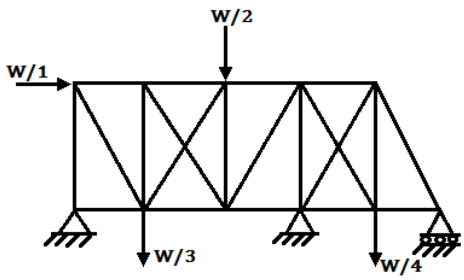
Q.3 Degree of kinematic indeterminacy of a pin joined plane frame is given by
a) 2j - r b) j – 2r
c) 3j – r d) None of these

Q.4 A pin-joined plane frame is unstable if
a) (m + r) < 2j b) m + r = 2j
c) (m + r) > 2j d) None of these

Q.5 The degree of static indeterminacy of a pin-joined space frame is given by
a) m + r – 2j b) m + r – 3j
c) 3m + r – 3j d) m + r + 3j

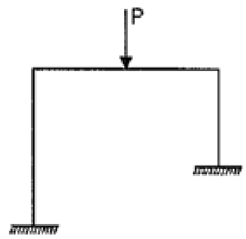
Q.6 The degree of kinematic indeterminacy of a pin-joined space frame is given by
a) 2j – r b) 3j - r
c) j – 3r d) j – 3r

Q.7 The degree of static indeterminacy of the pin-joined plane frame shown in figure is



a) 1 b) 2
c) 3 d) 4

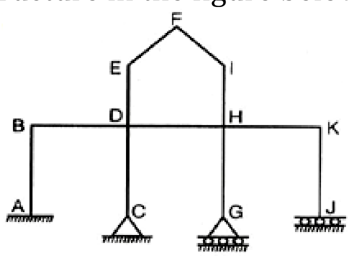
Q.8 The portal frame as shown in the given frame is statically indeterminate to the



a) first degree b) second degree
c) third degree d) None of these

Q.9 A perfect plane frame having n number of members and j number of joints should satisfy the relation
a) n < 2j - 3 b) n = 2j – 3
c) n > 2j - 3 d) n = 3 – 2j

Q.10 Neglecting axial deformation, the kinematic indeterminacy of the structure in the figure below is



a) 12 b) 14
c) 20 d) 22

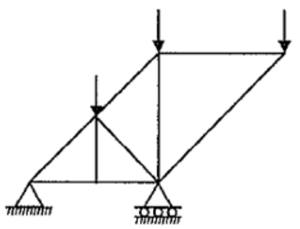
Q.11 Consider the following statements:

1) The displacement method is more useful when degree of kinematic indeterminacy is greater than the degree of static indeterminacy greater than the degree of static indeterminacy
2) The displacement method is more useful when degree of kinematic indeterminacy is less than the degree of static indeterminacy. Indeterminacy is less than the degree of static indeterminacy
3) The force method is more useful when degree of static indeterminacy is greater than the degree of kinematic indeterminacy.
4) The force method is more useful when degree of static indeterminacy is less than the degree of kinematic indeterminacy.

Which of the statements are correct?

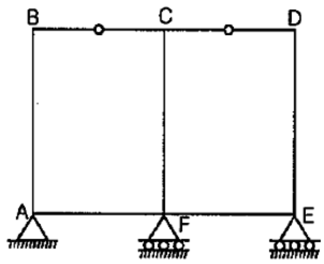
a) 1 and 3 b) 2 and 3
c) 1 and 4 d) 2 and 4

Q.12 The pin-joined frame shown in the figure is



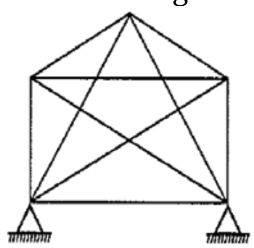
a) a perfect image
b) a redundant frame
c) a deficient frame
d) None of the above

Q.13 The degree of static indeterminacy of the rigid frame having two internal hinges as shown in the figure is



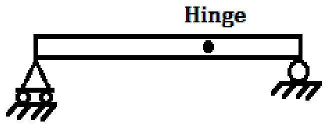
a) 8 b) 7
c) 6 d) 5

Q.14 What is the degree of static indeterminacy of the plane structure as shown in the figure below?



a) 3 b) 4
c) 5 d) 6

Q.15 The prismatic is show in the figure below.



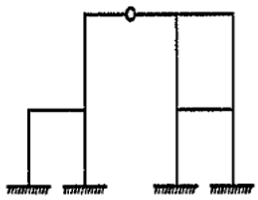
Consider the following statements:

1) The structure is unstable.
2) The bending moment is zero at supports and internal hinge.
3) It is a mechanism.
4) It is statically indeterminate.

Which of these statements are correct?

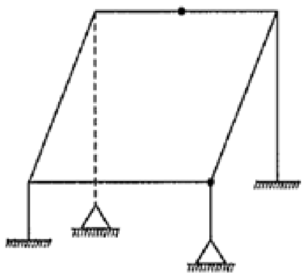
a) 1, 2, 3 and 4 b) 1, 2 and 3
c) 1 and 2 d) 3 and 4

Q.16 What is the statical indeterminacy for the frame shown below?



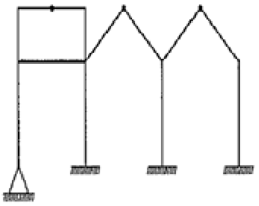
a) 12 b) 15
c) 11 d 14

Q.17 The statical indeterminacy for the given 3D frame is



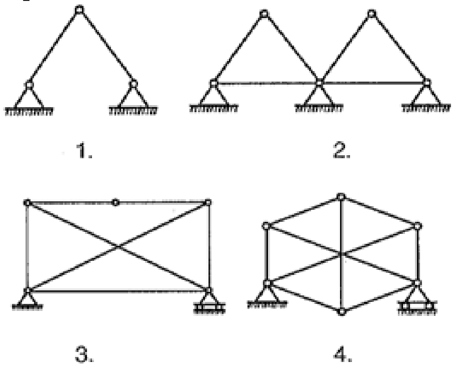
a) 8 b) 6
c) 9 d) 12

Q.18 The statical indeterminacy for the given 2D frame is



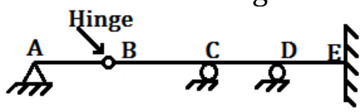
a) 8 b) 6
c) 9 d) 11

Q.19 Consider the following pin-joined plane frames



Which of these frames are stable?
a) 1, 2 and 3 b) both 3 and 4
c) 2, 3 and 4 d) both 2 and 4

Q.20 The kinetic indeterminacy of the beam as shown in figure is



a) 5 b) 9
c) 14 d) 15

ANSWER KEY:

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
(c)	(c)	(a)	(a)	(b)	(b)	(d)	(c)	(b)
10	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
(b)	(d)	(c)	(d)	(b)	(b)	(c)	(c)	(a)
19	**20**							
(c)	(b)							

EXPLANATIONS

Q.1 (c)
$D_{se} = R - r - r' = 12 - 3 - 3 = 6$
$D_{si} = 3C = 3(2) = 6$
$\mathbf{D_s} = 6 + 6 = \mathbf{12}$
(Note: r'=No of member connected at internal hinge – 1= 4-1 = 3)

Q.2 (c)

Q.3 (a)

Q.4 (a)

Q.5 (b)

Q.6 (b)

Q.7 (d)

No of members, m = 21
Number of joints, j = 11
$D_{se} = m + R - 2j$
$D_{se} = 21 + 5 - 2*11$
$D_{se} = 4$

Q.8 (c)

$D_{se} = R - r - r' = 6 - 3 - 0 = 3$
$D_{si} = 3C = 3(0) = 0$
$\mathbf{D_s} = 3 + 0 = \mathbf{3}$

Q.9 (b)
A perfect plane frame means a determinate structure, so
$n - (2j - 3) = 0$
$\therefore\ n = 2j - 3$

Q.10 (b)
$D_k = 3j - R$
$= 3(11) - 8 = 25$
$D_{knad} = D_k - m = 25 - 11 = \mathbf{14}$

Q.11 (d)
Force method is useful when
$D_s < D_K$
Displacement method is useful when $D_k < D_s$

Q.12 (c)
This is pinned jointed space frame
Degree of indeterminacy
No of members, m =9
Number of joints, j = 6
$D_{se} = m + R - 3j$
$D_{se} = 9 + 4 - 3*6$
$D_{se} = -5$
Since the degree of indeterminacy is Negative so it is a deficient frame.

Q.13 (d)
$D_{se} = R - r - r' = 4 - 3 - 2 = -1$
$D_{si} = 3C = 3(2) = 6$
$\mathbf{D_s} = -1 + 6 = \mathbf{5}$
(Note: r'=No of member connected at internal hinge – 1= 2-1 = 1
There are 2 similar internal hinge)

Q.14 (b)
For plane truss degree of indeterminacy
$D_s = m + r_e - 2j$
$r_e = 4;\ m = 10\ ; j = 5$
$D_s = 4 + 10 - 2 \times 5 = 4$

Q.15 (b)
The degree of indeterminacy $= 3 - 1 - 1 = -1$
So structure is deficient and unstable. It is a mechanism.

Q.16 (c)
$D_{se} = R - r - r' = 12 - 3 - 1 = 8$
$D_{si} = 3C = 3(1) = 3$
$\mathbf{D_s} = 8 + 3 = \mathbf{11}$
(Note: r'=No of member connected at internal hinge – 1= 2-1 = 1)

Q.17 (c)

D_{se} = R – r – r' = 18 – 6 - 9= 3

D_{si} = 6C = 6(1) = 6

D_s = 3 + 6 = **9**

(Note:r'=3(No of member connected at internal hinge – 1)

r'_1= 3(2-1) = 3

r'_2= 3(3-1) = 6)

Q.18 (a)

D_{se} = R – r – r' = 11 –3 - 3= 5

D_{si} = 3C = 3(1) = 3

D_s = 5 + 3 = **8**

(Note:r'=(No of member connected at internal hinge – 1= (2-1) =1

There are 3 similar internal hinge)

Q.19 (c)

Q.20 (b)

The kinematic indeterminacy of the beam is given by

$D_k = 3j - R + r_r$

Now, j = 5

$R = 7$

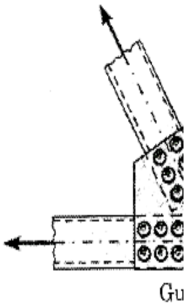
$r_r = 1$

$\therefore\ Dk = 3 \times 5 - 7 + 1 = 9$

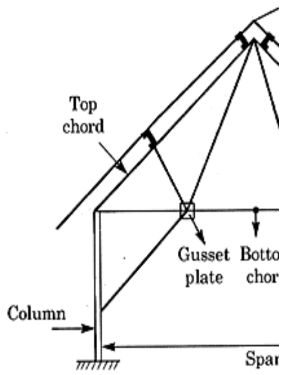
2 ANALYSIS OF TRUSS

2.1 Introduction:

A truss is a structure composed of slender members joined together at end points by bolting/ Riveting or welding. Ends of the members are joined to a common plate called gusset plate.



A roof truss and its component s



A typical Roof Truss

2.2 Assumptions for Design of Truss Members and Connection :

1. The members are joined together by smooth pins.
2. All loadings are applied to joints.
3. Self-weight of the members is negligible.
4. All members are straight.

2.3 Method of Analysis: (Statically Determinate and Stable Trusses)

There are two methods of analysis for statically determinate and stable trusses. They are:

1. Method of Joint
2. Method of Section
3. Tension coefficient method

2.3.1 Method of Joint:

In a planer-truss, at every joint there are two conditions of equilibrium

$$\sum F_x = 0$$

$$\sum F_y = 0$$

Since all the members at a joint are assumed to pass through a single point, moment about the joint will always be zero. Hence $\sum M = 0$ will not be of any consequence.

Sign Convention: Tension (+) ve, Compression (-) ve.

Analysis should start at joint having at least one known force and at most two unknown forces. For example in the following figure, if we take joint A then free body diagram of joint A can be drawn as in Figure (ii).

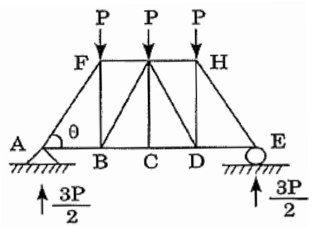
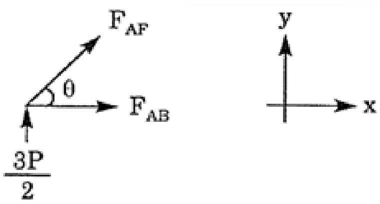


Fig. (i)



Now from the equilibrium of forces we have

$$\sum P_x = 0 \Rightarrow F_{AB} + F_{AF}\cos\theta = 0$$

$$\sum F_Y = 0 \Rightarrow F_{AF}\sin\theta + \left(\frac{3P}{2}\right) = 0$$

Thus two unknowns can be found out from two equilibrium equations. Likewise we can proceed to other joints and find out member forces by using equilibrium equation if no.

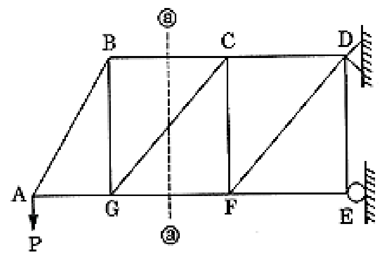
of unknown forces at the joint is at most two in number.

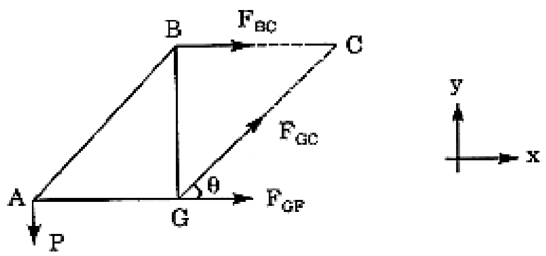
2.3.2 Method of Section

A section is cut through the truss such that its cuts at the maximum of three members in which forces are unknown.

Thus, if truss is cut into two parts at section (a)-(a), three equilibrium equations can be written for each part. If max no. of unknown forces in cut members are three, they can be found out.

Note: Hence max no. of member (in which forces are unknown) cut should be at most three.





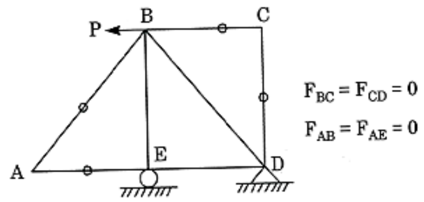
1. F_{gf} = Found out by taking moment about C of all forces in the cut part of truss. As F_{BC} and F_{GC} are passing through point 'C', there moment about 'O will be zero. Hence only one unknown i.e. F_{GF} remains, which can be found out.
2. Similarly, F_{BC} is found by taking moments about G or from equilibrium equations like

$$\sum F_X = 0 \text{ i.e., } F_{BC} + F_{GC}\cos\theta + F_{GF} = 0$$
$$\sum F_Y = 0 \text{ i.e., } F_{GC}\sin\theta = P$$

2.4 Zero Force Members:

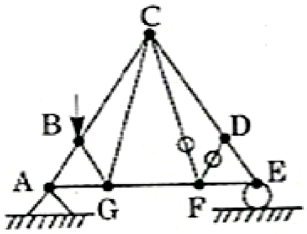
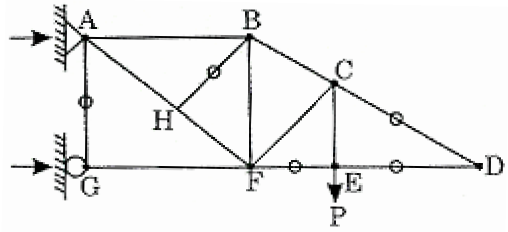
1. If only two non-collinear members exist at a truss joint and no external force or support reaction is applied to the joint, the members must be zero force members.

For example in the figure shown BC and CD are non-collinear members and no force exist at joint C, hence BC and CD are zero force members. Similarly, AB and AE are zero force members since A is not carrying any force and AB and AE are non-collinear.

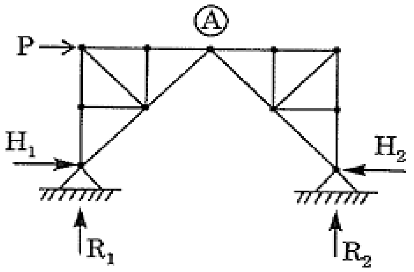


Note: These members (CB and CD) i.e. zero force members, are provided for stability of truss during construction and to provide support in case the applied loading changes.

2. If three members join at a point and out of them, two are collinear and also no external load acts at joint, the third member is a zero force member.

Example – 1	Example –2
 $F_{DF} = 0$ Since CD and DE are collinear and no load acts at D] $F_{FC} = 0$ Since GF and FE are collinear, Force in DF = 0 and no load acts at joint F]	 On similar lines as discussed above, $F_{BH} = 0,\ F_{DC} = F_{DE} = 0$ Also, since, F_{BH} = 0 and from equilibrium of joint E. $F_{ED} = F_{EF},\ F_{EF} = 0$ $F_{AG} = 0$ because at G no vertical force exist to balance force in GA.

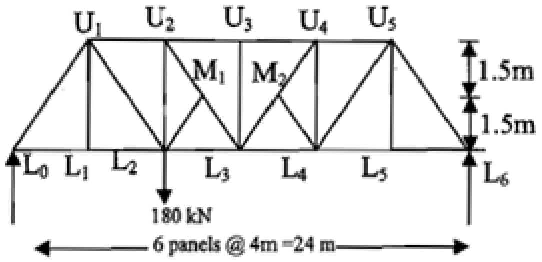
2.5 Truss with Internal Hinge:

	4 = no. of unknown support reactions $R_1, H_1 R_2, H_2$ 3 = no. of eq. of static equilibrium External indeterminacy $= 4 - 3 = 1$ Internal indeterminacy $= [m - (2J - 3)]$ $= [18 - (2 \text{ x } 11 - 3)] = -1$ **[m** = no. of members, **J** = no. of joints] **Overall indeterminacy = internal indeterminacy + external indeterminacy** = -1 + 1 = 0 The truss is overall determinate.

Note: In such cases, Hinge 'A' will provide additional condition for calculating external reactions i.e. BM at A = 0 i.e. moment of all forces either to the left of the section or to the right of the section is zero.

GATE QUESTIONS

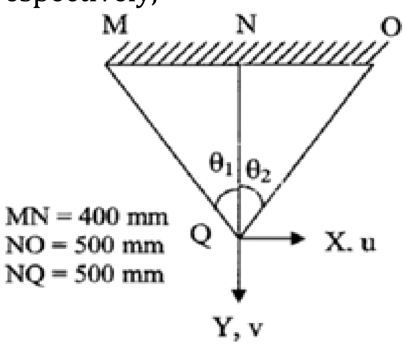
Q.1 A truss, as shown in the figure, is carrying 180 kN load at node L2. The force in the diagonal member M2U4 will be



a) 100 kN tension
b)100kN compression
c)80 kN tension
d)80 kN compression

[Gate–2003]

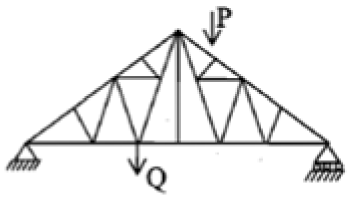
Q.2 In a redundant joint model, three bar members are pin connected at Q as shown in the figure. Under some load placed at Q, the elongation of the members MQ and OQ are found to be 48 mm and 35 mm respectively. Then the horizontal displacement `u' and the vertical displacement 'v' of the node Q, in mm, will be respectively,



a)-6.64and56.14 b) 6.64and56.14
c) 0.0 and 59.14 d) 59.41 and 0.0

[GATE-2003]

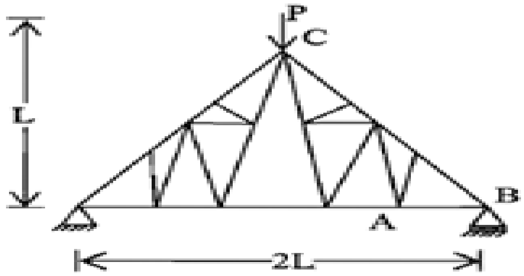
Q.3 For the plane truss shown in the figure, the number of zero force members for the given loading is



a) 4 b) 8
c) 11 d) 13

[GATE–2004]

Common Data for Q.4 and Q.5:
A truss is shown in the figure. Members are of equal cross section A and same modulus of elasticity E. A vertical force P is applied at point C.



Q.4 Force in the member AB of the truss is

a) $\frac{P}{\sqrt{2}}$ b) $\frac{P}{\sqrt{3}}$

c) $\frac{P}{2}$ d) P

[GATE-2005]

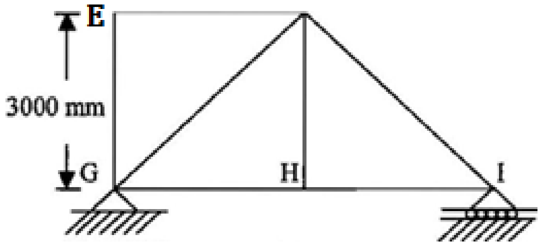
Q.5 Deflection of the point C is

a) $\left(\frac{2\sqrt{2}+1}{2}\right)\frac{PL}{EA}$ b) $\sqrt{2}\frac{PL}{EA}$

c) $\left(2\sqrt{2}+1\right)\frac{PL}{EA}$ d) $\left(\sqrt{2}+1\right)\frac{PL}{EA}$

[GATE-2005]

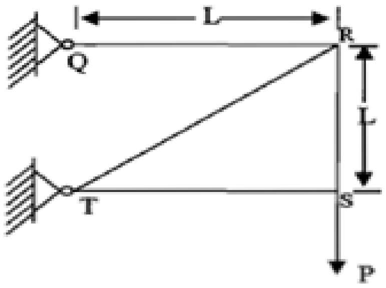
Q.6 The members EJ and IJ of a steel truss shown in the figure below are subjected to a temperature rise of 30°C. The coefficient of thermal expansion of steel is 0.000012 per °C per unit length. The displacement (mm) of joint E relative to joint H along the direction HE of the truss, is



a) 0.255 b)0.58
c) 0.764 d) 1.026

[GATE-2008]

Q.7 For the truss shown in the figure, the force in the member QR is

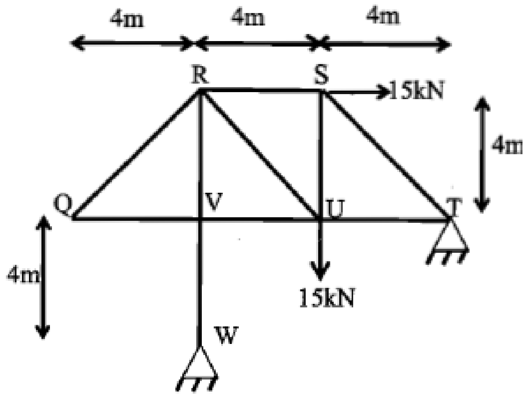


a) Zero b) $\frac{p}{\sqrt{2}}$

c) p d) $\sqrt{2}$

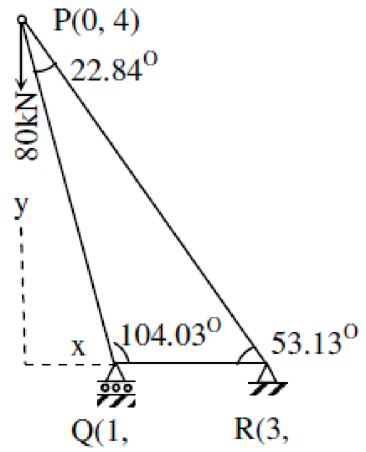
[GATE–2010]

Q.8 The pin-jointed 2-D truss is loaded with a horizontal force of 15 kN at joint S and another 15 kN vertical force at joint U, as shown. Find the force in member RS (in kN) and report your answer taking tension as positive & compression as negative.



[GATE-2013]

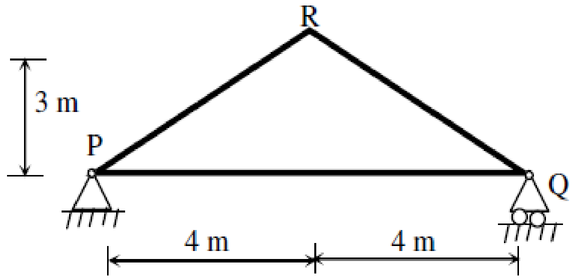
Q.9 Mathematical idealization of a crane has three bars with their vertices arranged as shown in the figure with a load of 80 kN hanging vertically. The coordinates of the vertices are given in parentheses. The force in the member QR, F_{QR} will be



a) 30 kN Compressive
b) 30 kN Tensile
c) 50 kN Compressive
d) 50 kN Tensile

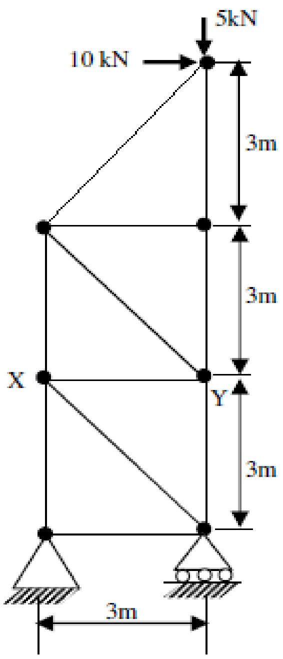
[GATE-2014]

Q.10 For the truss shown below, the member PQ is short by 3 mm. The magnitude of vertical displacement of joint R (in mm) is ___________



[GATE-2014]

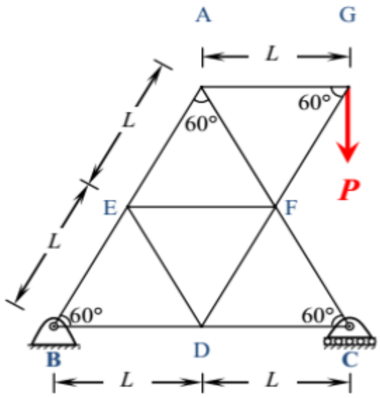
Q.11 For the 2D truss with the applied loads shown below, the strain energy in the member XY is____ kN-m. For member XY, assume AE = 30 kN, where A is cross-section area and E is the modulus of elasticity.



[GATE-2015]

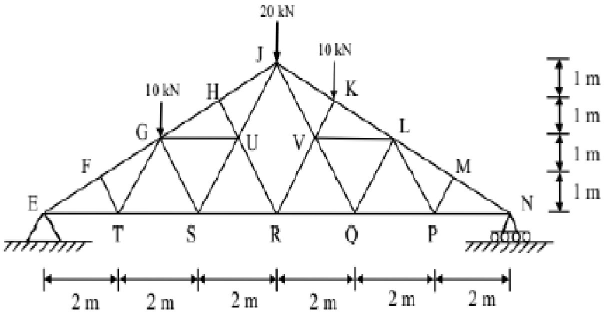
Q.12 Consider the plane truss with load P as shown in the figure. Let the horizontal and vertical reactions at the joint B be H_B and reaction at the joint C.
Which one of the following sets gives the correct values of VB, HB and VC?

a) $V_B = 0$; $H_B = 0$; $V_C = P$
b) $V_B = P/2$; $H_B = 0$; $V_C = P/2$
c) $V_B = P/2$; $H_B = P$ (sin 60°); $V_C = P/2$
d) $V_B = P$; $H_B = P$ (cos 60°); $V_C = 0$



[GATE-2016]

Q.13 A plane truss with applied loads is shown in the figure.

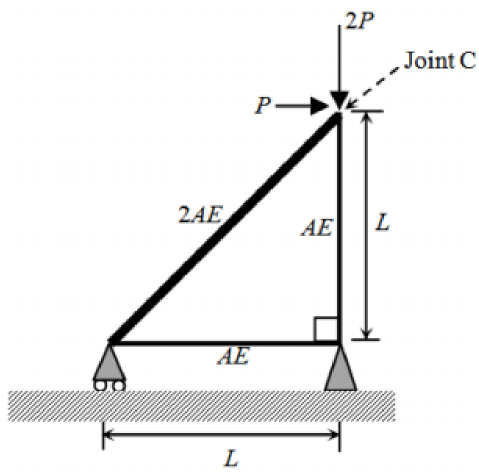


The members which do not carry any force are

a) FT, TG, HU, MP, PL
b) ET, GS, UR, VR, QL
c) FT, GS, HU, MP, QL
d) MP, PL, HU, FT, UR

[GATE-2016]

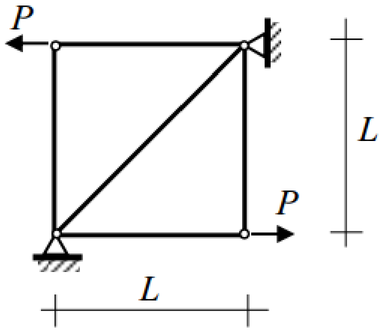
Q-14 Consider the deformable pinned jointed truss as shown in figure.



Given that E = 2 * 10^{11} N/m^2 , A = 10 mm^2 , L=1 m, P = 1 kN. The horizontal displacement at joint C (in mm up to one decimal) is_______

[GATE-2018]

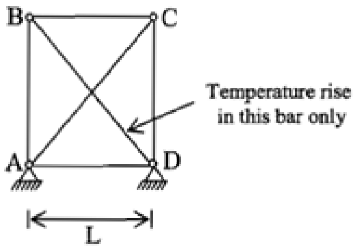
Q-15 Consider the member of planer truss having same cross sectional area (A) and Modulus of elasticity (E) .



For load shown on truss, the statement that represent the nature of forces in the member of truss is:

(A) There are 3 members are in tension and 2 members are in compression
(B) There are 2 members are in tension and 2 members are in compression, 1 zero force member
(C) There are 2 members are in tension and 1 member is in compression, 2 zero force member
(D) There are 2 members are in tension and, 3 zero force members

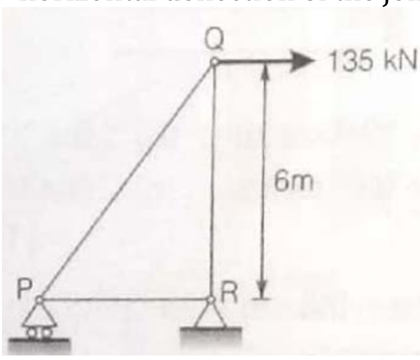
Q-16 Identify the FALSE statement from the following, pertaining to the effects due to a temperature rise ΔT of the bar BD alone in the plane truss shown below:



a) No reactions develop at supports A and D.
b) The bar BD will be subject to a compressive force.
c) The bar AC will be subject to a compressive force.
d) The bar BC will be subject to a tensile force.

[GATE-2001]

Q-17 The right triangular truss is made of members having equal cross sectional area of 1550mm^2 and Young's of 2x10^5 MPa. The horizontal deflection of the joint Q is



a) 2.47 mm b) 10.25 m
c) 14.31 mm d) 15.68 mm

[GATE-2007]

ANSWER KEY:

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
A	B	B	C	A	C	C	7.5
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
A	2	5	A	A	2.7	D	B
17							
D							

EXPLANATIONS

Q.1 (a)

The force in member

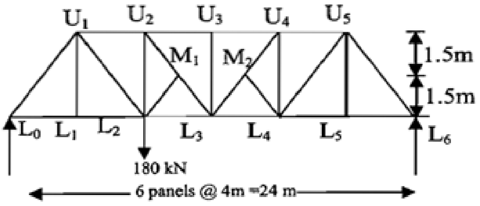
$M_2L_4 = 0$.

Hence remove this member for the sake of analysis.

Reactions,

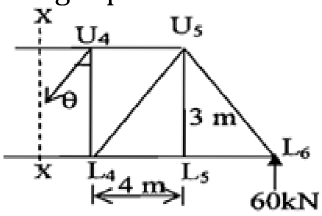
R_{L0}= 120 kN

R_{L6}= 60 kN



Use method of sections.

Apply a section 'XX' as shown. Consider right part of section



Apply ΣV = 0 for right part of section.

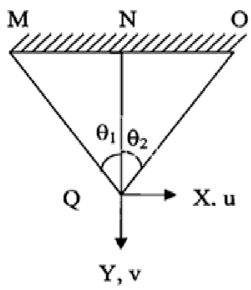
$$F_{U4}M_2 \cos\theta = 60$$

$$F_{U4}M_2 = \frac{60}{\cos\theta}$$

$$= \frac{60*5}{3} = 100\text{kN (Tension)}$$

(Force Pulling the Joint is Tension)

Q.2 (b)



$$\sin\theta_1 = \frac{4}{\sqrt{41}}, \cos\theta_1 = \frac{5}{\sqrt{41}}$$

$$\sin\theta_2 = \frac{1}{\sqrt{2}}, \cos\theta_2 = \frac{1}{\sqrt{2}}$$

Let the horizontal displacement of joint Q is 'u' and Vertical displacement is 'v' By resolving the displacement components in,

Along member 'OQ'

$U \sin\theta_2 + V \cos\theta_2 = 35$ _ _ _ _ (1)

Along member 'MQ'

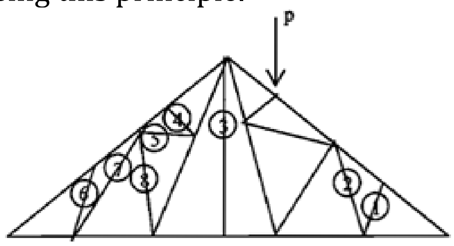
$U \sin\theta_1 + V \cos\theta_1 = 48$ _ _ _ _ (2)

Solving equations 1&2, we get,

u = 6.64 mm & v = 56.14 mm

Q.3 (b)

If at any joint, three forces act and if two of them are in the same line, then the third force must be zero. Using this principle.

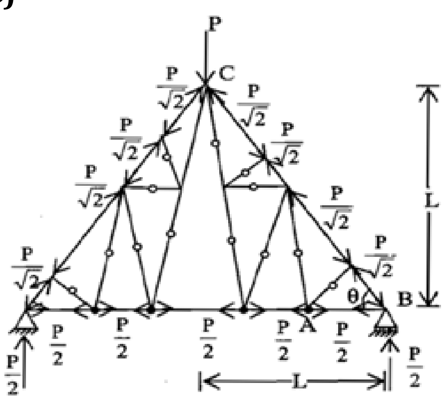


Dsc = 3 – 3 = 0

Dsi = 9 – 12 = 3 = 0

Total 8 members having zero forces. Pl follow the order of members as shown in fig for analysis.

Q.4 (c)



∟ABC = 45^0

Vertical reaction at support 'B' is

$\frac{P}{2}$ (upward)

Resolving vertically

At B, $\sin 45^0 = \frac{P}{2}$

$\therefore F_{BD} = \frac{P}{2}\sqrt{2} = \frac{P}{\sqrt{2}}$ (comp)

Resolving horizontally at B,

$F_{AB} = F_{BD}\cos 45^0$

$\frac{P}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{P}{2}$

Q.5 (a)

We know that if three forces act at a joint, and if two of them are in the same line, then the third force is zero. According to this rule, various members having zero force are marked in the figure.

Forces in other members are also shown in figure (F) Now remove the external load place a unit vertical load at 'C' Now forces in various members say

(K)

Now δ_V at 'C'

$$= \sum \frac{FKl}{AE}$$

$$= \frac{8\left(\frac{P}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)\left(\frac{L\sqrt{2}}{4}\right)}{AE} + \frac{4\left(\frac{P}{2}\right)\left(\frac{1}{2}\right)\left(\frac{L}{3}\right)}{AE} + \frac{\left(\frac{P}{2}\right)\left(\frac{1}{2}\right)\left(\frac{2L}{3}\right)}{AE}$$

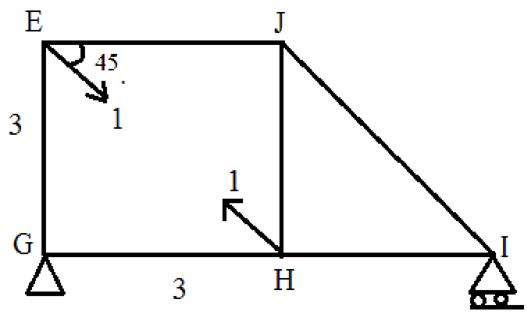
$$= \frac{PL\sqrt{2}}{AE} + \frac{PL}{2AE} = \frac{PL}{AE}\left(\sqrt{2} + \frac{1}{2}\right)$$

$$= \frac{PL}{AE}\left(\frac{2\sqrt{2}+1}{2}\right)$$

Q.6 (C)

As we need to calculate relative displacement between E and H, remove member between them and apply unit load.

Now as temperature is raised in EJ and JI, so calculate member force in these two members only,



$F_{EJ} = \frac{1}{\sqrt{2}} = 0.707$

$F_{JI} = 0$

Relative displacement

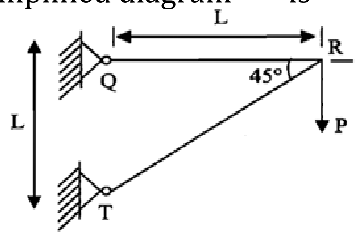
$= \sum FL\alpha T$

$=0.707*3000*0.000012*30$

$=0.764$ mm

Q.7 (c)

Analyzing at joint 'S' member SR and force 'P' are in the same line. Hence the third force 'TS' = 0. Further force in member RS = P. Now simplified diagram is



Apply $\sum V = 0$ at 'R'

$F_{RT} \sin 45° = P$

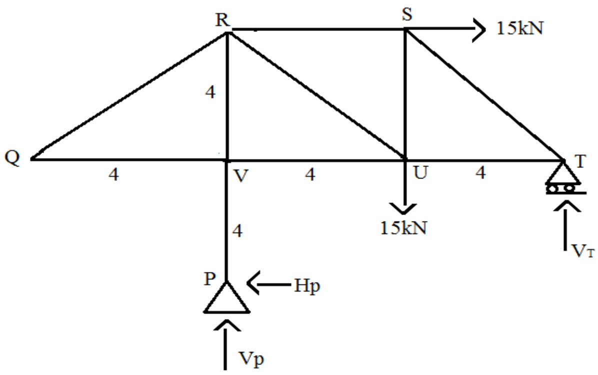
$\therefore F_{RT} = P\sqrt{2}$ (comp)

Apply $\sum H = 0$ at R

$F_{QR} = F_{RT} \cos 45°$

$= P\sqrt{2} \times 1/\sqrt{2} = = P$ (Tensile)

Q.8 (7.5)



$\sum M_P = 0$

$15*8 + 15*4 - 8*V_T = 0$

$V_T = 22.5$ kN

$\sum V = 0$

$V_P + V_T = 15$

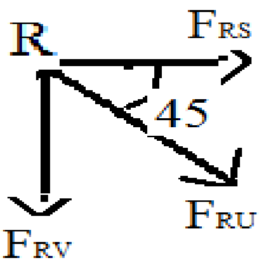
$V_P = -7.5$ kN

$F_{QR} = 0$ and $F_{QV} = 0$

<u>Joint V</u>

$F_{RV} = -V_P = 7.5$ kN

<u>Joint R</u>



$\sum V = 0$

$-F_{RV} - F_{RU} \sin 45 = 0$

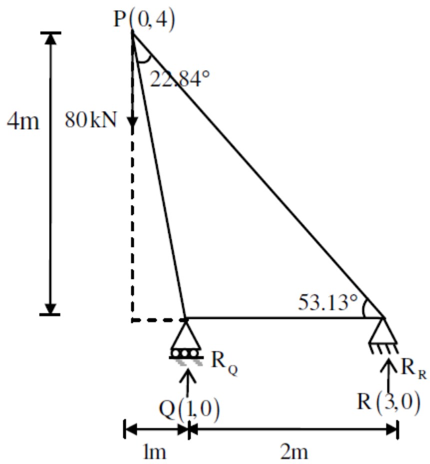
$F_{RV} = -7.5\sqrt{2}$ kN

$\sum H = 0$

$F_{RS} + F_{RU} \sin 45 = 0$

F_{RS} = 7.5 kN

Q.9 (a)



$\Sigma F_v = 0 \Rightarrow R_Q + R_R = 80$ ……..(i)

Taking moment about Q = 0

$\Rightarrow 80 \times 1 + R_R \times 2 = 0$

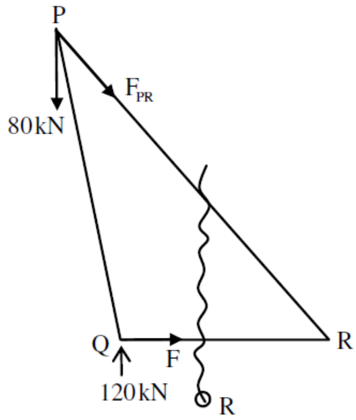
$\Rightarrow R_R = -40$ kN

From (i), we get $R_Q = 120$ kN

Taking moment about P = 0

$F_{QR} \times 4 + 120 \times 1 = 0$

$\Rightarrow$ $F_{QR}=-30$kN (-) mean compressive



Q.10 (2)

Since deflection at R is required. So, let us apply a virtual unit load at point R in upwards direction.

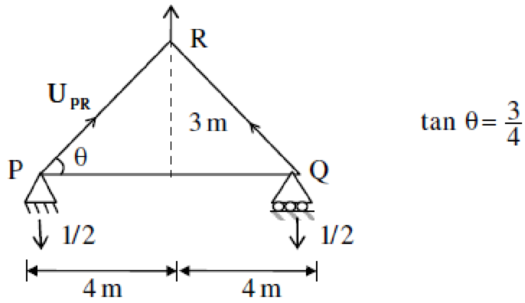
At point P $\Sigma F_v = 0 \Rightarrow U_{PR} \sin\theta = \frac{1}{2}$

$\Sigma F_H = 0 \Rightarrow U_{PR} \cos\theta + U_{PQ} = 0$

$\Rightarrow U_{PQ} = -U_{PR} \cos\theta$

$= -\frac{1}{2\sin\theta}.\cos\theta$

$= -\frac{1}{2} \times \frac{1}{\tan\theta} = \frac{1}{2} \times \frac{4}{3} = -2/3$

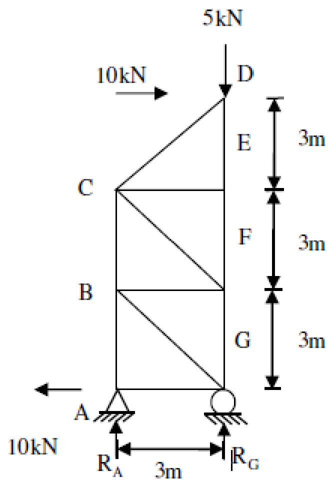


$\delta_R = U_{PQ} \times \lambda_{PQ}$

$= \frac{-2}{3} \times (-3)$ [∴ PQ is 3mm short]=2mm

So, deflection at R = 2mm (upwards)

Q.11 (5)

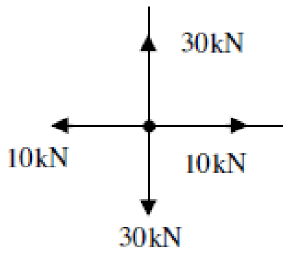


$$R_A \times 3 + 10 \times 9 = 0$$

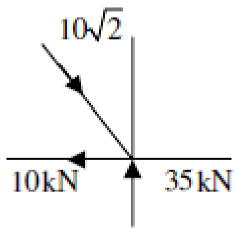
$\Rightarrow R_A$=-30 kN

R_G = 35 kN

Taking joint A



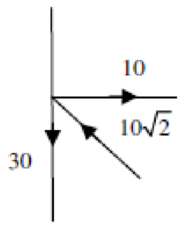
Joint G



Joint B

F_x = 10kN

$$U = \frac{F^2 \times L}{2A_E} = \frac{10 \times 10 \times 3}{2 \times 30} = 5kN - m$$

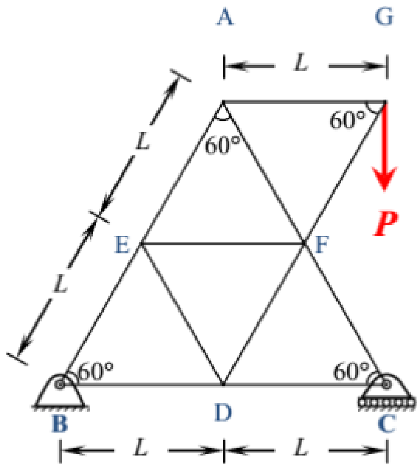


Q.12 (a)

$\Sigma F_H = 0 \Rightarrow H_B = 0$

$\Sigma M_c = 0 \Rightarrow V_B \times 2L = 0 \Rightarrow V_B = 0$

$\Sigma V = 0 \Rightarrow V_C = P$



Q.13 (a)

Conditions for zero force members are

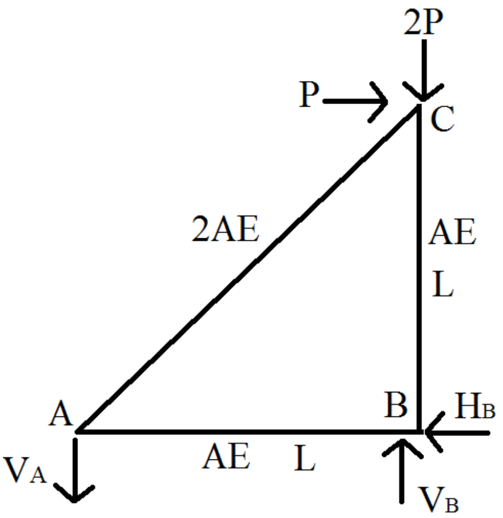
(i) The member meets at a joint and they are non-collinear and no external force acts at that joint. Both the members will be the zero force members.

(ii) When the members meet at joint and two are collinear and no external force act at the joint then third member will be zero force member. According to the above statements

We can say that

FT, TG, HU, MP and PL members are zero force members.

Q.14 (2.7)



$\sum M_B = 0$

$\therefore P*L - V_A*L = 0$

$\therefore \mathbf{V_A = P}$

Now,

$\sum V = 0$

$\therefore - V_A + V_B - 2P = 0$

$\therefore \mathbf{V_B = 3P}$

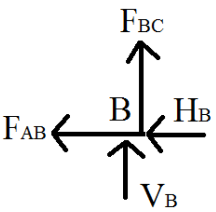
$\sum H = 0$

$\therefore -H_B + P = 0$

$\therefore \mathbf{H_B = P}$

Assume all members are in tension

Joint B



$\sum V = 0$

$\therefore V_B + F_{BC} = 0$

$\therefore 3P + F_{BC} = 0$

$\therefore \mathbf{F_{BC} = -3P}$

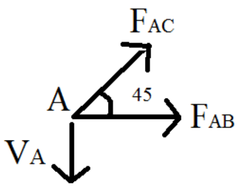
$\sum H = 0$

$\therefore -H_B - F_{AB} = 0$

$\therefore -P - F_{AB} = 0$

$\therefore \mathbf{F_{AB} = -P}$

Joint A



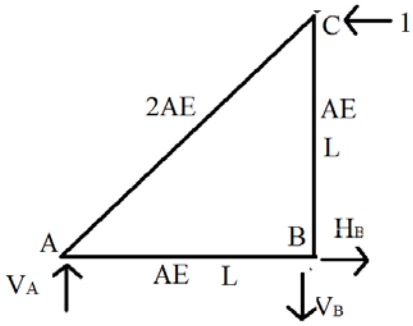
$\sum V = 0$

$\therefore -V_A + F_{AC} \sin 45 = 0$

$\therefore -P + F_{AC} \sin 45 = 0$

$\therefore \mathbf{F_{AC} = P\sqrt{2}}$

Now apply unit load at a point where deflection is required.



$\sum M_B = 0$

$\therefore -1*L + V_A*L = 0$

$\therefore \mathbf{V_A = 1}$

Now,

$\sum V = 0$

$\therefore V_A - V_B = 0$

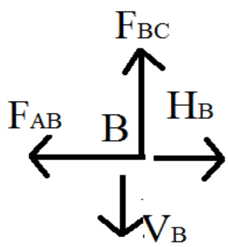
$\therefore \mathbf{V_B = 1}$

$\sum H = 0$

$\therefore H_B - 1 = 0$

$\therefore \mathbf{H_B = 1}$

Joint B



$\sum V = 0$

$\therefore -V_B + F_{BC} = 0$

$\therefore -1 + F_{BC} = 0$

$\therefore \mathbf{F_{BC} = 1}$

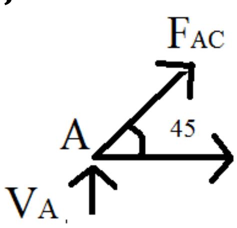
$\sum H = 0$

$\therefore H_B - F_{AB} = 0$

$\therefore 1 - F_{AB} = 0$

$\therefore \mathbf{F_{AB} = 1}$

Joint A



$\sum V = 0$

$\therefore V_A + F_{AC} \sin 45 = 0$

$\therefore 1 + F_{AC} \sin 45 = 0$

$\therefore \mathbf{F_{AC} = -\sqrt{2}}$

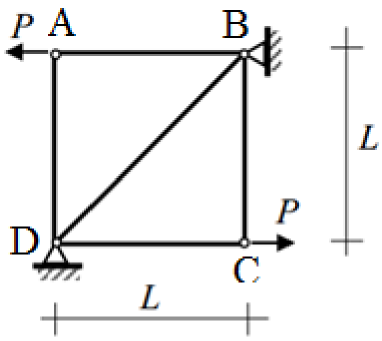
Part	L	P	K	AE	PKL/AE
AB	L	–P	1	AE	–PL/AE
AC	√2L	P√2	–√2	2AE	–√2 PL/AE
BC	L	–3P	1	AE	–3 PL/AE

$\delta_{HC} = \sum PKL/AE = -5.41PL/AE$

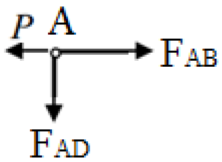
$$= \frac{-5.42*1000*1000}{10*2*10^5}$$

$= -2.705$ mm

Q.15 (D)

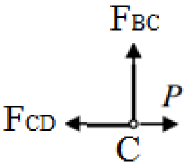


Joint A

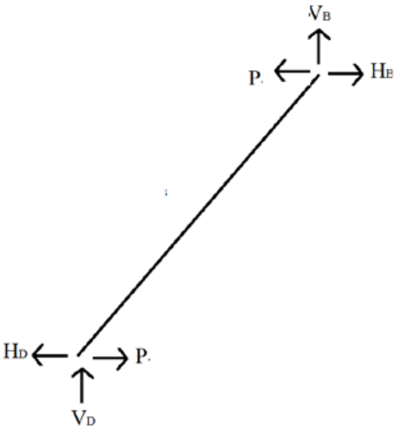


$\Sigma V = 0$
$\therefore F_{AD} = 0$

Joint C



$\Sigma V = 0$
$\therefore F_{BC} = 0$



Here force P is making couple of P*L which will be balanced by couple due to reactions H_D and H_B. It means inclined member will also carry zero force.

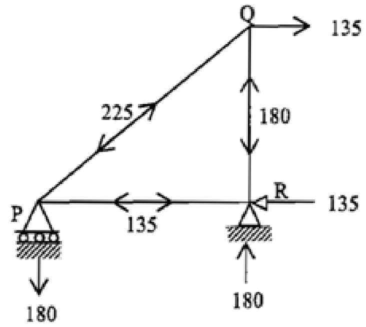
Q.16 (B)

As temperature increases, the bar `BD' will tend to elongate. But the Joints B & D will offer resistance. Hence the bar 'BD' will be in compression.

Q.17 (D)

Analyze the given truss for external load.

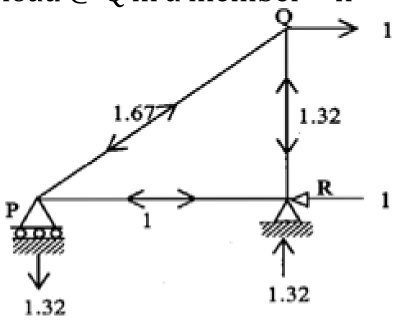
Say, Force due to external load in a member = P



Analyze the given truss for unit horizontal load at 'o'

Say, Force in members, compressive as +ve and tension as –ve.

Say, Force due to unit horizontal load @ Q in a member = k



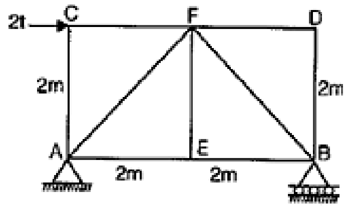
Member	P	k	*l*
PQ	225	1.67	7.5
QR	- 180	- 1.32	6
PR	- 135	- 1	4.5

$$Q\,\delta_{Qh} = \frac{\sum Pkl}{AE} = \frac{4862.025}{1550 \times 200}$$

= 0.01568 m = 15.68 mm

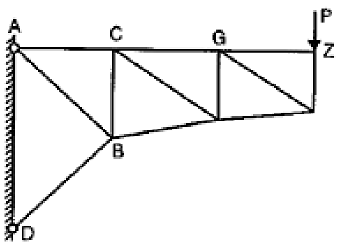
ASSIGNMENT

Q.1 A simply supported truss shown in the given figure carries load as shown. The force in member BE is



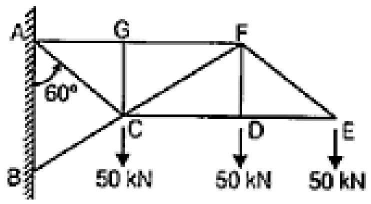
a) $\sqrt{2}$ t (tensile)
b) $\sqrt{2}$ t (compressive)
c) 1t (tensile)
d) 1t (compressive)

Q.2 A cantilever pin-jointed truss carries one load P at the point Z as shown in the given figure. The hinges in the vertical wall are at A and D. The truss has only three horizontal members AC, CG and GZ



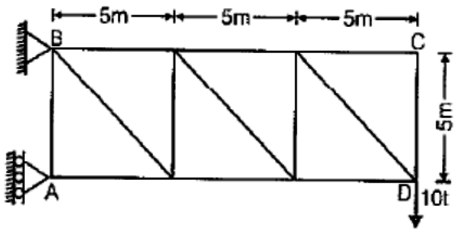
The nature of force in member AB
a) is tensile
b) is zero
c) is compressive
d) cannot be predicted

Q.3 The force in member FD in the given figure is



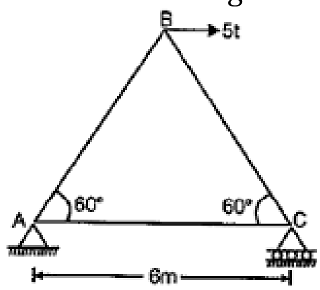
a) 50 kN compression
b) 50 kN tension
c) 150 kN tension
d) 100 kN compression

Q.4 In the cantilever truss shown in the figure the reaction at A is



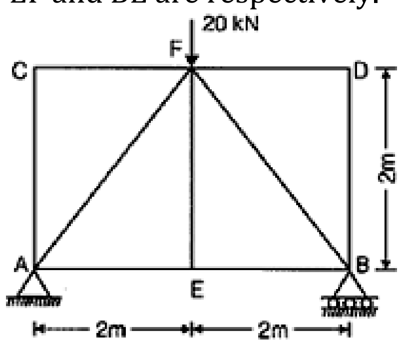
a) 10 t
b) 20 t
c) 30 t
d) 40 t

Q.5 Force in the member BC of the truss shown in the given figure



a) 5 t, tensile
b) zero
c) 2.88 t, compressive
d) 5 t, compressive

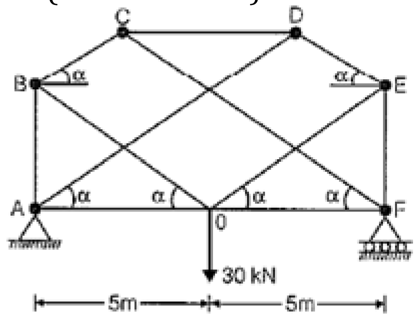
Q.6 A simply supported truss shown in the given figure carries a load if 20 kN at F the forces in the members EF and BE are respectively.



a) zero and 10 kN (compression)
b) zero and 10 kN (tension)

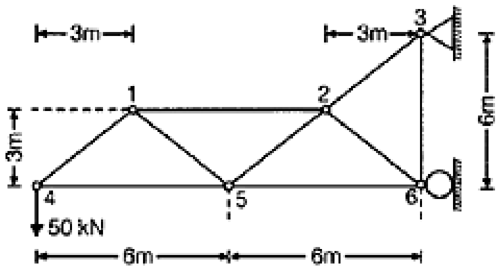
c) 10kN (tension) and 10kN (compression)
d) 10kN (compression) and 10kN (tension)

Q.7 Axial force in the member BC of the truss shown in the given figure is (where α = 30°)



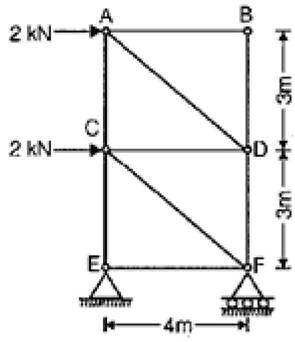
a) 15 kN b) $10\sqrt{3}$ kN
c) $15\sqrt{3}$ kN d) 30 kN

Q.8 Axial force in the members 1 – 2 and 1 – 5 of the truss shown in the figure are respectively.



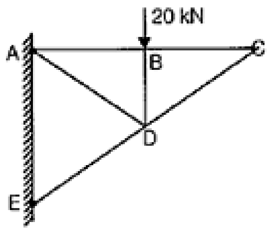
a) 50 kN (compressive) and 25 kN (tensile)
b) 25 kN (tensile) and $\frac{50}{\sqrt{2}}$ kN (compressive)
c) 50 kN (tensile) and $50\sqrt{2}$ (compressive)
d) 25 kN (compressive) and $\frac{50}{\sqrt{2}}$ kN (compressive)

Q.9 A pin-joined tower truss is loaded as shown in the below figure. The force induced in the member DF is



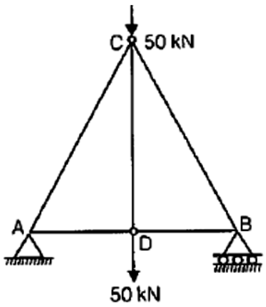
a) 1.5kN (tension)
b) 4.5 kN (tension)
c) 1.5 kN compression
d) 4.5 kN (compression)

Q.10 For the truss shown in the figure, which one of the following members has zero forced induced in it?



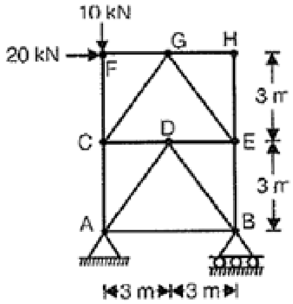
a) BC b) AD
c) DE d) BD

Q.11 The force induced in the vertical member CD of the symmetrical plane truss shown in the figure



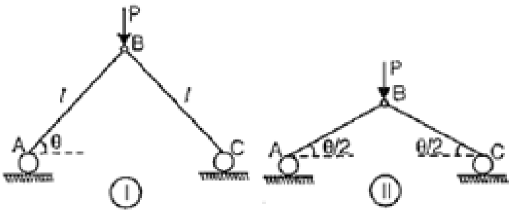
a) 50 kN (tension)
b) 100 kN (tension)
c) 50 kN (compression)
d) Zero

Q.12 A loaded pin-joined truss is shown in the given figure. The force in member AC is



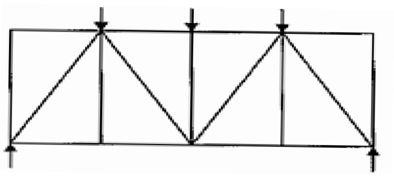
a) $10\sqrt{2}$ kN (Tensile)
b) 10 kN (Compressive)
c) zero
d) 10 kN (Tensile)

Q.13 Which of the following is the correct statements regarding the force and deflection at point B in trusses I and II shown in the figure?



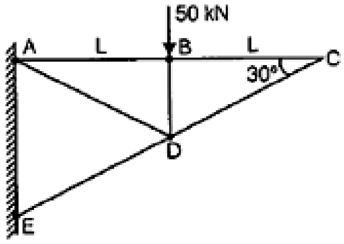
a) I will have less member force & less deflection at B compared t
b) I will have less member force and more deflection at B compared to II
c) I will have more member force & deflection at B compared to II
d) I will have more member force & less deflection at B compared to II

Q.14 In the plane truss shown below, how many members have zero force?



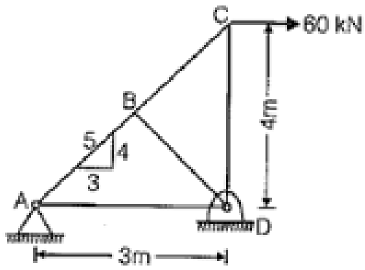
a) 3
b) 5
c) 7
d) 9

Q.15 In the pin-joined plane truss shown below, what is the magnitude and nature of force in member BC?



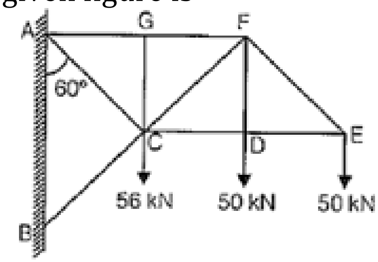
a) zero
b) 50 kN (tensile)
c) 50 kN (compressive)
d) $\frac{50\sqrt{2}}{2}$ kN (tensile)

Q.16 Due to horizontal pull 60 kN at C, what is the force induced in the member AB?



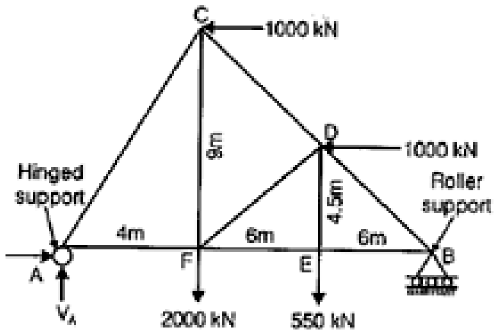
a) 0
b) 40 kN
c) 80 kN
d) 100 kN

Q.17 The force in member FD in the given figure is



a) 50 kN (compressive)
b) 50 kN (tensile)
c) 150 kN (tension)
d) 100 kN (compression)

Q.18 A truss ABC carries two horizontal and two vertical loads, as shown in the figure. The horizontal and vertical components of reactions at A will be



a) HA = 1000 kN; VA = 1706.25 kN
b) HA = 2000 kN; VA = 2550 kN
c) HA = 1000 kN; VA = 2350 kN
d) HA = 2000 kN; VA = 1706.25 kN

Q.19 What does the Williot-Mohr diagram yield?
a) Forces in members of a truss
b) Moments in a fixed beam
c) Reaction at the support
d) Joint displacement of a pin jointed plane frame

Q.20 Consider the following:
Assumption in the analysis of a plane truss.
1. The individual members are straight.
2. The individual members are connected by frictionless hinges.
3. The loads and reactions acts only at the joints.

Which of the following assumptions are valid?
a) 1 and 2
b) 1 and 3
c) 2 and 3
d) 1, 2 and 3

ANSWER KEY:

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
(c)	(c)	(b)	(c)	(d)	(b)	(d)	(c)	c
10	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
(a)	(a)	(c)	(a)	(c)	(a)	(d)	(b)	d
19	**20**							
(d)	(d)							

EXPLANATIONS

Q.1 (c)

At joint D there is no external force so force in FD and BD is zero.

Taking moment about A, the reaction at B

$$R_B = \frac{2\times2}{4} = 1t$$

Considering joint B, force in member BF

$F_{BF} = \sqrt{2}t$ (compressive)

Force in member BE

$F_{BE} = 1t$ (tensile)

Q.2 (c)

Q.3 (b)

Considering joint equilibrium at D. The force equilibrium equation in vertical direction gives FD = 50 kN tensile

Q.4 (c)

Only a horizontal reaction can exist at A.

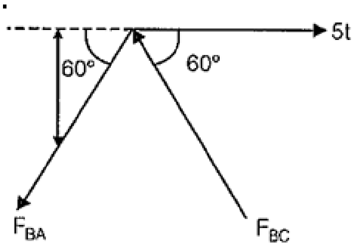
Taking moments about hinge.

$H_A \times 5 = 10 \times 15$

$H_A = 30t$

Q.5 (d)

By considering joint equilibrium at B.



$F_{BC} = F_{BA}$

$F_{BC}\cos 60^0 + F_{BA}\cos 60^0 = 5$

$F_{BC} = 5t$ (compressive)

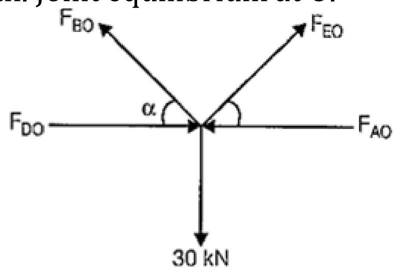
$F_{BA} = 5t$ (tensile)

Q.6 (b)

Force in EF is zero if joint equilibrium at E is considered force in BE will be Tensile, and its value will be 10 kN by considering joint equilibrium at B.

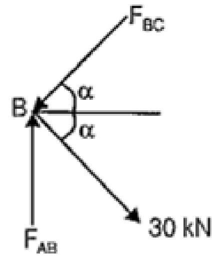
Q.7 (d)

The given truss is symmetrical. So force in member BO and EO will be same in magnitude and nature both. Joint equilibrium at O.



$$F_{bo} = \frac{15}{\sin 30^0} = 30 \text{ kN}$$

Joint equilibrium at B.



$\therefore F_{BC} = 30$ KN

Q.8 (d)

Cutting a section through 1-2; 2-5; and 5-6.

Taking moment of left part about 5.

$$F_{1\text{-}2} = \frac{50\times6}{3} = 100 \text{ kN (Tensile)}$$

Cut a section through 1-2, 1-5 and 4-5 and balance vertical force for left part only.

$$\frac{F_{1\text{-}5}}{\sqrt{2}} = 50$$

$\therefore F_{1\text{-}5} = 50\sqrt{2}$ kN (Compressive)

Q.9 (c)

Q.10 (a)

Cutting a section through AC, CD and DF and taking moment of upper portion About C. The forces in the members AC and CD meet at C so they not produce
Any moment. The force in member DF.

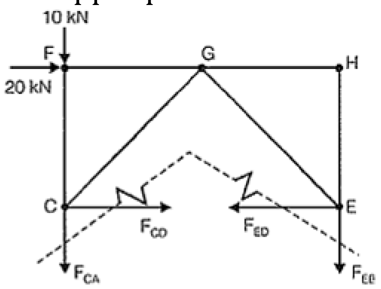
$$F_{DF} = \frac{2\times3}{4} = 1.5\text{kN (compression)}$$

Q.11 (a)

At join D, the equilibrium shows the force in member CD is 50 kN tensile.

Q.12 (c)

Cutting a section through members CA, CD, DE and EB and taking moment of
The upper part about E.



$$\therefore\ F_{CA}\times 6 + 10\times 6 - 20\times 3 = 0$$

$$\therefore\ F_{CA} = 0$$

Q.13 (a)

Force in AB or AC [Consider joint B]

Case-I: $\frac{P}{2}\sin\theta$

Case-II: $\frac{P}{2}\sin\left(\frac{\theta}{2}\right)$

as $\sin\theta > \sin\frac{\theta}{2}$

So in case I force in members will be less than that in case II.
From symmetry there will be only vertical deflection of joint B.

$$U = U_{AB} + U_{BC} = \frac{F^2 l}{2AE}\times 2 = \frac{F^2 l}{AE}$$

$$U_1 = \frac{P^2 l}{4AE\sin^2\theta}$$

$$\Delta_1 = \frac{\delta V_1}{\delta P} = \frac{Pl}{2AE\sin^2\theta}$$

$$\Delta_{1t} = \frac{Pl}{2\ AE\sin^2\left(\frac{\theta}{2}\right)}$$

Thus $\Delta_{1t} > \Delta_1$

Q.14 (c)

Six members having zero forces are obvious and seventh member can be from top chord member between the applied loads.

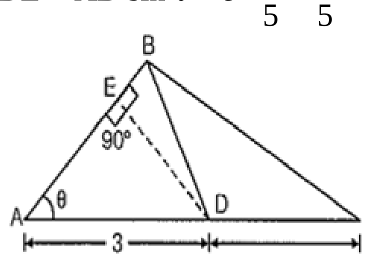
Q.15 (a)

At joint C there is no external force so force in member BC is zero.

Q.16 (d)

Cut a section through members AB, BD and CD, and take moment of upper part about D.

$$DE = AD\sin\theta = 3\times\frac{4}{5} = \frac{12}{5}$$



$$DE\times F_{AB} = 60\times 4$$

$$F_{AB} = \frac{60\times4\times5}{12} = 100$$

Q.17 (b)

Q.18 (d)

Q.19 (d)

Q.20 (d)

For plane truss assumptions are:

i. Weight of all members are

neglected. All members are straight.

ii. All connections are smooth, frictionless pins.

iii. External loads are applied directly to the pin joints.

iv. All frames are perfect, hence statically determine.

i

3 ARCH

3.1 Introduction:

Arches are a structure which eliminates tensile stresses in spanning great amount of open space. All the forces are resolved into compressive stresses.

It also helps to reduce bending moment in long span structure.

3.2 Types of Arches:

Types of arches based on number of hinges

1. Two hinged arch

2. Third hinged arch

3. Fixed arch or hinge less arch

- A three hinged arch is a statically determinate structure where the rest two aches are statically indeterminate
- In bridge construction, especially is railroad bridges, the more frequently used aches the two hinged and the fixed end ones.

3.3 Advantages of Arch over beam

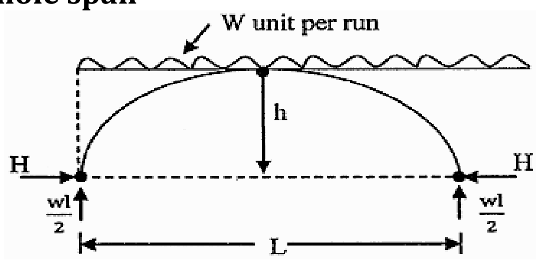
- Section required for arch is small compare to beam for same span and loading as bending moment at any section is less in arch compare to beam.

3.4 Three Hinged Arches

- A three hinged arch is a statically determinate structure, having a hinged at each abutment or springing, and also at the crown.
- In three hinged arch three equations are available from static equilibrium and one additional equation is available from the fact that B.M at the hinge at the crown is zero.

Standard cases

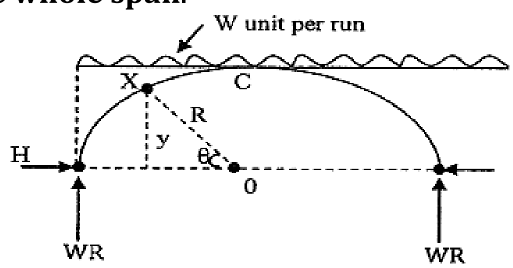
Case 1: A three hinged parabolic arch of span L rise 'h' carries a udl of w over the whole span



a) The horizontal reaction at each support is $H = \frac{WL^2}{8h}$

b) BMxx = 0
SFxx =0

Case 2: A three hinged semi-circular arch of radius 'R' carries a udl of w on the whole span.

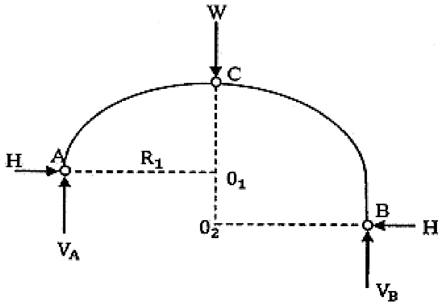


a) The horizontal thrust at each end; $H = \frac{WR}{2}$

b) The maximum bending moment occurs at $\theta = 30°$

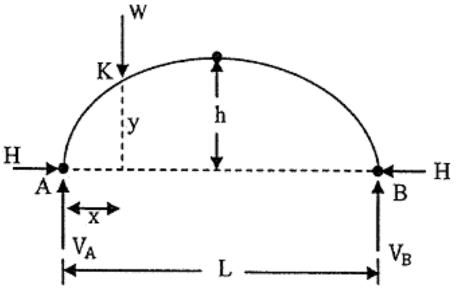
$$M_{max} = \frac{WR^2}{8} \quad \text{(Hogging)}$$

Case3: A three hinged arch consisting of two quadrant parts AC and CB of radii R_1 and R_2. The arch carries a concentrated load of W on the crown



$$V_A = V_B = H = \frac{W}{2}$$

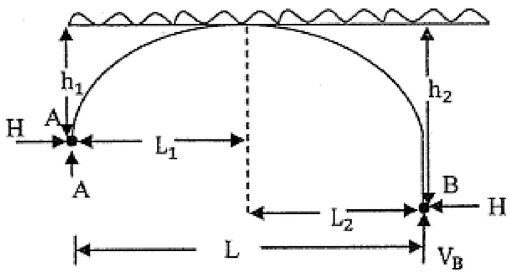
Case 4 : A symmetrical three-hinged parabolic arch of span land rise carries a point load, w, which may be placed anywhere on the span



$$H = \frac{Wx}{2h}$$

The absolute maximum bending moment occurs at a distance of $\frac{1}{2\sqrt{3}}$ on either side of the crown.

Case 5 : A three hinged parabolic arch of span 1 has its abutments at depth h_1 and h_2 below the crown the arch carries an udl of w per unit length over the whole span



The horizontal thrust at each support is given by

$$H = \frac{WI^2}{2\left(\sqrt{h_1} + \sqrt{h_2}\right)^2}$$

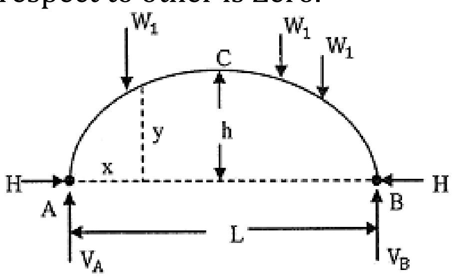
Case 6 :

A three hinged parabolic arch of span 1 has its abutments A and B at depth h_1 and h_2 below the crown C. The arch carries a concentrated load W at the crown. The horizontal thrust at each support is given by

$$H = \frac{WI^2}{2\left(\sqrt{h_1} + \sqrt{h_2}\right)^2}$$

3.5 Two Hinged Arches:

Two hinged arch is an indeterminate structure. V_a and V_b can be determined by taking moment about either end. The horizontal thrust at each support may be determined from the condition that the horizontal displacement of the either hinge with respect to other is Zero.

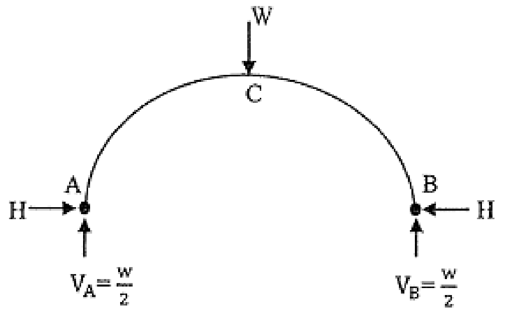


$$H = \frac{\int \frac{M.yd_s}{EI}}{\int \frac{y^2.d_s}{EI}}$$

Where, M is beam moment

Standard Cases

Case 1: A two hinged semi-circular arch of radius R carries a concentrated load W at the crown

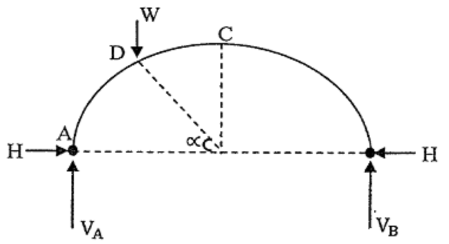


The horizontal thrust on each support is given by

$$H = \frac{W}{\pi}$$

Case 2 :

A two hinged semi-circular arch of radius R carries a load W at a section, the radius vector corresponding to which makes an angle ∝ with the horizontal.

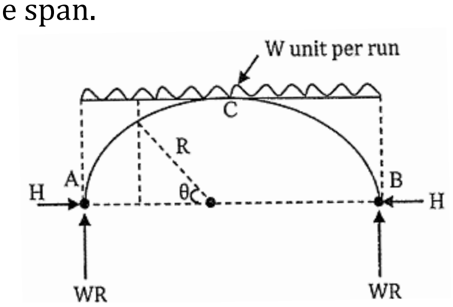


The horizontal thrust at each support is given by

$$H = \frac{W}{\pi}\sin^2 \alpha$$

Case 3 :

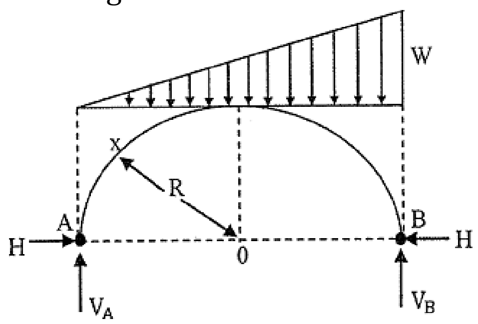
A two hinged semi-circular arch of radius R carries a udl W per unit length over the whole span.



The horizontal thrust at each support is given by $H = \frac{4}{3}\frac{WR}{\pi}$

Case 4 :

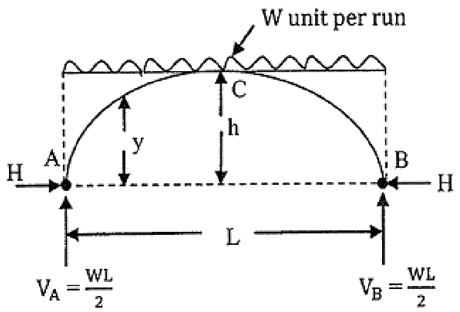
A two hinged semi-circular arch of radius R carries a distributed load uniformly varying from zero at the left end to w per unit run at the right end.



The horizontal thrust at each support for this case is $H = \frac{2}{3}\frac{WR}{\pi}$

Case 5 :

A two hinged parabolic arch carries a udl of W per unit run on entire span of the span of the arch is L and its rise is h.



The horizontal thrust at each is given by $H = \frac{WL^2}{8h}$

BM = 0 (everywhere)

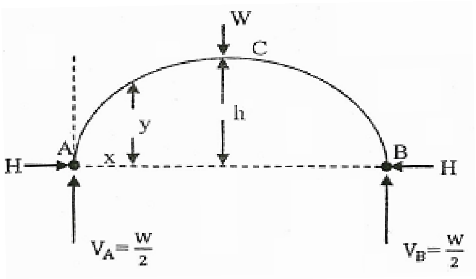
Case 6 :

When half of the parabolic arch is loaded by udl. Then the horizontal reaction at support is given by $H = \frac{WL^2}{16h}$

Case 7 : When two hinged parabolic arch carries uniformly varying distributed load, for zero to w the horizontal thrust is given by $H = \frac{WL^2}{16h}$

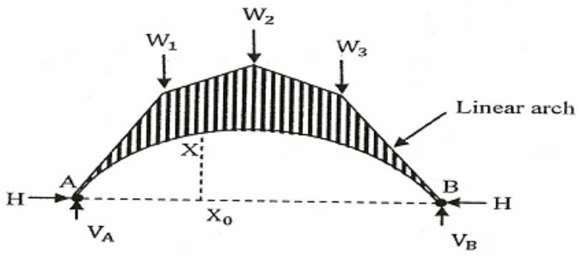
Case 8 :

A to hinged parabolic arch of span L and rise h carries a concentrated load W at the crown



The horizontal thrust at

The Linear Arch or Theoretical Arch:



a) The different members of the linear arch are subjected to axial compressive forces only. The joints of this linear arch are in equilibrium
b) The shape of linear arch follows the shape of the free bending moment diagram for a beam of the same spam and subjected to the same loading
c) The B.M at any section of an arch is proportional to the ordinate or the intercept between the given arch and the linear arch. This principle is called Eddy's theorem
d) The actual BM at the section X is proportional to the ordinate x_1x.

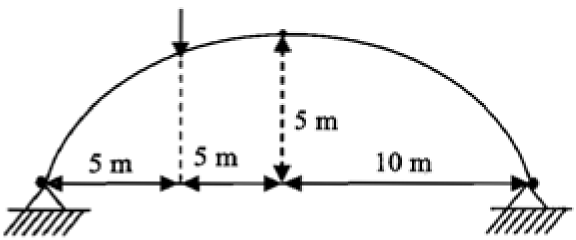
GATE QUESTIONS

Q.1 A three hinged parabolic arch ABC has a span of 20 m and a central rise of 4 m. The arch has hinges at the ends and at the centre. A train of two point loads of 20 kN and 10 kN, 5 m apart, crosses this arch from left to right, with 20 kN load leading_ The maximum thrust induced at the supports is

a) 25.00 kN b) 28.13 kN
c) 31.25 kN d) 32.81 kN

[GATE – 2004]

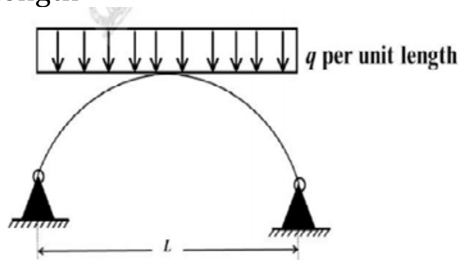
Q.2 A three hinged parabolic arch having a span of 20 m and a rise of 4 m carries a point load of 10 kN at quarter span from the left end as shown in the figure. The resultant reaction at the left support and its inclination with the horizontal are respectively



a) 9.01 KN and 56.31°
b) 9.01 kN and 33.69°
c) 7.50 kN and 56.31°
d) 2.50 kW and 33.69°

[GATE-2010]

Q.3 A figure shows two hinged parabolic arch of span L subjected to UDL over entire span of intensity q per unit length



The maximum bending moment in the arch equal to

(a) $\frac{qL^2}{8}$ (b) $\frac{qL^2}{12}$

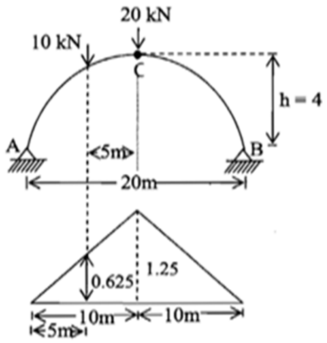
(c)Zero (d) $\frac{qL^2}{10}$

ANSWER KEY:

1	2	3
(c)	(a)	(c)

EXPLANATIONS

Q.1 (c)



ILD for Horizontal thrust at Supports.

Central ordinate

$$= \frac{l}{4h} = \frac{20}{4 \times 4} = 1.25$$

At quarter point,

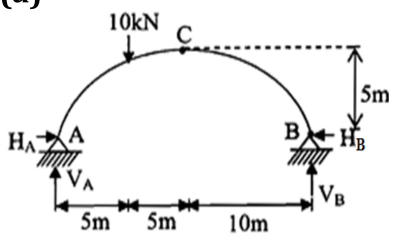
$$\text{Ordinate } = \frac{1.25}{2} = 0.625$$

According to ILD, Maximum thrust is induced at supports if the leading load 20 kN is at the crown.

∴ Horizontal thrust at supports

H=20×1.25 + 10×0625 = 31.25 kN

Q.2 (a)



Apply $\sum M_A = 0$

from right.

$$V_B \times 20 = 10 \times 5$$

$$\therefore V_B = 2.5 \text{ kN} \uparrow$$

$$V_A + V_B = 10 \text{ kN}$$

$$\therefore V_A = 10 - 2.5 = 7.5 \text{ kN} \uparrow$$

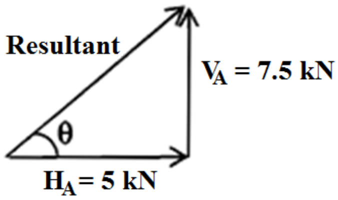
Applying moment at central hinge 'C',

$\sum M_C = 0$ from right,

$$V_B \times 10 = H_B \times 5$$

$$\therefore H_B = \frac{2.5 \times 10}{5} = 5\text{kN} \leftarrow$$

$$H_A = 5\text{kN} \rightarrow$$



Resultant Reaction,

$$R = \sqrt{H_A^2} + \sqrt{V_A^2}$$

$$= \sqrt{5^2 + 7.5^2} = 9.01 \text{ kN}$$

Inclination of Resultant with

$$\text{Horizontal} = \tan^{-1} \frac{7.5}{5} = 56.31°$$

$= 56.31°$ With horizontal.

Q.3 (c)

A two hinged parabolic arch carries a udl of q per unit run on entire span of the span of the arch is L, BM is zero everywhere.

ASSIGNMENT

Q.1 In a two hinged arch an increase in temperature induces

a) no bending moment in the arch rib
b) Uniform bending moment in the arch rib
c) Maximum bending moment at the crown
d) Minimum bending moment at the crown

Q.2 Which one of the following is associated with the rib shortening in arches either due to change in temperature or lack of fit to cause stress in the arch members?

a) Only two-hinged arches and not three-hinged arches
b) Two and three-hinged arches
c) Two-hinged arches made of reinforced concrete only.
d) Only three-hinged arches but not two-hinged arches.

Q.3 A three-hinged parabolic arch rib of span L and crown rise 'h' carries a uniformly distributed superimposed load of intensity 'w' per unit length. The hinges are located on two abutments at the same level and the third hinge at a quarter span location from left hand abutment. The horizontal thrust on the abutment is

a) $\frac{wL^2}{4h}$ b) $\frac{wL^2}{6h}$
c) $\frac{wL^2}{8h}$ d) $\frac{wL^2}{12h}$

Q.4 A symmetrical two-hinged parabolic arch rib has a span of 32 m between abutment pins at the same level and a central rise of 5 m. When a rolling load of 100 kN crosses the span, the maximum horizontal thrust at the hinges will be

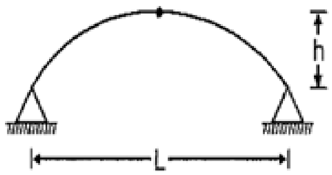
a) 100 kN b) 125 kN
c) 160 kN d) 240 kN

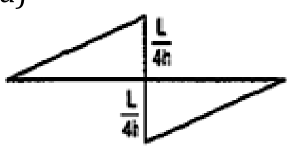
Q.5 A uniformly distributed load of 2 kN/m covers left half of the span of a three-hinged parabolic arch, span 40 m and central rise 10 m. Which of the following statements related to different functions at the loaded quarter point are correct?

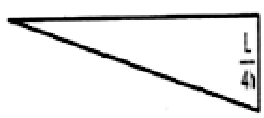
1. The slope is $\tan^{-1}\left(\frac{1}{2}\right)$
2. The normal thrust is $6\sqrt{5}$ KN
3. The shear force is not zero
4. The bending moment is zero.

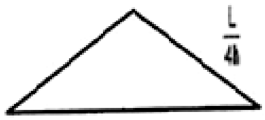
Select the correct answer using the codes given below.
Codes:
a) 1, 2 and 4 b) 2 and 3
c) 1 and 3 d) 3 and 4

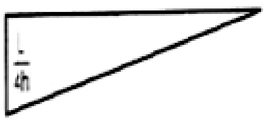
Q.6 A three hinged arch is shown in the below figure. The influence line for the horizontal thrust is



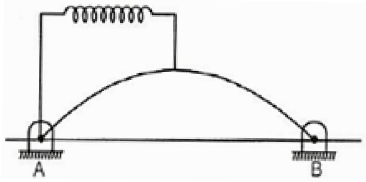
a)


b)


c) 

d) 

Q.7 For the two hinged parabolic arch as shown in the figure below, which one of the following diagram represents the shape of the bending moment variation?



a) 

b) 

c) 

d) 

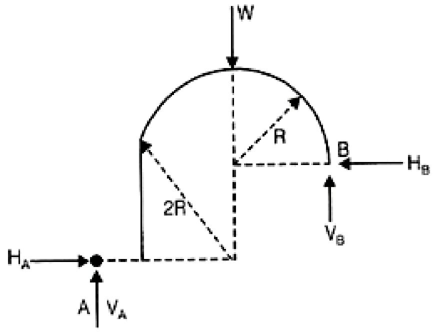
Q.8 A circular three-pinned arch of span 30 m and a rise of 8 m is hinged at the crown and springing. It carries a horizontal load of 100 kN per vertical metre on the left side. The horizontal thrust at the right springing will be

a) 200 kN b) 400 kN
c) 600 KN d) 800 kN

Q.9 A symmetrical three-hinged parabolic arch of span L and rise h is hinged at springing and crown. It is subjected to a UDL w throughout the span. What is the bending moment at a section L/4 from the left support?

a) $wL^2/8$ b) $wL^2/16$
c) $wL^3/8h$ d) Zero

Q.10 A three-hinged circular arch ACB is formed by two quadrants of circles AC & BC of radii 2R & R respectively with C as crown as shown in the figure. Consider the following in respect of the reactive forces developed at supports A & B due to concentrated load at the crown:



1. Line of action of reaction RA at A is inclined at 45° to the horizontal.
2. $V_a = V_B = W/2$
3. $H_B = 2H_A$
4. $H_A = H_B = W/2$

Which of the above is/are correct?

a) 1 and 3 b) 2 and 3
c) 1, 2 and 4 d) 4 only

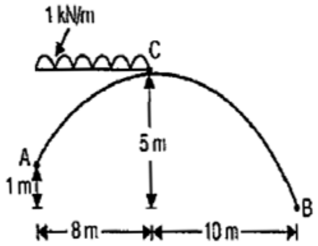
Q.11 A parabolic arch, symmetrical with hinges at centre and ends, carries a point load p at a distance x from left support. The arch has a span of 20 m and rise of 5 m. What is the value of 'x' if the left hinge reaction is inclined with a slope of two vertical to one horizontal?

a) 8 m b) 5 m
c) 4 m d) 2.5 m

Q.12 A symmetrical circular arch of span 25 m with a central rise of 5 m is hinged at crown and springing. It carries a point load on the arch at a distance of 6.35 m from the left hinge. If the inclination of the thrust at the right hinge is θ measured from horizontal, then what is the value of tan θ?

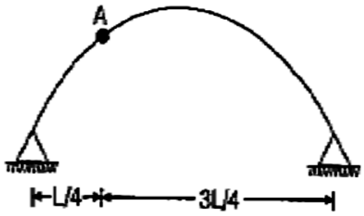
a) 2.5
b) 1.0
c) 0.4
d) Not possible to calculate with the given data

Q.13 The horizontal thrust at support A in a three hinged arch shown in the given figure is



a) 2 KN | b) 4 KN
c) 8 KN | d) 10 KN

Q.14 For the semi-circular two-hinged arch shown in the figure below a moment 50 t-cm applied at B produces a displacement load of 10 t is applied at A, the rotation at B in the arch will be



a) 0.1 | b) 0.001
c) 0.0001 | d) 0.01

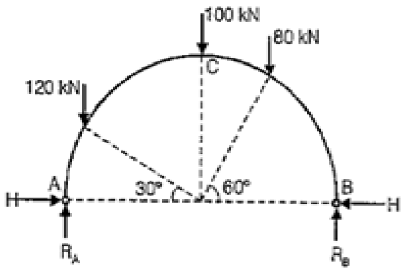
Q.15 A three hinged semi-circular arch of radius R carries a uniformly distributed load w per unit run over the whole span. The horizontal thrust is

a) wR | b) $\frac{wR}{2}$
c) $\frac{4wR}{3\pi}$2t | d) $\frac{2wR}{3\pi}$1t

Q.16 The horizontal thrust due to rise in temperature in a semi-circular two hinged arch of radius R is proportional to

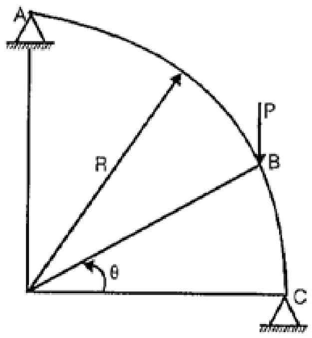
a) R | b) R^2
c) 1/R | d) $1/R^2$

Q.17 A two hinged semi-circular arch of radius 20 meters carries a load system shown in figure. The horizontal thrust at each support is (Assume uniform flexural rigidity)



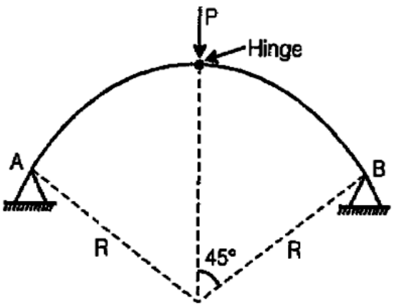
a) $\frac{180}{\pi}$KN | b) $\frac{190}{\pi}$KN4t
c) $\frac{200}{\pi}$KN2t | d) None of these

Q.18 A three hinged arch is shown in the figure is quarter of a circle. If the vertical and horizontal components of reaction at A are equal, the value of θ is



a) 60° | b) 45°
c) 30° | d) None of these

Q.19 A three hinged arch is subjected to point load at crown. The magnitude of horizontal reaction at A is



a) $\frac{P}{2\left(\sqrt{2}-1\right)}$ | b) $\frac{P}{2}$
c) $\sqrt{2}-1$ | d) $\frac{P}{2\sqrt{2}}$

Q.20 An arch is used in long span bridges because there is

a) considerable reduction in horizontal thrust
b) considerable reduction in bending moment
c) reduction in shear and bending moment
d) no change in shear and bending moment

ANSWER KEY:

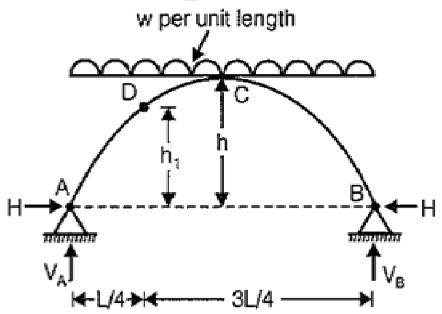
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
(c)	(a)	(c)	(b)	(c)	(c)	(b)	(a)	(d)
10	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
(c)	(c)	(c)	(b)	(a)	(b)	(d)	(b)	(b)
19	**20**							
(a)	(b)							

EXPLANATIONS

Q.1 **(c)**
Increase in temperature in a two hinge arch will cause horizontal thrust only.
Moment due to horizontal thrust is –Py
So maximum bending moment will be at crown as crown has highest value of y.

Q.2 **(a)**
In the case of two hinged and fixed arches rib shortens by an amount $\frac{HL}{AE}$
There is always horizontal compressive thrust due to temperature increase or lack of fit in the case of two hinged arches. While in three hinged arches no such thrust exists and only the shape will change at crown.

Q.3 **(c)**
$$V_A = V_B = \frac{WL}{2}$$



$$\therefore \frac{h}{(L/2)^2} = \frac{h - h1}{(L/4)^2}$$
$$\Rightarrow \quad 4(h - h_1) = h$$
$$3h = 4h_1$$
$$h_1 = \frac{3h}{4}$$
Taking moment about hinge at
$$V_A \times \frac{L}{4} - H \times h_1 - \frac{wL}{4} \times \frac{L}{8} = 0$$
$$\Rightarrow \frac{wL}{2} \times \frac{L}{4} - H \times \frac{3h}{4} - \frac{wL^2}{32} = 0$$
$$\therefore \; H = \frac{wL^2}{8h}$$

Q.4 **(b)**
For unit load maximum horizontal thrust
$$= \frac{25L}{128h}$$
For 100 KN load maximum horizontal thrust
$$= \frac{25 \times 32}{128 \times 5} \times 100$$
$$= \frac{5}{4} \times 100 = 125 \text{ kN}$$

Q.5 **(c)**
$$H = \frac{wL^2}{16h} = 20KN$$
$$V_A = \frac{3}{8} wL = 30KN$$
The equation of parabolic arch y
$$= \frac{4h}{L^2} \times (L - X)$$
$$\sin\theta = \frac{1}{\sqrt{5}} \cos\theta = \frac{2}{\sqrt{5}}$$
$$\therefore \frac{dy}{dx} = \frac{4h}{L^2}(L - 2X)$$
At $X = \frac{L}{4}$
$$\tan\theta = \frac{2h}{L}$$
For given arch $\theta = \tan^{-1}(1/2)$
So statement (1) is correct B.M.D
$$\sin\theta = \frac{1}{\sqrt{5}} \cos\theta = \frac{2}{\sqrt{5}}$$

$M = \frac{WX}{8}(L - 2X)$ for loaded half

At $X = \frac{L}{4}$

$M = \frac{W}{8} \times \frac{L}{4} \times \frac{L}{2} = \frac{wL^2}{64} = 50 kN\text{-}m$

So bending moment is not zero and statement (4) is wrong

Normal thrust $N = V_A \sin\theta + H\cos\theta$

$= 14\sqrt{5}$ kN

Radial shear $V_r = V_A \cos\theta - H\sin\theta$

$= 8\sqrt{5} kN$

Q.6 (c)
The horizontal reaction varies linearly from any end up to the crown.

Q.7 (b)
The horizontal thrust influence line in two-hinged arches is a fourth order parabola.

However bending moment at any point

= Beam B.M – Thrust B.M

Horizontal thrust due to udl through the span

$= \frac{wL^2}{8h}$

For half span loading shown in the figure, the symmetry shows that horizontal thrust

$H = \frac{wL^2}{16h}$

B.M for loaded half portion taking x positive from A

$H = \frac{3}{8} wlx - \frac{wx^2}{2} - \frac{wl^2}{16h} \times \frac{4h}{l^2} \times (l - x)$

$H = \frac{wlx}{8} - \frac{wx^2}{4} = \frac{wx}{8} \times (l - 2x)$

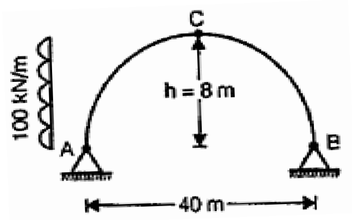
$x \le \frac{l}{2}$

For right half $M = -\frac{wx}{8}(l - 2x)$

With x positive from B.

These expressions are same as that for three hinged arch in the same case.

Q.8 (a)



The vertical reaction At B is

$V_B \times 40 = 100 \times 8 \times 4$

$V_B = 80$ kN

For horizontal thrust at B taking moment of right part about crown (c)

$V_B \times 20 = H \times 8$

$\therefore H = \frac{80 \times 20}{8} = 200KN$

Q.9 (d)
For a three hinged parabolic arch, carrying UDL over the entire span, bending moment is zero at all the sections.

Q.10 (c)
The moment at crown being zero. Consider AC

$H_A.2R = V_A.2R \Rightarrow H_A = V_A$

For BC $H_B = V_B$

As $H_A = H_B$ so $V_A = V_B$

Now $V_A + V_B = W$

so $V_A = V_B = W/2$

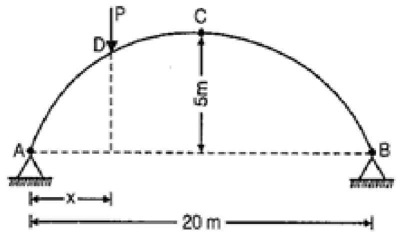
Q.11 (c)
Vertical at A

$Va = \frac{P(20 - x)}{20}$

Horizontal thrust

$H = \frac{V_B \times 10}{5} = 2V_B$

$\therefore \frac{2P_X}{20}$



Given that left hinge reaction is inclined with a slope of two vertical on one horizontal.

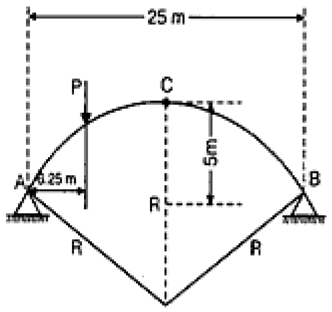
$\therefore$ VA = 2H

$$\frac{P(20-x)}{20} = \frac{4px}{20}$$

(20 - x) = 4x

X = 4m

Q.12 (c)



Vertical reaction at A

$$V_A = \frac{p\times(25\text{-}6.25)}{25} = 0.75P$$

Vertical reaction at B $V_B = 0.25P$

Horizontal thrust

$$H = \frac{12.5V_B}{5} = 0.625P$$

$$\therefore \tan\theta = \frac{VB}{H} = \frac{0.25P}{0.625P} = 0.4$$

Q.13 (b)

The vertical reaction at $B = V_B$ and the horizontal thrust at A and B be H

Taking moment of forces on right segment about C

$V_B \times 10 = H \times 5$

$H = 2V_B$

$V_B = 0.5H$

For left segment taking moment about C.

$V_A \times 8 - H \times 4 - 8 \times 4 = 0$

$V_A = 0.5H + 4$

But $V_A = V_B = 8kN$

So H = 4kN

Q.14 (a)

From Maxwell-Betti's theorem

$P_A \Delta_{AB} = M_{BA}\theta_{BA}$

$$\therefore \theta_{BA} = \frac{10\times0.5}{30}$$

= 0.1 radians

Q.15 (b)

Q.16 (d)

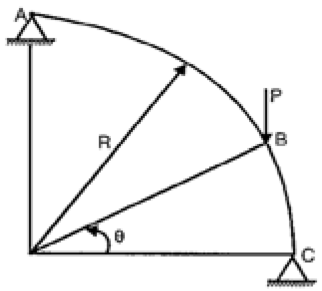
It is given by $H = \dfrac{4EI\alpha t}{\pi R^2}$

Q.17 (b)

$$H = \frac{120}{\pi}\sin^2 30^\circ + \frac{100}{\pi} + \frac{80}{\pi}\sin^2 60^\circ$$

$$= \frac{190}{\pi} kN$$

Q.18 (b)



Values of horizontal and vertical components of reaction at A are equal at θ = 45° since the vertical and horizontal components are given by sin θ and cos θ respectively, and sin θ = cos θ at 45° therefore the components are equal

Q.19 (a)

Vertical reaction at both support = $\dfrac{P}{2}$ (by symmetry)

Now taking moment about hinge at crown from LHS = 0

$$\Rightarrow \frac{P}{2}\times\frac{P}{2} - H\left(R-\frac{R}{\sqrt{2}}\right) = 0$$

$$\Rightarrow H = \frac{P}{2(\sqrt{2}-1)}$$

Q.20 (b)

GATEFLIX

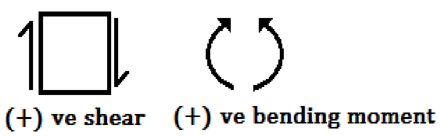
4 INFLUENCE LINE DIAGRAM

4.1 Introduction

- An influence line represents the variation of the reaction, shear, moments or deflection at a specified point in a member as a concentrated unit force moves over the member.
- Influence lines represent the effect of a moving load only at a specified point on a member, whereas shear and moment diagram represents the effect of fixed loads at all points along the member.
- Thus influence line helps in deciding, at a glance, where should the moving loads be placed on the structure so that it creates greatest influence at the specified point.

4.2 Sign Convention

- Influence line for reaction → (+) ve, if reaction acts upward.
- Influence line for shear and moment



Influence lines for statically determinate structures consist of straight-line segment.

Influence lines for statically indeterminate structures will consist of curved-line segments.

The following example illustrates the construction of influence line diagram.

4.3 Maximum S.F. and B.M. Values due to Moving Loads

A designer finds the position of moving; loads for which shear force and bending moment values are maximum. Influence line diagrams are used for this purpose. In this article, the following types of loads on a simply supported girder are considered:

1. Single point load
2. Uniformly distributed load longer than the span
3. Uniformly distributed load shorter than the span
4. A train of point loads.

4.4 Beam Subjected to a Single Point Load W

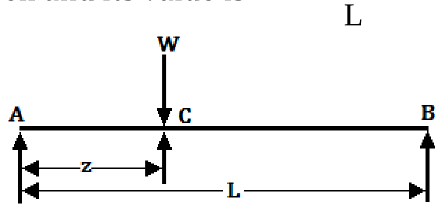
(a) Maximum S.F. & B.M. at a given point

- Let W be the moving load and the values of maximum S.F. and B.M. required be at C, which is at a distance z from A. ILD for S.F. and B.M. are as shown in Figure.

  It is clear that maximum negative S.F. occurs when the load is just to the left of section C and its value is $=\frac{Wz}{L}$.

  Similarly, maximum positive S.F. occurs when the load is just to the right of the section and its value is $= W\left[\frac{L-z}{L}\right]$.

  From ILD for moment M_C, it is clear that maximum bending moment will occur when the load is on the section itself and its value is $=\frac{Wz(L-z)}{L}$



Simply supported beam with given point C

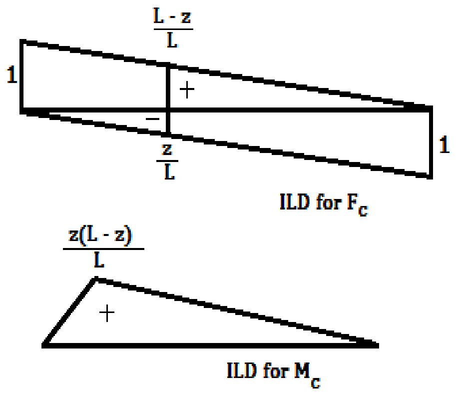


Fig. (1)

(b) Absolute maximum values anywhere in the beam

- For achieving this, ordinate of ILD should be maximum. Negative S.F. ordinate is maximum when z = L and is equal to 1.
- Therefore, absolute maximum negative S.F. = W and it occurs at B.
- For positive S.F., ILD ordinate has maximum value of 1, when z = 0; i.e., at A. Absolute maximum S.F. = W.
- Ordinate of ILD for moment is maximum when $z = \frac{L}{2}$. Hence, when a load is at mid-span, absolute maximum moment occurs and its value is $\frac{WL}{4}$

4.5 Uniformly Distributed Load Longer Than the Span

Let a uniformly distributed load of intensity w move from left to right.

(a) Maximum S.F. and B.M. at given Sections

- Load intensity and the area of ILD over loaded length give the value of stress resultant (SF/BM). Referring again to Figure (1)
- Negative S.F. is maximum, when the load covers portion AC only.
- Maximum negative F_C = ω x Area of ILD for F_C in length AC
  $$= w\left(\frac{1}{2}\right)z\left(\frac{z}{L}\right) = \frac{wz^2}{2L}$$
- Positive S.F. is maximum when the uniformly distributed load occupies the portion CB only and Maximum positive $F_C = \omega \times$ Area of ILD for F_C in length CB
  $$= w\left(\frac{1}{2}\right)(L-z)\left(\frac{L-z}{L}\right) = \frac{w(L-z)^2}{2L}$$
- From Figure, it is clear that maximum moment at C will be, when the udl covers entire span,
  $M_{,C.max} = \omega \times$ Area of ILD for M_c
  $$= w\left(\frac{1}{2}\right)L\frac{z(L-z)}{L} = \frac{wz(L-z)}{2}$$

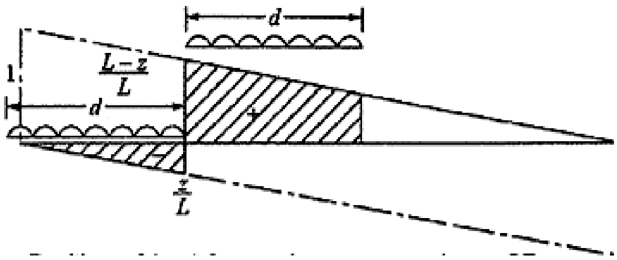
(b) Absolute maximum values anywhere in the beam

- Negative S.F. is maximum when z = L, i.e. at B when the load occupies entire span AB
  Absolute maximum negative S.F.
  $$= w\times\frac{1}{2}L\frac{z(L-z)}{L} = \frac{wz(L-z)}{2}$$
- Similarly, maximum positive S.F. occurs when z = 0; i.e., at A when the load occupies entire span AB
  Maximum positive S.F.
  $$= w\times\frac{1}{2}\times1\times L = \frac{wL}{2}$$
- Maximum moment at any section
  $$= \frac{1}{2}\times\frac{wz(L-z)}{L}\times L = \frac{1}{2}\times wz(L-z)$$
  This is maximum, when $z = \frac{L}{2}$ i.e., at mid-span
  Absolute maximum moment,
  $$= \frac{1}{2}\times w\times\frac{L}{2}\times(L-L/2) = \frac{wL^2}{8} \text{ at mid span}$$

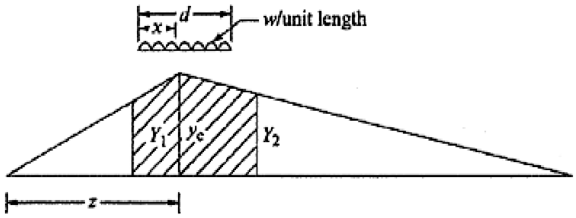
4.6 Uniformly Distributed Load Smaller Than the Span

- Let the length of uniformly distributed load ω/unit length is d. Let it move from left to right over beam AB of span L. We are considering the case when, $d < L$. Now, position of this load for maximum shear force and bending moment at section C are to be determined.

Simply supported beam with given point C



Position of load for maximum +ve & –ve SF



Position of load for maximum moment

- From ILD for shear force at C, it is clear that maximum shear force will develop when the head of the load reaches the section.
- For maximum positive shear force the tail of the udl should reach the section.
- Maximum bending moment will develop at C when the load is partly to the left of the section and partly to the right of section. Let the position of the section be as shown in Figure. Referring to this figure.

$$M_c = \omega X \frac{x(y_1 + y_c)}{2} + \omega(d - x)\frac{(y_c + y_2)}{2}$$

For M_C to be maximum

$$\frac{dM_c}{dx} = 0 = \frac{\omega(y_1 + y_c)}{2} - \frac{\omega(y_c + y_2)}{2}$$

i.e.,

Thus, moment at C will be maximum when the ordinates of ILD for M_c at head and tail of the udl are equal.

Now, $y_1 = y_2$

i.e.,

$$\frac{(z - x)}{z} y_C = \frac{(L - z) - (d - x)}{L - z} y_C$$

$$\therefore (z - x)(L - z) = z(L - z - d + x)$$

$$Lz - z^2 - Lx + xz = Lz - z^2 - dz + xz$$

i.e., $Lx = dz$

or $\frac{x}{d} = \frac{z}{L}$

i.e. Bending moment at a section is maximum when the load is so placed that the section divides the load in the same ratio as it divides the span.

Once the position of moving load is identified for maximum values the required values can be easily found.

Position for absolute maximum moment

Obviously for this y_c should be maximum

Now,

$$y_c = \frac{z(L - z)}{L}$$

For y_c to be maximum,

$$\frac{dy_c}{dx} = 0 = L - 2z$$

Or $z = \frac{L}{2}$

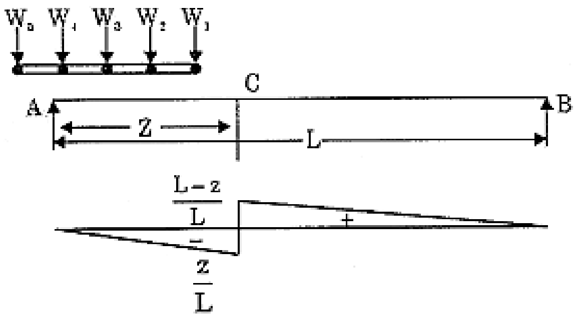
i.e. Absolute maximum moment occurs at mid-span. The position of the load is to be such that the section divides the load in the same ratio as it divides the span which means that for absolute maximum moment C.G. of the load will be at the mid-span.

4.7 A Train of Concentrated Load

A train of concentrated loads moving over a simply supported beam from left to right is shown in Figure. It is required to find:

a) Maximum shear force at C
b) Maximum bending moment at C
c) Absolute maximum shear force in the beam
d) Absolute maximum bending moment in the beam.

(a) Maximum shear force at C



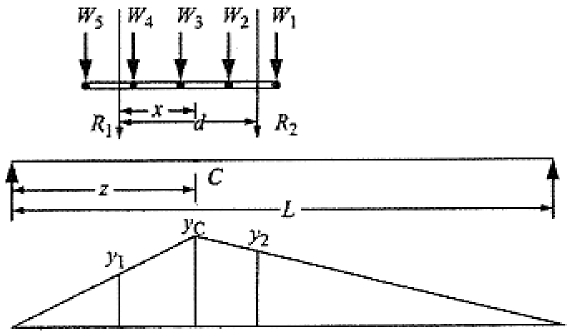
- Influence line diagram for shear force at C is shown in Figure. As soon as W_1 enters the span negative shear force develops at C. It increases as the load moves on. Some more loads may enter the span and hence, the rate of increase in S.F. goes up. This will continue till the load W_1 reaches the section C.
- As soon as W_1 crosses section C, it contributes to positive shear, thus, reducing the negative shear. Hence, there will be a drop in shear force value.
- Further movement causes more increase in shear force till the second load reaches C. There is a second peak value and a sudden drop, when the second load crosses.
- Thus, shear force will have a peak value whenever a load is on the section. Highest value among these peak values is to be selected.
- By two or three trial values, it is possible to get maximum negative shear force value.
- It is to be noted that for maximum negative shear force, most of the loads are to the left of the section.
- Similarly, for maximum positive shear force, there are peak values whenever a load comes on the section and the maximum value is obtained when most of the loads are to the right of the section.

(b) Maximum bending moment

- Let R_1 be the resultant of the loads on the left of the section and R_2 be resultant of the loads on the right of the section.

Distance between R_1 and R_2 be d and R_1 be at a distance x from C.

Let ordinate of ILD for moment at C be y_1 under R_1 and y_2 under R_2 and maximum ordinate at C be y_C.



ILD for M_C

$$M_c = R_1y_1 + R_2y_2$$

$$= R_1\left(\frac{z-x}{z}\right)y_c + R_2\frac{(L-z)-(d-x)}{L-z}\times y_c$$

For M_c to be maximum

$$\frac{dM_c}{dx} = 0 = -\frac{R_1y_c}{z} - R_2\left(\frac{y_c}{L-z}\right)\times y_c$$

$$\frac{R_1}{z} = \frac{R_2}{L-z}$$

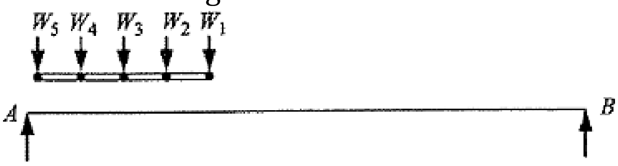
- i.e., the average load on the left-side portion of the beam is same as the average load on the right side portion of the beam.
- But, seldom have we got exactly equal average load on both sides of the section. For example, when load W_1 is to the left of the section, the average load on left side may be heavier. When it just rolls over the section, the average load on right-hand side may become heavier. Hence, the above condition for maximum bending moment can be interpreted as the bending moment is maximum when that load is on the section.
- Thus, due to a train of moving loads on a simply supported beam, maximum moment at the given section develops when the load W_1 is on the section where the load W_1 is such that as it rolls on the section and comes to the other side, heavier portion of the beam

becomes lighter and lighter portion becomes heavier.

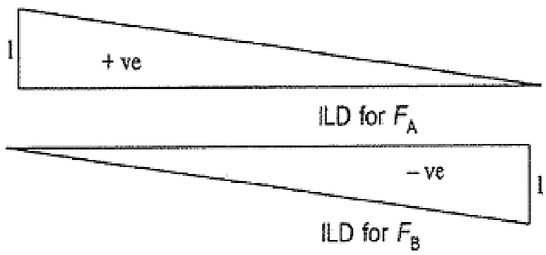
- In case of some load entering and some leaving the span, the change of portion heavier becoming lighter and lighter portion becoming heavier may happen under more than one particular load. All such cases are to be considered to identify which position gives maximum moment at the section.

(c) Absolute maximum shear force

At any section, influence line ordinate for negative shear is $\left(\frac{z}{L}\right)$ and for positive shear it is $\left(\frac{L-z}{L}\right)$. Hence, when z = 0, i.e., at support A, ILD ordinate for positive shear force is maximum (= 1) and when z-L, i.e., at support B, ILD ordinate for negative shear force is maximum (= 1). For shear force at support sections A and B are as shown in Figure.



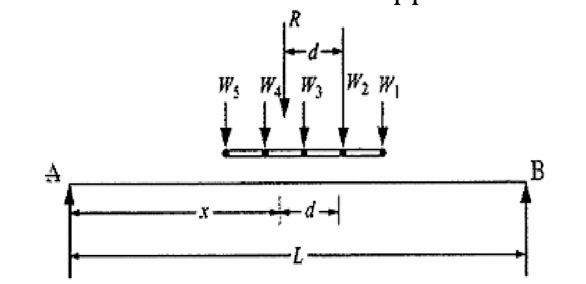
Simply supported beam subjected to a train of load



- Obviously maximum shear force occurs when one of the load is on support A.
- When the load starts moving from left to right, contribution of leading loads to shear force at A decreases but more number of loads may come on the beam and they will contribute to additional shear.
- However, no general conclusions can be drawn to say whether increase due to additional load is more or decrease due to the reduced contribution from leading loads is more. It needs a few trials to arrive at conclusions. However, it can be definitely said that one of the loads should be on the support A to get absolute maximum positive SF.
- Similarly, to get absolute maximum negative shear force, one of the loads should be on support B (just to the left of the section) and a few trials may be required to get absolute maximum negative shear force which occurs at support B.

(d) Maximum moment under a load

- Let a train of concentrated loads $W_1, W_2, W_3...$ move on a simply supported beam AB from left to right as shown in Figure. Now, the condition for moment to be maximum under wheel load W_2 is required. Let R be the resultant of all loads. Let its distance from W_2 be d and from support A be x.



Simply supported beam subjected to a train of load

Now, $R_A = \frac{R(L-x)}{L}$

Therefore, moment under load W_2

$$M = R_A(x+d) - Rd$$

$$= R\left[\frac{L-x}{L}\right](x+d) - Rd$$

$$= \frac{R}{L}\left[Lx - x^2 + Ld - xd\right] - Rd$$

For the moment to be maximum

$$\frac{dM}{dx} = 0 = \frac{R}{L}[L - 2x - d]$$

or $x = \frac{L}{2} - \frac{d}{2}$

Distance of W_2 from A

$= x + d = \frac{L}{2} - \frac{d}{2} + d = \frac{L}{2} + \frac{d}{2}$

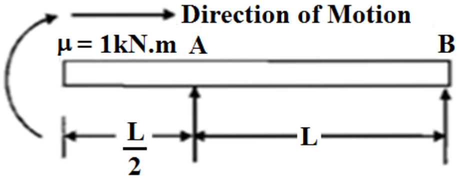
Thus, we can conclude, for moment to be maximum under any particular load, the load and the resultant should be equidistant from the mid-span.

(e) Absolute maximum bending moment

- Influence line diagram ordinate for bending moment is maximum at the center of span. Hence, bending moment will be maximum near the center of the span when heavier loads are near to the center.
- Since, the maximum moment always occurs under a wheel load, it can be concluded that absolute maximum moment occur under one of the loads when the resultant of all the loads and the load under consideration are equidistant from the center of the beam.
- The maximum moment under possible loads can be evaluated and the maximum of these selected as absolute maximum.

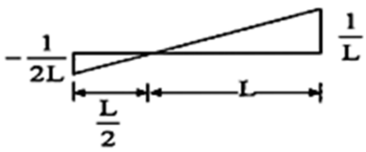
GATE QUESTIONS

Q.1 A simply supported beam with an overhang is traversed by a unit concentrated moment from the left to the right as shown below:

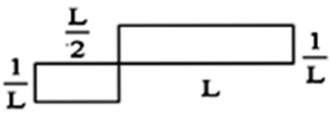


The influence line for reaction at B is given by

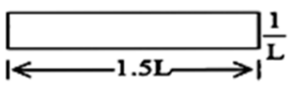
a)



b)



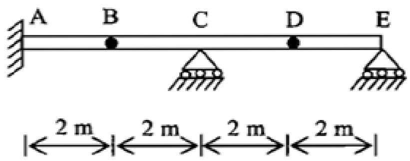
c)



d) Zero every where

[GATE- 2000]

Q.2 Identify, from the following, the correct value of the bending moment MA (in kNm units) at the fixed end A in the statically determinate beam shown below (with internal hinges at B and D), when a uniformly distributed load of 10 kN/m is placed on the spans. (Hint: Sketching the influence line for MA or applying the Principle of Virtual Displacements makes the solution easy).



a) -80 b) -40

c) 0 d) +40

[GATE - 2001]

Common Data for Q.3, Q.4 & 5:

A beam PQRS is 18 m long and is simply supported at points. Q and R 10m apart. Overhangs PQ and RS are 3m and 5m respectively. a train of two point loads of 150 kN and 100 kN, 5 m apart, crosses this beam from left to right with 100 k1NT load leading.

Q.3 The maximum sagging moment under the 150 kN load anywhere is

a) 500 kNm b) 450 kNm
c) 400 kNm d) 375 kNm

[GATE – 2003]

Q.4 During the passage of the loads, the maximum and the minimum reactions at supports `R' in kN, are respectively

a) 300 and -30 b) 300 and -25
c) 225 and -30 d) 225 and -25

[GATE – 2003]

Q.5 The maximum hogging moment in the beam anywhere is

a) 300 kNm b) 450 kNm
c) 500 kNm d) 750 kNm

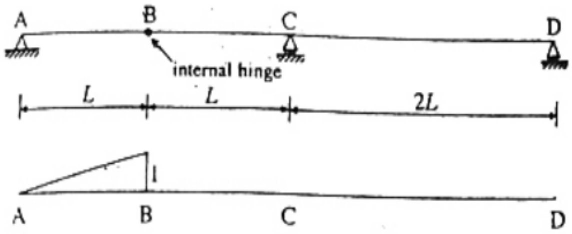
[GATE – 2003]

Q.6 Muller Breslau Principle in structural analysis is used for

a) Drawing influence line diagram for any force function
b) Writing virtual work equation
c) Superposition of load effects
d) None of these

[GATE-2003]

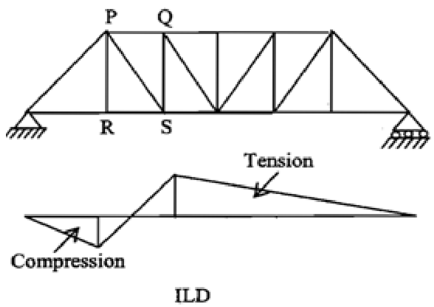
Q.7 Consider the beam A_BCD and the influence line as shown below. The influence line pertains to



a) Reaction at A, R_A
b) Shear force B, V_B
c) Shear force on the left of C, V^-_c
d) Shear force on the right of C, V^+_c

[GATE - 2006]

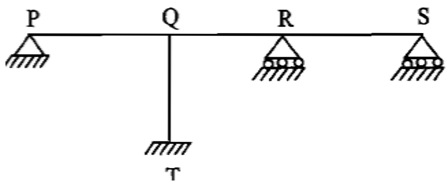
Q.8 The influence line diagram (I L D) shown is for the member



a) PS
b) RS
c) PQ
d) QS

[GATE - 2007]

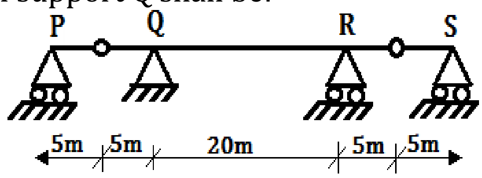
Q.9 The span(s) to be loaded uniformly for maximum positive (upward) reaction at support P, as shown in the figure below, is (are)



a) PQ only
b) PQ and QR
c) QR. and RS
d) PQ and RS

[GATE - 2008]

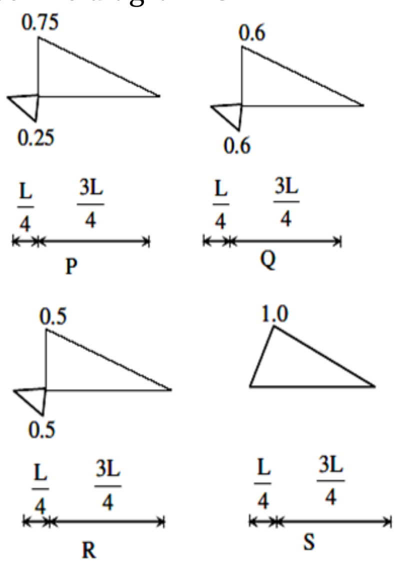
Q.10 Beam PQRS has internal hinges in spans PQ and RS as shown. The beam may be subjected to a moving distributed vertical load of maximum intensity 4 kN/m of any length anywhere on the beam. The maximum absolute value of the shear force (in kN) that can occur due to this loading just to the right of support Q shall be:



a) 30
b) 40
c) 45
d) 55

[GATE-2013]

11. In a beam of length L, four possible influence line diagrams for shear force at a section located at a distance of $\frac{L}{4}$ of from the left end support (marked as P, Q, R and S) are shown below. The correct influence line diagram is

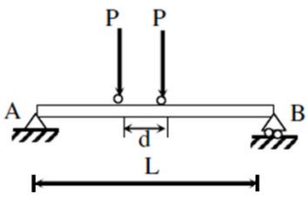


a) P
b) Q
c) R
d) S

[GATE-2014]

12. A simply supported beam AB of span, L = 24 m is subjected to two wheel loads acting at a distance, d =

5 m apart as shown in the figure below. Each wheel transmits a load, P = 3 kN and may occupy any position along the beam. If the beam is an I-section having section modulus, S = 16.2 cm^3, the maximum bending stress (in GPa) due to the wheel loads is _______.



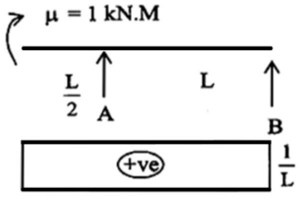
[GATE-2015]

ANSWER KEY:

1	2	3	4	5	6	7	8	9	10	11	12
C	C	C	A	C	A	B	A	D	C	A	1.78

EXPLANATIONS

Q.1 (c)



I.L.D of R_B

Whatever may be the position of couple, reactions are same at supports of s.s. beam with overhang.

$\sum M_A = 0$

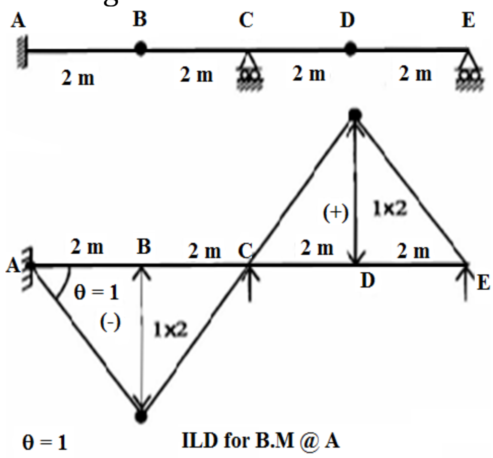
$R_B \times L = \mu = 1$

$\therefore R_B = \frac{1}{L}$

Q.2 (c)

Use Muller Breslau principle. To draw ILD for M_A, release the moment at 'A' by placing a moment hinges at 'A'.

Draw the deflected profile by allowing unit rotation at 'A'



$\therefore$ ordinate at 'B' $\frac{y}{2} = 1$

$\therefore y = 2$

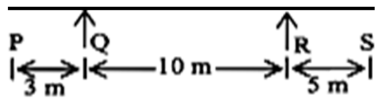
Similar ordinate at 'D' = 2

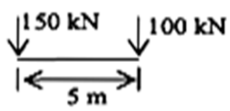
B.M. at A = intensity of loading × area under the loading

$$= 10 \times \left(\frac{1}{2} \times 4 \times 2\right) + 10\left(-\frac{1}{2} \times 4 \times 2\right) = 0$$

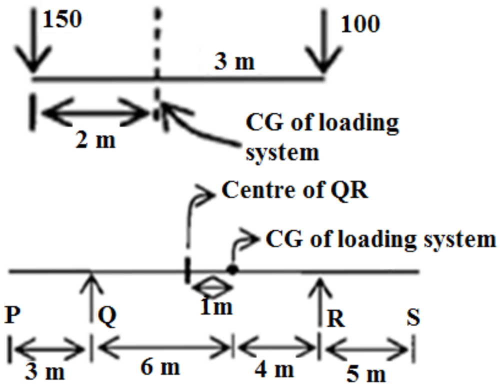
'Zero' can be visualized by seeing the diagram also.

Q.3 (c)





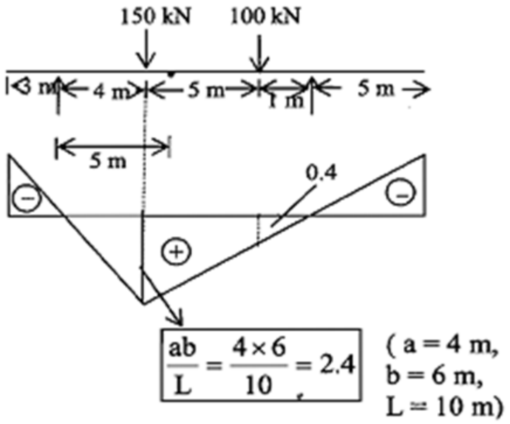
(1) ILD for maximum sagging moment under 150 kN C.G. of Loading system



$$\overline{X} = \frac{100 \times 5 + 150 \times 0}{100 + 150}$$

= 2 m from 150 KN load

For maximum sagging moment both C.G of load system and the chosen load 150 kN shall be placed at equal distances on either side of centre of span 'QR' as shown below.



ILD for maximum sagging moment

From similar triangle $\frac{2.4}{6} = \frac{y}{1}$

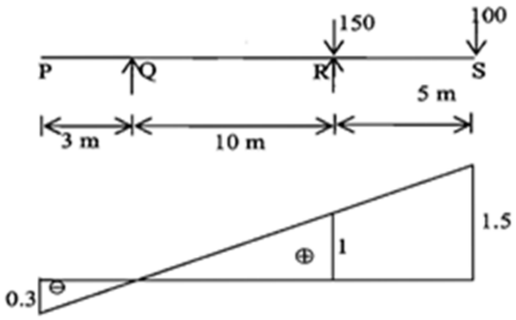
$\therefore$ y = 0.4 under 100 kN load.

Maximum sagging moment

= 150 $\times$ 2.4 + 100 $\times$ 0.4 = 400 kNm

Q.4 **(a)**

ILD for maximum and minimum reaction



ILD for Reaction at R

Maximum reaction

= 150 $\times$ 1 + 100 $\times$ 1.5 = 300 kN

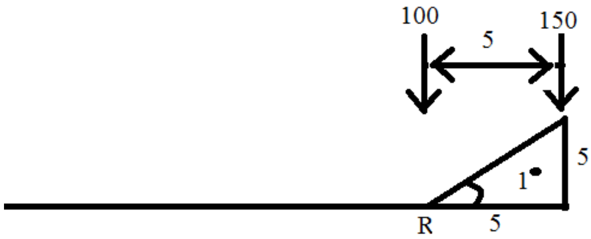
Minimum reaction we will get if leading load is at point P.

According to I.L.D. minimum reaction = - 100$\times$ 0.3 = - 30kN

Q.5 **(c)**

Having maximum overhang available near support R, Maximum hogging moment occurs at support R.

Apply unit moment at support R

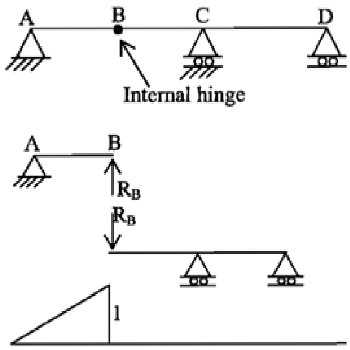


BM_{max} = 150*5 = 750 kNm

Q.6 **(a)**

Muller Breslau's principle can be used for drawing ILD of any force function not only of statically determinate structures, but also of indeterminate structures. It uses deflection profiles as the basis for ILD.

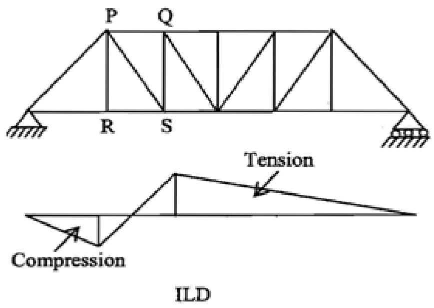
Q.7 **(b)**



If the unit load is at 'B'. Rb = 1. The reaction at a support is also equal to S.F at support. Hence considering free body diagram of AB, the ILD shown is of S.F at B.

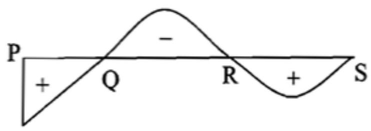
Q.8 **(a)**

The ILD is changing, sign in the region 'R S' which is focal length. The ILD changes sign either for diagonal members or vertical members. The focal length shown is of diagonal 'PS'. Hence the ILD shown is of Members 'PS'.



Q.9 **(d)**

"According to Muller Breslau's principle, the ILD for any force of a given structure is given by the deflection profile by release of that force to some scale". For getting ILD for reaction at 'P' release vertical force and allow it to deflect to some scale.



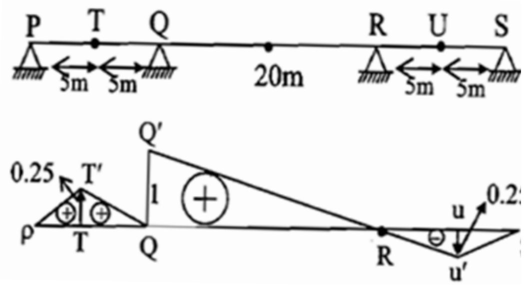
ILD for reaction at 'P' :

The reaction is positive for loads between placed on spans 'PQ' and 'RS'.

∴ Load is to be placed on span PQ & RS, in order to have maximum vertical reaction at 'P'.

Q.10 **(c)**

Apply Muller Breslau's principles. Release the S.F just right of 'Q' by making a cut. Give a unit displacement to just the right of 'Q' by doing so the slop to the right of 'Q' and left of 'Q' should be same, as per Muller Breslau. The deflected profile; by giving nit displacement as discussed above is shown below.



In 20m the ordinate is 1

In 5m the ordinate is 0.25

To have the maximum S.F just right of 'Q', place u.d.l on span 'PR'.

Hence

Max S.f = Intensity of u.d.l [area of I.L.D under U.D.L]

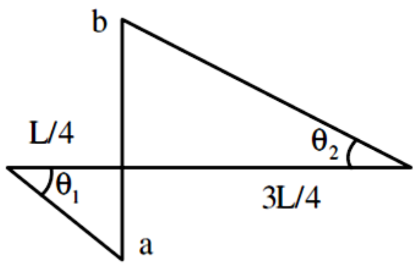
$$= 4\left[\frac{1}{2}\times 10\times 0.25+\frac{1}{2}\times 20\times 1\right]$$

$$= 4\,[1.25+10] = 4\times 11.25$$

Max S.F = 45 kN

11. **(a)**

$\theta_1 = \theta_2$



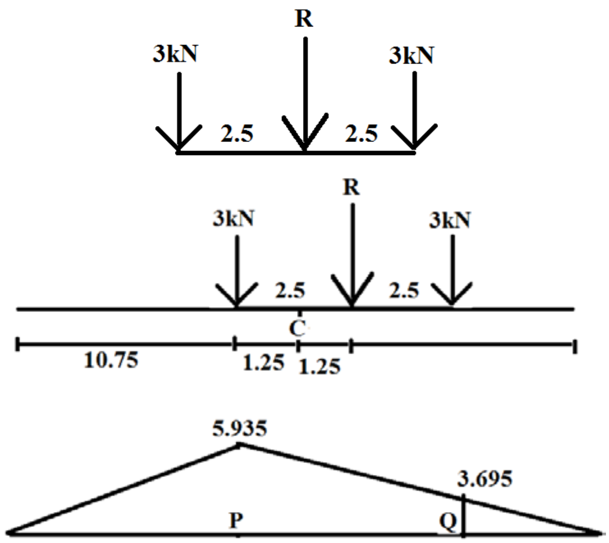
So, $\dfrac{a}{L/4} = \dfrac{b}{3L/4} \Rightarrow b = 3a$

Also $a + b = 1 \Rightarrow a = 1/4 = 0.25$

$\Rightarrow b = 3/4 = 0.75$

Q.12 **(1.78)**

For maximum bending moment both C.G of load system and the chosen load 3 kN shall be placed at equal distances on either side of centre of span 'QR' as shown below



$$\text{Max ordinate at P} = \frac{X(L-X)}{L}$$

$$= \frac{10.75(24-10.75)}{24}$$

$$= 5.935$$

$$\text{Ordinate at Q} = \frac{5.935*8.25}{13.25}$$

$$= 3.695$$

BM_{max} =3*5.935 + 3*3.696
=28.89 kNm

$$\sigma = \frac{M}{I} \cdot y = \frac{M}{Z}$$

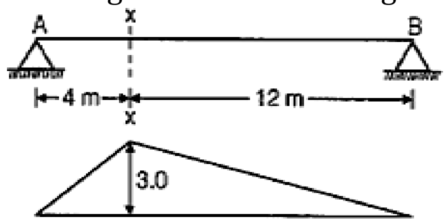
$$= \frac{28.89 \times 10^6}{16.2 \times 10^3}$$

$$= 1783.3 MPa$$

$$= 1.783 GPa$$

ASSIGNMENT

Q.1 The influence line diagram for bending moment at section X(M_x), at a distance of 4 m from the left support of a simply supported girder AB is shown in figure below. A uniformly distributed load of intensity 2 t/m longer than the span crosses the girder from left to right.



Influence line for M_x

The maximum bending moment at section X is equal to

a) 12 tm b) 24 tm
c) 48 tm d) 96 tm

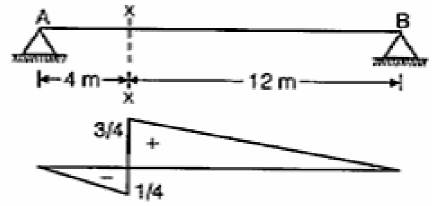
Q.2 The Muller-Breslau principle can be used to

1. Determine the shape of the influence line
2. Indicate the parts of the structure to be loaded to obtain the maximum effect
3. Calculate the ordinates of the influence lines

Which of these statements is/are correct?

a) only 1 b) both 1 and 2
c) both 2 and 3 d) 1, 2 and 3

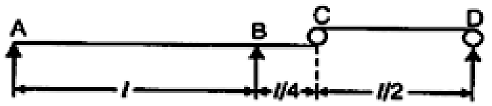
Q.3 The influence line for shear at section x(Fx) at a distance of 4 m from the left supported girder AB is shown in figure



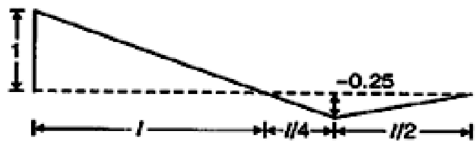
The shear force at section X due to a uniformly distributed dead load of intensity 2 t/m covering the entire span will be

a) 8t b) 4t
c) 2t d) 1t

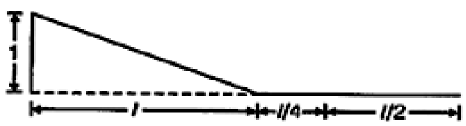
Q.4 A beam with cantilevered arm BC supporting a freely supported end span CD is shown in the figure. Which one of the figures represents the influence line for shear force at A



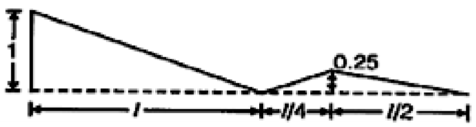
a)



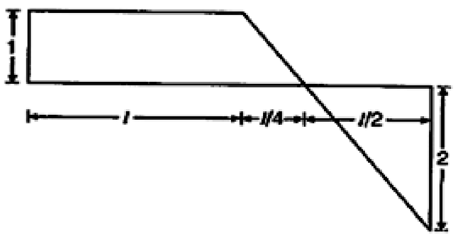
b)



c)



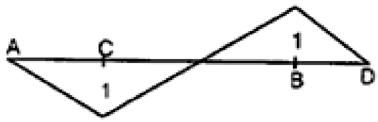
d)



Q.5 Which one of the following represents the correct influence line for bending moment at point C for the beam shown in the figure below.

a)

b)



c)

d)

Q.6 Three wheel loads 10t, 26t and 24t spaced 2 m apart roll on a girder from left to right with the 10t load leading. The girder has a span of 20 meter. For the condition of maximum bending moment at a section 8 meter from the left end

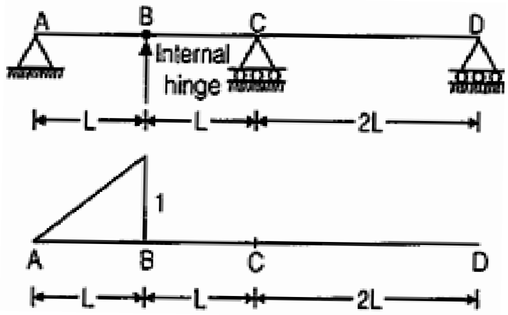
a) The 10t load should be placed at the section.

b) The 26t load should be placed at the section.

c) The 24t load should be placed at the section.

d) Either the 26t load or the 24t load should be placed at the section.

Q.7 Consider the beam ABCD and the influence line as shown below. The influence line pertains to



a) reaction at A_1 R_a

b) shear force at B_1 V_B

c) shear force on the left of C_1 V_{c-}

d) shear force on the left of C_1 V_{c+}

Q.8 The absolute maximum bending Moment in a simply supported beam of span 20 m due to a moving UDL of 4 t/m spanning over 5 m is

a) 87.5 t-m at the support

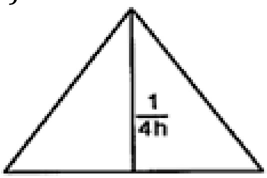
b) 87.5 t-m near the midpoint

c) 3.5 t-m at the midpoint

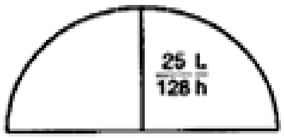
d) 87.5 t-m at the midpoint

Q.9 The influence line for horizontal thrust of a two-hinged parabolic arch of span ‘/ and rise ‘h’ will be shown in

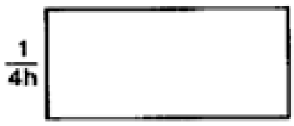
a)



b)



c)



d) None of the above

Q.10 A simply supported beam with overhangs is shown in the figure.

The influence line diagram for shear in respect of a section just to the right of the support 'A' will be as shown in

a)

b)

c)

d)

Q.11 Consider the following statements:

1. An influence line for a function (example: moment shear force, reaction, deflection) in a structure is a curve which shows its variation at a particular section of the structure for various positions of a moving unit load.
2. The influence line for bending moment/shear force must not be confused with bending moment diagram and shear force diagram for the structure,
3. The bending moment of diagram and shear force diagram shows the moment/shear values at all the sections of the structure. The influence line diagram for BM/SF is always drawn for moving unit price load and for a particular section only

Of these statements:

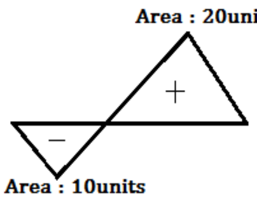
a) 1, 2 and 3 are correct
b) 1 and 2 are correct
c) 2 and 3 are correct
d) 1 alone is correct

Q.12 Which one of the following equations represents influence line of fixed end moment at B of the fixed beam AB of length l with origin at A?

a) $\frac{x^2(l-x)}{l^2}$ b) $\frac{x(l-x)^2}{l^2}$

c) $\frac{x(l-x)}{l}$ d) $\frac{x^2}{l^2}$

Q.13 Influence line diagram for a truss member is shown in the above figure.

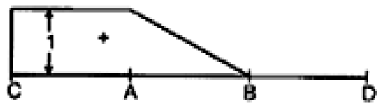


Positive values indicate tension. Dead load of the truss is 20kN/m and the live load is 10kN/m. Live load is longer than the span. Maximum tensile force in the member is:

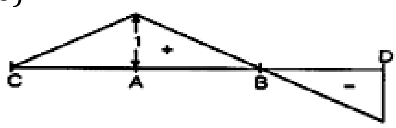
a) 600kN b) 400kN
c) 300kN d) 200kN

Q.14 Which one of the following is the influence line for reaction at A of the beam shown in the figure?

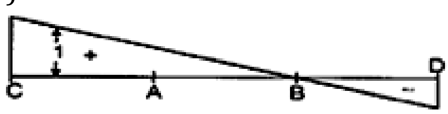
a)



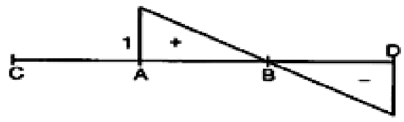
b)



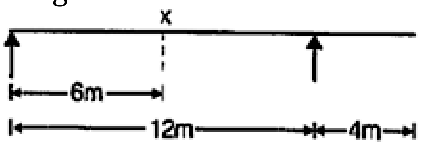
c)



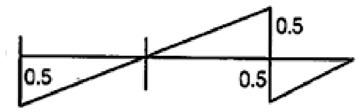
d)



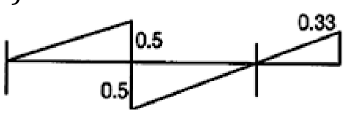
Q.15 Select the correct influence line diagram for shear force at x of the following beam.



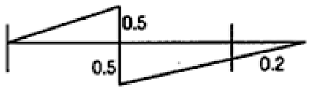
a)



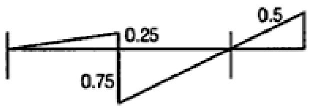
b)



c)



d)



Q.16 The maximum bending moment at the left quarter point of a simple beam due to crossing of UDL of length shorter than the span in the direction left to right, would occur after the load has just crossed the section by

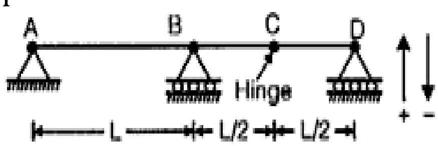
a) one-fourth of its length

b) half of its length

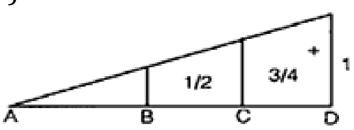
c) three-fourth of its length

d) its full length

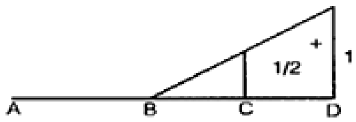
Q.17 For the continuous beam shown in figure. The influence line diagram for support reaction at D is best represents as



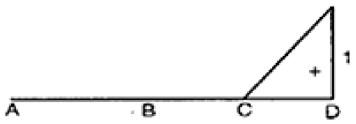
a)



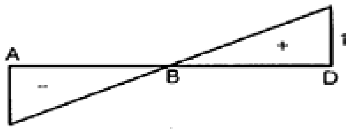
b)



c)



d)



Q.18 Consider the following statements:

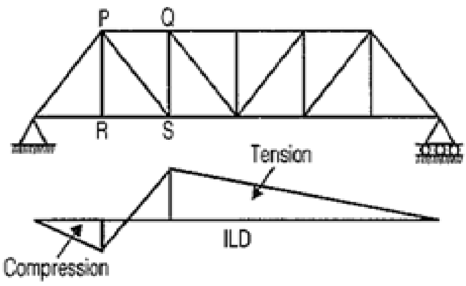
1. Influence Line Diagram (ILD) for SF at the fixed end of a cantilever and SFD due to unit load at the free end are same.
2. ILD for BM at the fixed end of a cantilever and BMD due to unit load at the free end are same.

Which of these statements is/are correct?

a) Only 1 b)Only 2

c) Both 1 and 2 d)Neither 1 nor 2

Q.19 The influence line diagram (ILD) shown is for the member



a) PS b) RS

c) PQ d) QS

Q.20 A uniformly distributed line of 60 kN per meter run of length 5 meters on a girder of span 16 meters. What is the maximum positive shear force at a section 6 meters from the left end?

a) 140.625 kN b) 65.625 kN

c) 90.625 kN d) 45.625 kN

ANSWER KEY:

1	2	3	4	5	6	7	8	9
(c)	(d)	(a)	(a)	(d)	(b)	(b)	(d)	(a)
10	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
(a)	(a)	(a)	(b)	(c)	(b)	(c)	(c)	(a)
19	**20**							
(a)	(a)							

EXPLANATIONS

Q. 1 (c)
The maximum bending moment at section-X will occur when UDL occupies the whole span.
∴ Maximum bending = Area under the influence line diagram

$$= \frac{1}{2}\times 4\times 3\times 2+\frac{1}{2}\times 12\times 3\times 2$$

$$= \frac{24+72}{2} = 48 \text{ t}-\text{m}$$

Q.2 (d)

Q.3 (a)
SF will be equal to the area under the influence line diagram

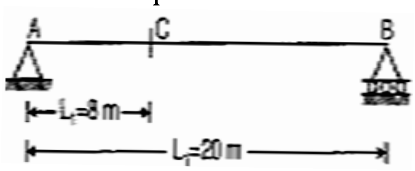
$$SF = 2\left[\frac{1}{2}\times 12\times\frac{3}{4}-\frac{1}{2}\times 4\times\frac{1}{4}\right]$$

$$=2\left[4.5-0.5\right]=8 \text{ t}$$

Q.4 (a)

Q.5 (d)

Q.6 (b)
Maximum bending moment at a section occurs when a particular load is on the section which changes the ratio $R_1/L_1 > R/L$ to $R_1/L_1 < R/L$ as the load passes over the section



Where $R_1 \rightarrow$ resultant of load on left side of section
Resultant of all loads (R)
$=10+26+24=60t$
$R/L=60/20=3t/m$
When 10t load crosses section C
$R_1=26+24=50t$

$R_1/L_1=50/8=6.25\ t/m > R/L$
When 26t load crosses the section C
R1 = 24t
$R_1/L_1=24/8=3t/m=R/L$
It means that maximum bending moment is obtained when 26t load is on the section

Q. 7 (b)

Q.8 (d)
Absolute maximum bending moment will occur at the centre when the loads is spread equally on either side of the centre.

$$M_{max} = \frac{wa}{4}\left(1-\frac{a}{2}\right)$$

$$M_{max} = \frac{4\times 5}{4}\left(20-\frac{5}{2}\right)$$

$=5\times 17.5=87.5$ t-m centre

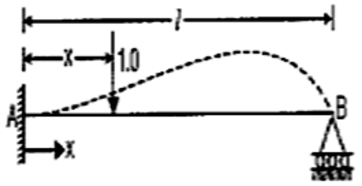
Q.9 (a)
Figure in (a) is the I.L.D of horizontal thrust for three hinged arch.

Q.10 (a)

Q.11 (a)

Q.12 (a)
For the influence line of fixed end moment at B, release the moment at B and give unit rotation, the deformed shape will represent the influence line

GATEFLIX



$$FEM_B = \frac{x^2(l - x)}{l^2}$$

Due to unit load at x distance from A.

Q.13 (b)

Tensile force due to D.L

$= (20 - 10) \times 20 = 200$ kN

Maximum Tensile force due to L.L

$= 20 \times 10 = 200$ kN

Thus maximum tensile force

$= 200 + 200$

$= 400$ kN

Q.14 (c)

Q.15 (b)

Maximum shear force is at either of the support due to a point load

Q.16 (c)

The load should be positioned such that section divides the span and load in the same ratio.

Q.17 (c)

The ILD for support reaction at D can be obtained by giving unit displacement in the direction of reaction. The deflected shape of beam will represent ILD as in figure(c).

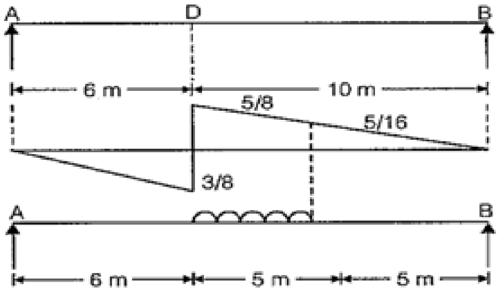
Q.18 (a)

ILD for BM at fixed end will have maximum ordinate when the unit load is at free end. While the BMD due to unit load at free end will have zero ordinate at free end and maximize ordinate at fixed end.

Q.19 (a)

Q.20 (a)

We must first draw the influence line diagram for the SF at the section D



For maximum positive SF at D, the loading should be applied as shown in the figure. Maximum positive = load × area of ILD

SF at D intensity covered by the load

$$= 60 \times \frac{5}{2}\left(\frac{5}{8} + \frac{5}{16}\right) \text{kN}$$

$$= \frac{1125}{8} \text{kN} = 140.625 \text{ kN}$$

5 MOMENT DISTRIBUTION METHOD

5.1 Introduction

- Moment distribution method is the most suitable manual method for analysis of continuous beams and plane frames. The method was presented by Prof. Hardy Cross of USA in 1929.
- The method consists in solving indirectly the equations of equilibrium as formulated in slope deflection method without finding the displacements. This is an iterative procedure. This is also known as relaxation method.

5.2 Basic Concept

In this method the analysis begins by assuming each joint in the structure to be fixed. Then, by unlocking and locking each joint in succession, The **internal moments** at the joints are distributed and balanced until the joints have rotated to then final or nearly final positions). The following examples has been given to illustrate the basic concept.

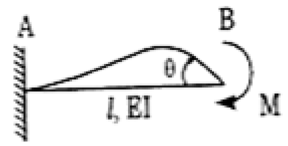
5.3 Sign Convention

Clockwise moment is taken as (+) ve, anticlockwise moment is taken as (-) ve i.e. the sign convention is same as that in slope deflection method for member end moment calculation.

5.4 Stiffness

Stiffness of member is moment required to produce unit rotation at a joint.

a) Far end Fixed



$$\theta = \frac{Ml}{4EI}$$

$$\text{Stiffness} \frac{M}{\theta} = K = \frac{4EI}{l}$$

b) Far end Hinged

$$\theta = \frac{Ml}{3EI}$$

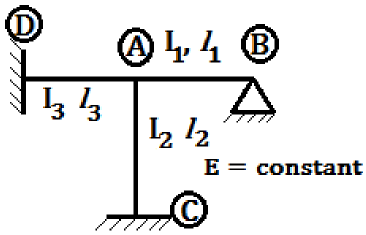
$$\text{Stiffness} \frac{M}{\theta} = K = \frac{3EI}{l}$$

5.5 Distribution Factor (D.F.): If moment is applied at a joint which distributes to connecting members in the proportion of their stiffness.
Hence, DF of a member is

$$DF = \frac{\text{Stiffness of member}}{\text{Total stiffness of all members at the joint}}$$

OR

$$DF = \frac{\text{Relative stiffness of member}}{\text{Total relative stiffness at the joint}}$$



If a moment 'M' is applied at joint A' due to which the joint rotates by θ_A then, Moment distributed in AD

$$M_{AD} = \frac{4EI_3\theta_A}{l_3}$$

Moment distributed in AC

$$M_{AC} = \frac{4EI_2\theta_A}{l_2}$$

Moment distribution in AB

$$M_{AB} = \frac{3EI_1\theta_A}{l_1}$$

$$M_{AD} : M_{AC} : M_{AB} = \frac{I_3}{l_3} : \frac{I_2}{l_2} : \frac{3}{4}\frac{I_1}{l_1}$$

$\frac{I_3}{l_3}$ = Relative stiffness of AD

$\frac{I_2}{l_2}$ = Relative stiffness of AC

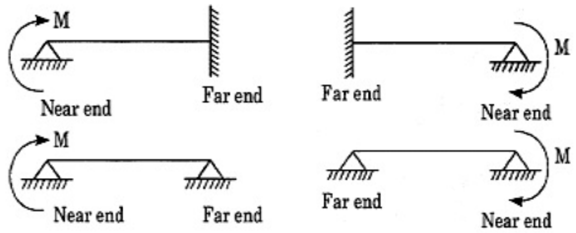
$\frac{3}{4}\frac{I_1}{l_1}$ = Relative stiffness of AB

$\Rightarrow$ Relative stiffness when far end is fixed $= \frac{I}{l}$

$\Rightarrow$ Relative stiffness when far end is hinged $= \frac{3}{4}\frac{I}{l}$

Note:

Meaning of far end



Stiffness of member AD

$$K_{AD} = \frac{4EI_3}{l_3}$$

Stiffness of member AC

$$K_{AC} = \frac{4EI_2}{l_2}$$

Stiffness of member AB

$$K_{AB} = \frac{3EI_1}{l_1}$$

$$\sum K = K_{AB} + K_{AC} + K_{AD}$$

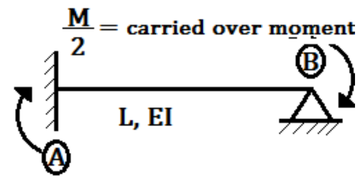
DF for AB, $= \frac{K_{AB}}{\sum K}$

DF for AC, $= \frac{K_{AC}}{\sum K}$

DF for AD, $= \frac{K_{AD}}{\sum K}$

5.6 Carry over Factor

$\Rightarrow$ Carry over Factor $= \frac{\text{Carried over moment}}{\text{Applied moment}}$



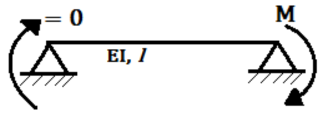
EI = constant throughout AB

M = applied Moment

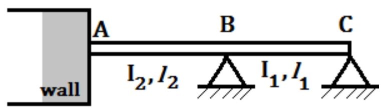
When moment 'M' is applied at pin,

Moment carried over to fixed far end $= \frac{M}{2}$

However, in the following case, carry over factor = 0.



5.7 Distribution Factor for Fixed and Pin Ends



At end 'A' two members, Wall and AB joins. Hence, distribution factor at

$$A = \frac{K_{AB}}{\infty + K_{AB}} = 0$$

DF of fixed end = 0

Note:

Wall has infinite stiffness i.e. for all applied moments; rotation is zero as in case of fixed ends $\frac{M_{AB}}{0} = \infty$

At end 'C' there does only one member exist at joint C, i.e. CB

$\Rightarrow$ D.F at C $= \frac{K_{CB}}{K_{CB}} = 1$

$\Rightarrow$ $\boxed{\text{D.F. of hinged end } = 1}$

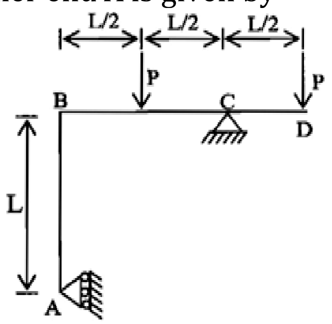
From these we conclude that **wall** or **fixed ends** does not rotate and absorbs all the moment carried over to it

5.8 Causes of sway of frames

- Unsymmetrical loading
- Different end condition od column
- Non uniform section(change in EI)
- Unsymmetrical outline
- Horizontal loading
- Settlement of supports

GATE QUESTIONS

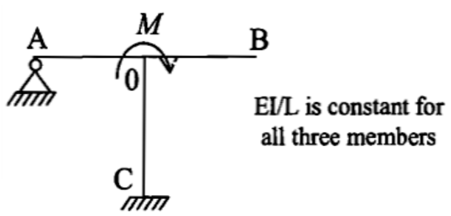
Q.1 A frame ABCD is supported by a roller at A and is on a hinge at C as shown below: The reaction at the roller end A is given by



a) P b) 2P
c) P/2 d) Zero

[GATE-2000]

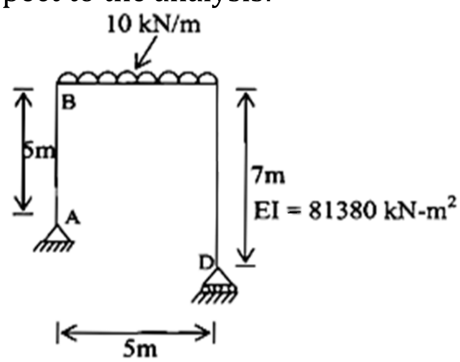
Q.2 The frame below shows three beam elements OA, OB and OC, with identical length L and flexural rigidity EI, subject to an external moment M applied at the rigid joint O. The correct set of bending moments $\{M_{OA}, M_{OB}, M_{OC}\}$



a) {3M/8, M/8, 4M/8}
b) {3M/11, 4M/11, 4M/11}
c) {M/3, M/3, M/3}
d) {3M/7, 0, 4M/7}

[GATE-2001]

Q.3 The plane frame below is analysed by neglecting axial deformations. Following statements are made with respect to the analysis:

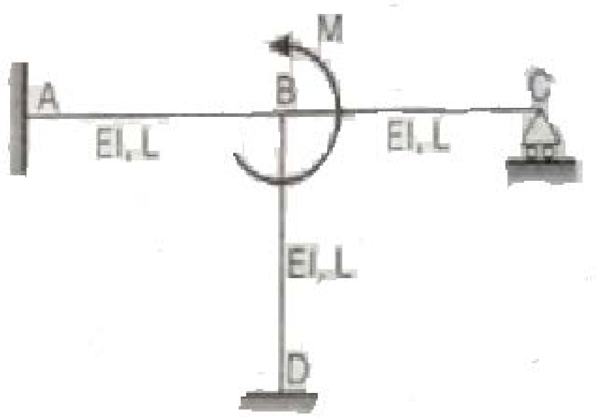


1. Column AB carries axial force only.
2. Vertical deflection at the center of beam BC is 1 mm. With reference to the above statements, which of the following applies?

a) Both the statements are true
b) Statement 1 is true but 2 is false
c) Statement 2 is true but 1 is false
d) Both the statements are false

[GATE - 2004]

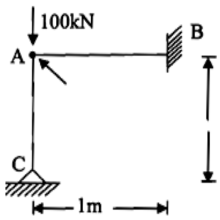
Q.4 All members of the frame shown below have the same flexural rigidity EI and length E. if a moment M is applied at joint B, the rotation of the joint is



a)ML/12El b)ML/11El
c)ML/8EI d)ML/7El

[GATE – 2005]

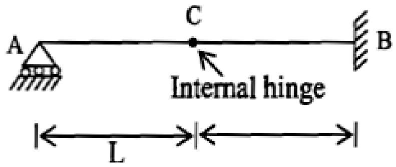
Q.5 Vertical reaction developed at B in the frame below due to the applied load of 100 kW (with 150, 000 mm2 cross-sectional area and 3.125 x 109 mm4 moment of inertia for both members) is



a) 5.9 kN b) 30.2 kN
c) 66.3 kN d) 94.1 kN

[GATE -2006]

Q.6 Carry-over factor CAB for the beam shown in the figure below is



a) 1 / 4 b) 1 / 2
c) 3 / 4 d) 1

[GATE – 2006]

Common .Data for Q.11 and Q.12:
A two span continuous beam having equal spans each of length L is subjected to a uniformly distributed load w per unit length. The beam has constant flexural rigidity.

Q.7 The reaction at the middle support is

a) wL b) 5wL/2
c) 5wL/4 d) 5wL/8

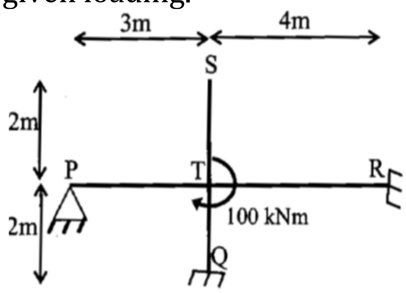
[GATE-2007]

Q.8 The bending moment at the middle support is

a) $wL^2/4$ b) $wL^2/8$
c) 5wL/4 d) 5wL/8

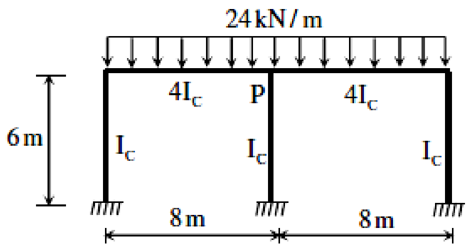
[GATE - 2007]

Q.9 All members in the rigid-jointed frame shown are prismatic and have the same flexural stiffness EL Find the magnitude of the bending moment at Q (in kNm) due to the given loading.



[GATE-2013]

Q.10 Considering the symmetry of a rigid frame as shown below, the magnitude of the bending moment (in kNm) at P (preferably using the moment distribution method) is

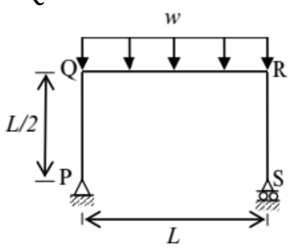


a) 170 b) 172
c) 176 d) 178

[GATE-2014]

Q.11 The portal frame shown in the figure is subjected to a uniformly distributed vertical load w (per unit length).

The bending moment in the beam at the joint 'Q' is



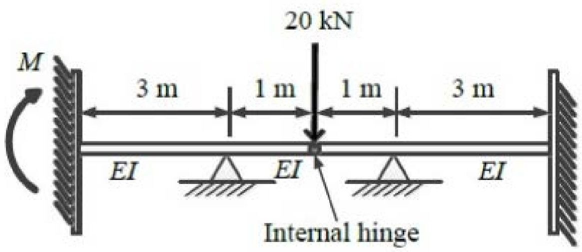
The bending moment in the beam at the joint 'Q' is

a) zero b) $\frac{wL^2}{24}$(hogging)

c) $\frac{wL^2}{12}$(hogging) d) $\frac{wL^2}{8}$(hogging)

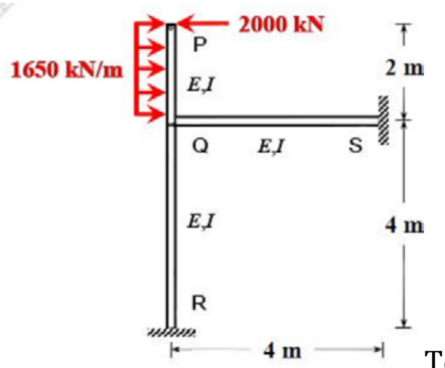
[GATE-2016]

Q.12 For beam shown below, the value of support moment is______kNm



[GATE-2015-1]

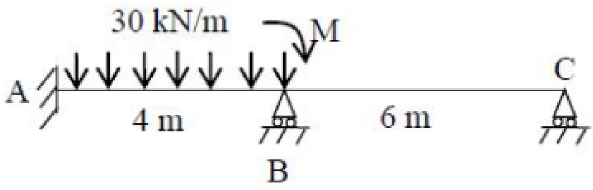
Q.13 consider portal frame shown in figure. Assume E=2.5*10^4 MPa , I = 8*10^8 mm^4 for all member of frames.



Total rotation (in degree up to one decimal place) at the rigid joint Q would be____

[GATE-2017-2]

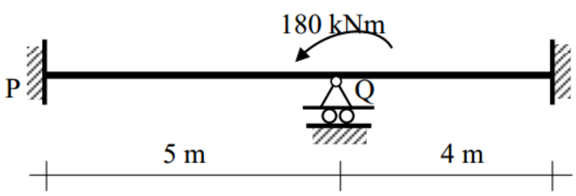
Q.14 The value of moment M in beam ABC shown in figure such that joint B does not rotate.



The value of support reaction (in kN) at B should be equal to_____

[GATE-2017-1]

Q.15 A prismatic beam PQR of flexural rigidity EI= 1*10^4 kN.mm^2 is subjected to moment of 180 kNm at Q as shown in figure.



The rotation at Q (up to two decimal places) is _____

[GATE-2018-2]

ANSWER KEY:

1	2	3	4	5	6	7	8	9
D	D	A	B	A	D	C	B	25
10	**11**	**12**	**13**	**14**	**15**			
C	A	5	1°	60	0.01			

EXPLANATIONS

Q.1 (d)

Apply $\Sigma M_c = 0$

Say horizontal reaction at A as H_A.

$$H_A . L + \frac{P.L}{2} = \frac{P.L}{2}$$

$\therefore H_A = 0$

Hence reaction at Roller end A = 0

Q.2 (d)

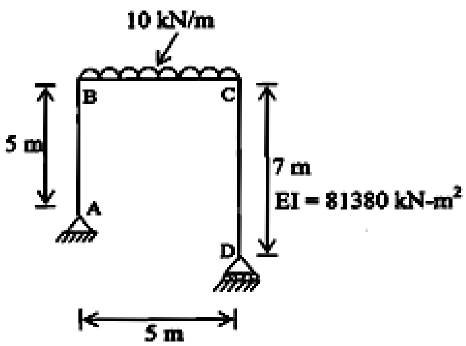
Joint	Part	K	$\sum K$	DF=$\frac{K}{\sum K}$	Member Moment =DF*M
O	OA	$\frac{3EI}{l}$	$\frac{7EI}{l}$	$\frac{3}{7}$	$\frac{3}{7}M$
	OB	0		0	0
	OC	$\frac{4EI}{l}$		$\frac{4}{7}$	$\frac{4}{7}M$

Q.3 (a)

The frame has no horizontal loads.

Hence no horizontal reaction at 'A'.

$\therefore$ Column AB is subjected to axial force only.



Vertical deflection at the centre of the span

$$= \frac{5WL^4}{384EI} = \frac{5\times10\times5^2}{384\times813} = 1\text{ mm}$$

Q.4 (b)

Joint	Part	K	$\sum K$
B	BA	$\frac{4EI}{l}$	$\frac{11EI}{l}$
	BC	$\frac{3EI}{l}$	
	BD	$\frac{4EI}{l}$	

$\because$ rotation at B, $\theta_B = \frac{M}{K} = \frac{M.L}{11EL}$

Q.5 (a)

At joint 'A'

Deflection in Beam = Contraction in Column

$$\because \frac{(100-R).L^3}{3EI} = \frac{R.L}{AE}$$

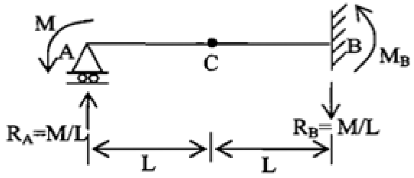
$$\frac{(100-R)\times(1000)^2}{3\times3.125\times10^9} = \frac{R}{150,000}$$

R = 5.88 kN (say) 5.9 kN

Q.6 (d)

Carry over factor

$$C_{AB} = \frac{\text{Moment developed at far end}}{\text{Moment applied at near end}}$$



let us apply moment 'M' at A

for R_A ; take moment at C = 0

$\therefore \Sigma M_c = 0 \quad \therefore R_A \times L = M$

$R_A = M/L$ (upward)

& $R_B = \frac{M}{L}$ (downward)

Again $\Sigma M_c = 0$ from right side

$\therefore M_B = R_B \times L$

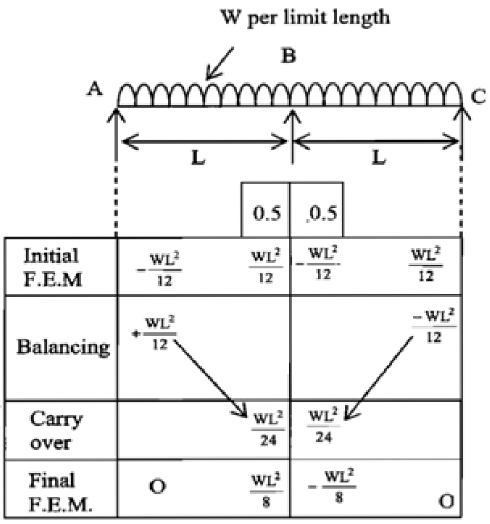
$\therefore M_B = \frac{M}{L} \times L$

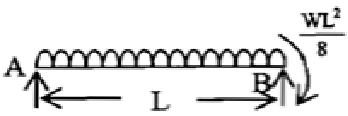
$\therefore M_B = M$

Carry over factor

$= \frac{\text{Moment at B}}{\text{Moment at A}} = \frac{M}{M} = 1$

Q.7 (c)

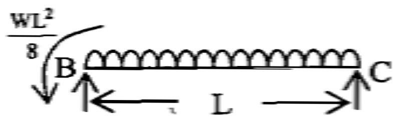




Take moment at A, $\Sigma M_A = 0$

$$R_B \times L = \frac{WL^2}{8} + \frac{WL^2}{2}$$

$$\therefore R_B = \frac{5WL^2}{8}$$



Take moment at C, $\Sigma M_C = 0$

$$\therefore \text{Total } R_B = \frac{5WL}{8} + \frac{5WL}{8} = \frac{5WL}{4}$$

$$R_B \times L = \frac{WL^2}{8} + \frac{WL^2}{2}$$

$$\therefore R_B = \frac{5WL^2}{8}$$

The given continuous beam can be treated as two symmetrical propped cantilevers.

Each span $R_B = \frac{5WL}{8}$

(It is a standard case, which can be remembered)

Q.8 (b) As per above solution

Q.9 (25kNm)

Joint	Part	K	∑K	DF= $\frac{K}{\sum K}$	Member Moment =DF*M
T	TP	$\frac{3EI}{3}$	4EI	$\frac{1}{4}$	$\frac{1}{4}M$
	TQ	$\frac{4EI}{2}$		$\frac{2}{4}$	$\frac{2}{4}M$
	TR	$\frac{4EI}{4}$		$\frac{1}{4}$	$\frac{1}{4}M$
	TS	0		0	0

$$\text{Moment transfer at TQ} = \frac{2}{4}M = \frac{2}{4}*100$$

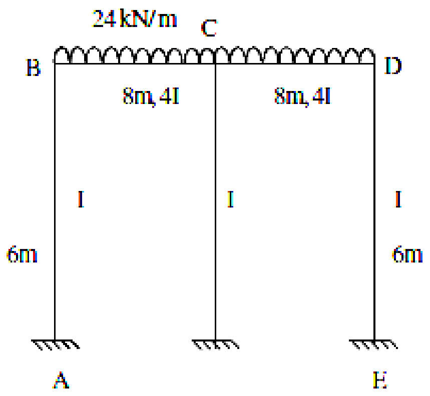
$$= 50\text{kNm}$$

Carry over factor = 1/2

$$\text{Moment transfer at Q} = \frac{50}{2}$$

$$= 25\text{kNm}$$

Q.10 (c)



Axis of symmetry is passing through a column; hence it can be treated as

Member Stiffness D.F

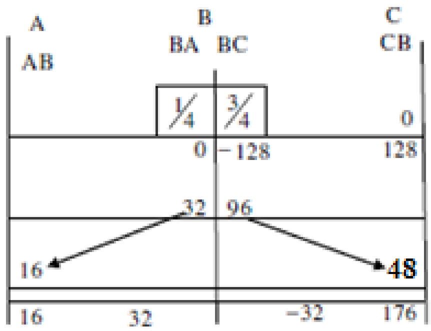
$$BA \quad \frac{4EI}{6} \quad \frac{1}{4}$$

$$BC \quad \frac{4EI(4I)}{8} \quad \frac{3}{4}$$

FE11:

$$M_{BC} = \frac{-WL^2}{12} = \frac{-24\times8\times8}{12} = -128kNM$$

M_{CB} = +128 kNM



BM at C = 176 kNm

Q.11 (a)

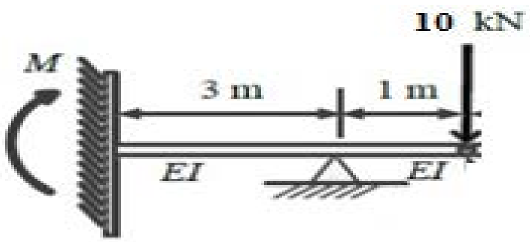
V_P=wl/2

V_S=wl/2

So, H_P=0

$\Rightarrow M_Q$ =0

Q.12 (5kNm)



Moment at hinge is 10*1=10kNm

Carry over moment at fixed end = 10/2

=5kNm

Q.13 (1°)

Moment at joint Q=2000*2-1650*2*1

=700 kNm

$$\sum K = \frac{4EI}{4} + \frac{4EI}{4} = 2EI$$

$$EI = 2.5*10^4 \frac{N}{mm^2} * 8*10^8 mm^4$$

$$EI = 2.5*10^4*10^3 \frac{kN}{m^2} * 8*10^8*10^{-12} m^4$$

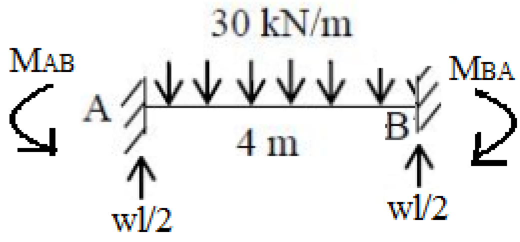
$$EI = 20000 kN.m^2$$

$$\theta = \frac{M}{K} = \frac{700}{2EI} = \frac{350}{EI} = \frac{350}{20000}$$

$$\theta = 0.0175 * \frac{180}{\pi} = 1^\circ$$

Q.14 (60 kN)

As joint B is not getting rotates means it has to treat as Fixed where rotation is always 0. Now, beam is treated as fixed beam.



R_B =wl/2 =30*4/2 = 60 kN

(Reaction at B will not come from BC span as there is not any load on span)

Q.15 (0.01 rad)

$$\sum K = \frac{4EI}{5} + \frac{4EI}{4} = 1.8EI$$

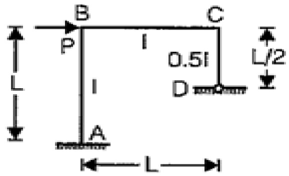
$$EI = 10000 kN.m^2$$

$$\theta = \frac{M}{K} = \frac{180}{1.8EI} = \frac{180}{1.8*10000} = 0.01 rad$$

ASSIGNMENT

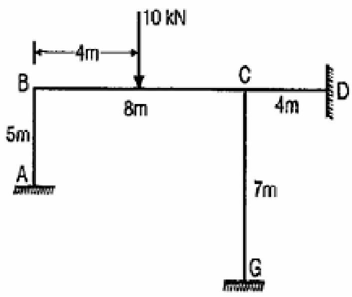
Q.1 The given figure shows a portal

frame with one end fixed and another end fixed and other hinged. The ration of the fixed end moments $\frac{M_{BA}}{M_{CD}}$ due to side sway will be



a) 1.0 b) 2.0
c) 2.5 d) 3.0

Q.2 The distribution factors for members CB, CD and CG for the frame shown in the figure (EI constant) will be respectively

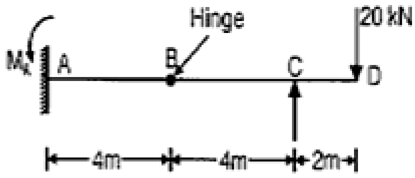


a) 0.24, 0.28 and 0.48
b) 0.24, 0.48 and 0.28
c) 0.48, 0.24 and 0.28
d) 0.28, 0.48 and 0.24

Q.3 A fixed beam AB is subjected to a triangular load varying from zero at end A to w per unit length at end B. The ration of fixed end moment at B to A will be

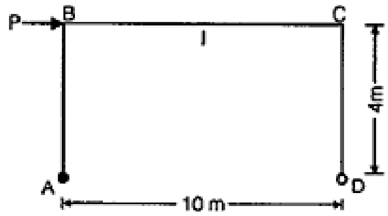
a) $\frac{1}{2}$ b) $\frac{1}{3}$
c) $\frac{2}{3}$ d) $\frac{3}{2}$

Q.4 The fixed end moment M_A for the beam shown in the figure is



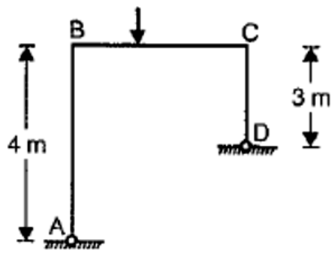
a) +40 kN.m b) -40 kN.m
c) +80 kN.m d) -80 kN.m

Q.5 A portal frame is shown in the given figure. If $\theta_B = \theta_B = \frac{400}{EI}$ radian, then the value of moment at B will be



a) 120 kNm b) 240 kNm
c) 360 kNM d) 480 kNm

Q.6 For the frame as shown in figure below, the final end moment M_{BC} has been calculated as −40 kN-m. What is the end moment M_{CD} ?

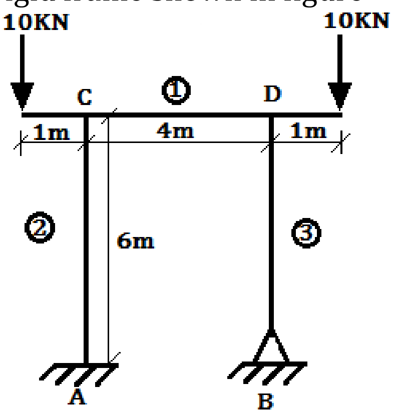


a) +40kN-M b) -40kN-M
c) +30kN-M d) -30kN-M

Q.7 A horizontal fixed beam AB of span 6 m has uniform flexural rigidity of 4200 kN/ m². During loading the support B sinks downwards by 25 mm. The moment induced at the end A is

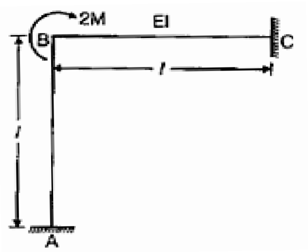
a) 17.5 kN m(Anticlockwise)
b) 17.5 kN m(Clockwise)
c) 105 kN m(Anticlockwise)
d) 105 kN m(Clockwise)

Q.8 The possible direction of sway of the rigid frame shown in figure



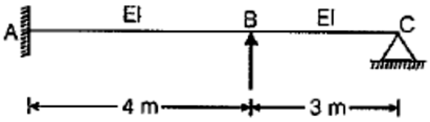
a) is towards left
b) is towards right
c) does not exist as there is no sway
d) cannot be ascertained

Q.9 Members AB and BC in the figure shown are identical. Due to a moment 2M applied at B, what is the value of axial force in the member AB?



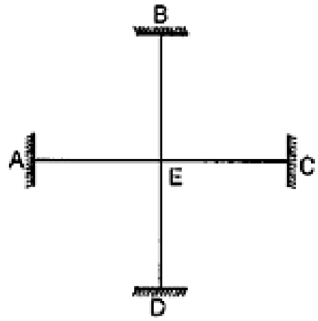
a) M/l (compression)
b) M/ l (tension)
c) 1.5 M/ l (compression)
d) 1.5 M/ l (tension)

Q.10 What are the distribution factors at joint B for the members BA and BC respectively in the figure?



a) 0.57 and 0.43 b) 0.43 and 0.57
c) 0.50 and 0.50 d) 0.36 and 0.64

Q.11 Four identical beams AE, BE, CE and DE have been rigidly joined at E. The point C slip and routes along with member firmly fixed at E



Which one among the following is correct?

a) There is no moment on the members
b) Except at C, there is no moment on the members of frame
c) Except at C and E for member EC, no moment will be there on other members
d) All the members are subjected to moment.

Q.12 A uniform simply supported beam is subjected to a clockwise moment M at the left end. What is the moment required at the right end so that rotation of the right end is zero?

a) 2M b) M
c) M/2 d) M/3

ANSWER KEY:

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
(a)	(b)	(d)	(b)	(b)	(d)	(a)	(c)
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
(d)	(c)	(d)	(c)				

EXPLANATIONS

Q.1 (a)

Due to sway, the deflection of point B will be equal to that of point C.

$$M_{BA} = \frac{6El\Delta}{L^2}$$

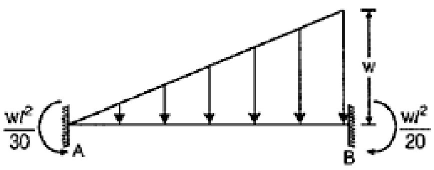
$$M_{CD} = \frac{3E(0.5l)\Delta}{(L/2)^2} = \frac{6el\Delta}{L^2}$$

$$\therefore \frac{M_{BA}}{M_{CD}} = 1.0$$

Q.2 (b)

Member	Relative stiffness	Distribution Factor
CB	El/8	0.24
CD	El/4	0.48
CG	El/7	0.28
Total	$\frac{29El}{56}$	1.00

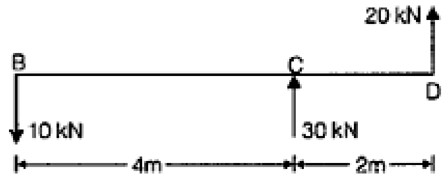
Q.3 (d)



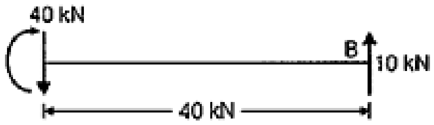
$$\frac{M_B}{M_A} = \frac{3}{2}$$

Q.4 (b)

For part B.D.



For Part AB



$\therefore$ $M_A = -40$ kN-m (Hogging)

Q.5 (b)

The deformed shape of structure will have translation at B and C

$$M_{BC} = 0 + \frac{2EI}{10}(2\theta_B + \theta_C)$$

$$= \frac{2EI}{10} \times 3 \times \frac{400}{EI} = 240 \text{ kN-m}$$

Q.6 (d)

The shear force at end A should be equal and opposite to shear force at D. Let -40 kN-m denotes the clockwise moment so moment at end B of column AB is 40 kN-m anticlockwise and the shear force at A is 10 kN towards left.

Therefore shear force at D is 10 kN towards right. Thus end moment at C is 30 kN-m clockwise or -30 kN-m.

Q.7 (a)

$$M_{AB} = \frac{6EI\delta}{L^2} = \frac{6 * 4200 * 0.025}{6^2}$$

$$M_{AB} = 17.5(anticlockwise)$$

Q.8 (c)

Q.9 (d)

The distribution factors for member AB and BC at B will be 0.5 for both.

Thus member end moment at BC is M.

Moment at C = $\frac{M}{2}$

Reaction at B $R_B = \frac{1.5M}{l}$

This reaction is away from B so axial force is member AB is tensile. This can be found from body diagram of AB. Similarly axial force in member BC will also be $\frac{1.5M}{l}$ but compressive.

$\therefore\ M' = \frac{M}{2}$

Q.10 (c)

Distribution Factors $D_{BA} = \frac{k_{BA}}{k_{BA} + k_{Bc}}$,

$D_{BC} = \frac{k_{BA}}{k_{BA} + k_{Bc}}$

For propped cantilever portion BA

$k_{BA} = \frac{4EI}{4} = EI$

For simply supported portion BC

$k_{BC} = \frac{3EI}{3} = EI$

Thus $D_{BA} = D_{BC} = 0.5$

Q.11 (d)

Since the member are firmly fixed at E. It is a rigid joint at which joint at which all the members will have same rotation. So from slope deflection equations all the members will have moment.

Q.12 (c)

Rotation	Left End	Right End
Clockwise	$\frac{ML}{3EI}$	$\frac{ML}{6EI}$

Moment (M) (clockwise) (anticlockwise) at left end to keep rotation at right and zero, a moment should be applied in anticlockwise direction. Let the moment in ‘M’

$\frac{ML}{3EI} = \frac{ML}{6EI}$ (for zero rotation)

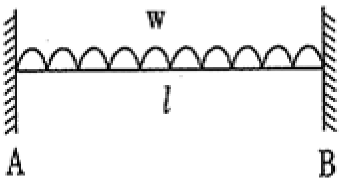
6 SLOPE DEFLECTION METHOD

6.1 Introduction

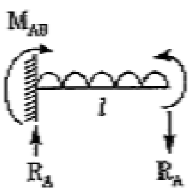
- Slope deflection method is useful to analyses indeterminate structures like continuous beams and plane frames.
- The unknowns in this method are degree of freedom i.e. slope (0) and deflection (A).
- Combined these slopes and deflections are known as displacements. Thus slope-deflection method is a displacement method.
- This is a classical method on which moment distribution method, Kani's method and stiffness matrix method are based.

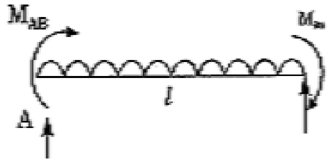
  Analysis of a beam and frame means determination of bending moment and shear force throughout the length of the member i.e. determination of BMD and SFD for the member. BMB and SFD for a member of structure can be drawn if we know the internal end moments of a member.

For example: if we have a fixed beam as shown below.



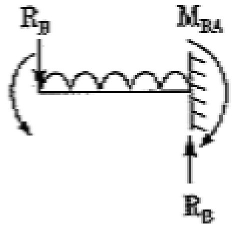
The BMD and SFD can be found if we know the internal end moments of member i.e. M_{AB} and M_{BA}





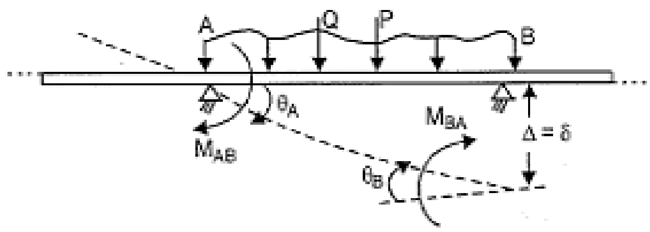
$$R_A = \frac{\omega l}{2} - \frac{M_{AB} + M_{BA}}{l}$$

$$R_B = \frac{\omega l}{2} + \frac{M_{AB} + M_{BA}}{l}$$



- Thus, in slope deflection method we establish a relationship between the degree of freedom (0, A) and the member end moments. This relationship is called slope deflection relationship.
- Finally by using equilibrium equation we find the slope deflection relationship to obtain the member moments.
- To find out slope-deflection relationship, method of superposition is used.

Example: (Continuous Beam)



$\Delta =$ deflection of joint B with respect to joint A

θ_A and θ_B = rotation of A and B

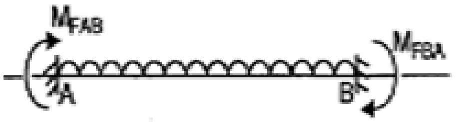
M_{AB} = Internal member end moment at 'A'

M_{BA} = Internal member end moment at 'B'

To find out the effect of **external loading, rotation** θ_A, θ_B and **displacement** $'\delta'$ on internal moments, we follow method of superposition.

6.2 Fundamental of slope deflection equation:

1. Consider all the joints to be fixed. The member end moment due to external loads is MFAB and MFBA

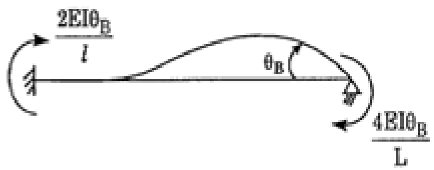


Effect of external load

2. Allow the support B to settle with respect to A. The member end moments generated are as show below.

3. Allow end A to rotate. The member end moments are as shown below.

4. Allow end B to rotate. The member end moments are as shown below.



5. Axial deformation is neglected.
6. Shear deformations are neglected.

6.3 Sign Convention: (For calculation of member end moment).

- Clockwise moment is taken as (+) ve.
- Clockwise rotation is taken as (+) ve.
- If 'δ' gives clockwise rotation, it is taken as positive ((+) ve). For example in the above discussion if end support *B* settles by amount 'δ' with respect to A, it is taken as (+) ve. Thus, from the principal of superposition, we have

$$M_{AB} = M_{FAB} + \frac{4EI\theta_A}{l} + \frac{2EI\theta_B}{l} - \frac{6EI\delta}{l^2}$$

$$M_{BA} = M_{FBA} + \frac{2EI\theta_A}{l} + \frac{4EI\theta_B}{l} - \frac{6EI\delta}{l^2}$$

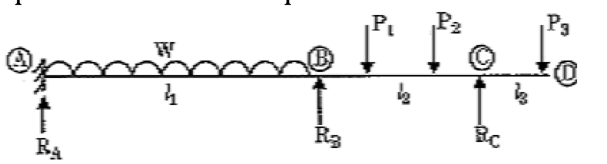
$$\boxed{\begin{aligned} M_{AB} &= M_{FAB} + \frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{3\delta}{l}\right) \\ M_{BA} &= M_{FBA} + \frac{2EI}{l}\left(2\theta_B + \theta_A - \frac{3\delta}{l}\right) \end{aligned}} \quad .....(A)$$

This equation A is called slope deflection equation.

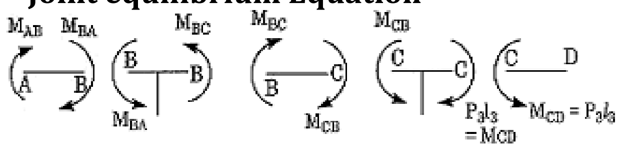
6.4 Equilibrium Equation

- We know that in slope deflection method; slope deflection equations are written which relates member end moments to the slopes and deflections (i.e. degree of freedom of the structure). Hence, if values, of slopes and deflections are known, member end moments are calculated.
- To find out the slope and deflections, equilibrium equations are to be written. No. of such equilibrium equations, required for complete solution is equal to the no. of degree of freedom of the structure.
- The equilibrium equations are classified as:
  1. Joint equilibrium equation.
  2. Shear equations
- Joint equilibrium equations are written to find out the values of 0 and shear equations are written to determine unknown displacement (δ).

The following example, illustrates how do we write down the joint equilibrium equation and shear equation.



Joint equilibrium Equation



Equilibrium of Joint B $\boxed{M_{AB} + M_{BC} = 0}$ ....(i)

Equilibrium of Joint C $\boxed{M_{CB} - P_3l_3 = 0}$ ....(ii)

Shear equations

$$R_A + R_B + R_C = Wl_1 + P_1 + P_2 + P_3$$

Note:

a) If unknown displacement (δ) is horizontal, the shear equation is $\sum F_H = 0$

b) If unknown displacement (δ) is vertical, the shear equation is $\sum F_v = 0$

Where

EF_H = Summation of all horizontal forces = 0

EF_V = Summation of all vertical forces = 0.

6.5 Steps for Analysis in Slope Deflection Method

a) Computation of fixed end moments:
The formulae for fixed end actions for various load cases are given in Table on the next page. The sign convention followed is that for moments, i.e. clockwise positive and anticlockwise - negative.

b) Relate member end moments to joint displacement:
The end moments are as follows :

$$M_{AB} = \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\delta}{L}\right) + M_{FAB}$$

$$M_{BA} = \frac{2EI}{L}\left(\theta_A + 2\theta_B - \frac{3\delta}{L}\right) + M_{FBA}$$

These are also known as slope deflection equations.

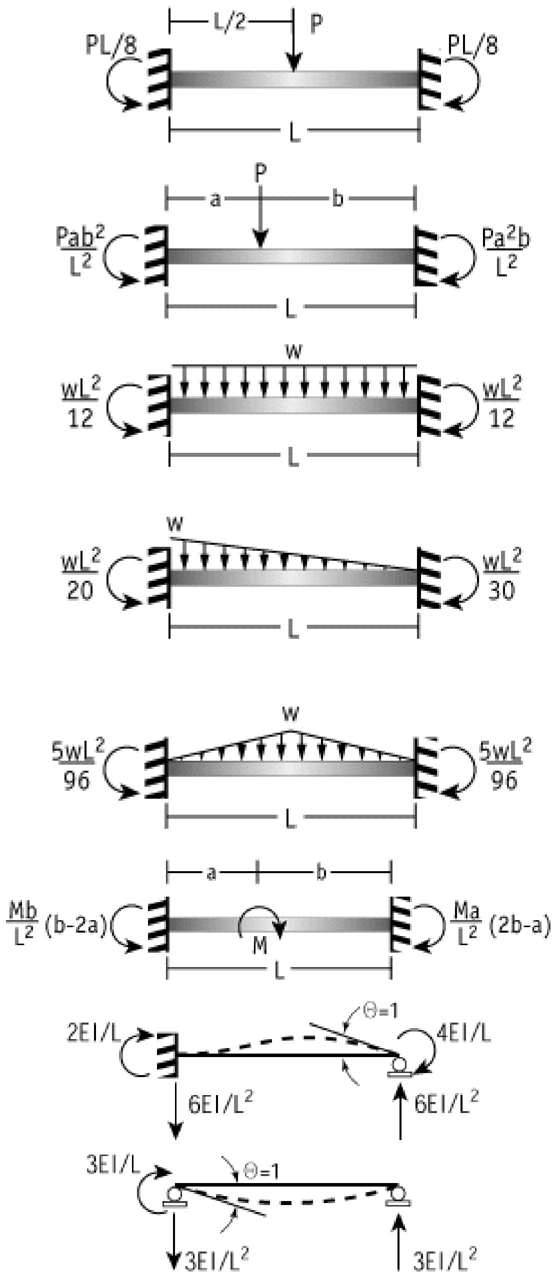
c) Formulate equilibrium equations:
These are obtained by making algebraic sum of moments at each joint as zero. In case of frames with sway, additional equations are obtained considering shear condition.

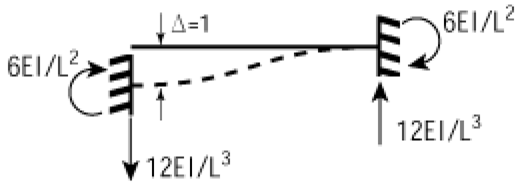
d) Solve the equations:
This will give displacements (primary unknowns), i.e. θ_A, θ_B, δ etc.

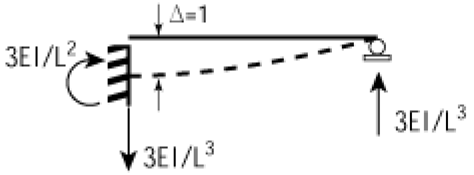
e) Back-substitution:
In the expressions for end moments formed in step (b), substitute values of known displacements as obtained in step (d). This gives final end moments for each member. Form this, the support reactions can also be calculated.

(f) Sketch shear force diagram (SFD) and bending moment diagram (BMD).

6.6 Examples of Fixed end moments



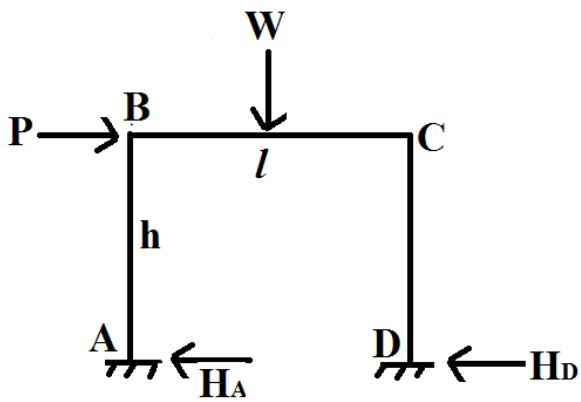




$$H_A + H_B = P$$

$$\frac{M_{AB} + M_{BA}}{h} + \frac{M_{CD} + M_{DC}}{h} - P = 0$$

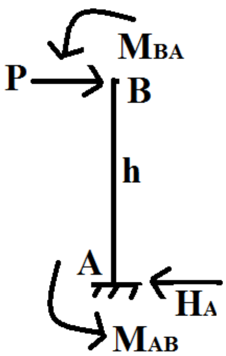
6.7 Shear equation in frame



$$\sum H = 0$$

$$P - H_A - H_B = 0$$

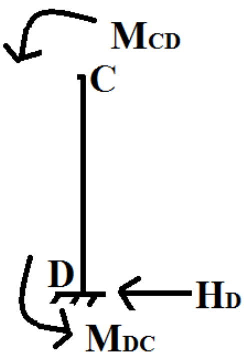
$$H_A + H_B = P$$



$$\sum M_B = 0$$

$$H_A.h - M_{AB} - M_{BA} = 0$$

$$H_A = \frac{M_{AB} + M_{BA}}{h}$$



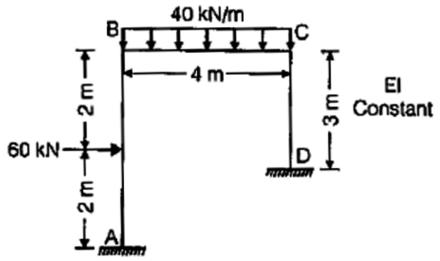
$$\sum M_C = 0$$

$$H_D.h - M_{CD} - M_{DC} = 0$$

$$H_D = \frac{M_{CD} + M_{DC}}{h}$$

ASSIGNMENT

Q.1 The figure given above shows a rigid frame. If D is a lateral translation of the joints, slope deflection equation for the member BA be written as:



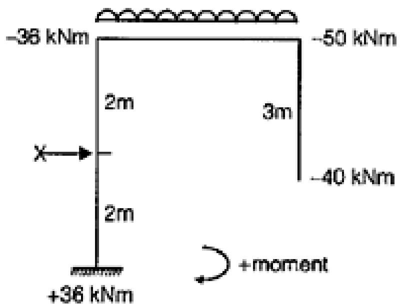
a) $M_{BA} = -30 + \frac{2EI}{4}\left(2\theta - \frac{3\Delta}{4}\right)$

b) $M_{BA} = -30 - \frac{2EI}{4}\left(2\theta + \frac{3\Delta}{4}\right)$

c) $M_{BA} = 30 + \frac{2EI}{4}\left(2\theta - \frac{3\Delta}{4}\right)$

d) $M_{BA} = 30 + \frac{2EI}{4}\left(2\theta + \frac{3\Delta}{4}\right)$

Q.2 Final moment values got through the analysis of a portal frame have been shown in the figure. What is the value of X?



a) 96 kN b) 60 kN
c) 30 kN d) 24 kN

ANSWER KEY:

1	2	3	4	5	6	7	8
c	b						

EXPLANATIONS

Q.1 **(c)**

Taking:

FEM_{BA} = 30 kN-m

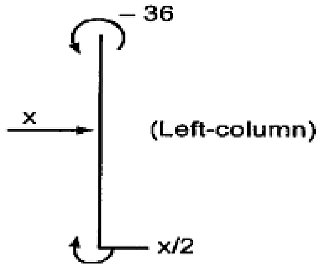
Sign of moment due to sinking of support B is already considered in derivation of equation.

Q.2 **(b)**

Free-body diagram of columns

Applying horizontal force equilibrium condition for frame

$$30 + \frac{x}{2} - x = 0$$



X = 60 N

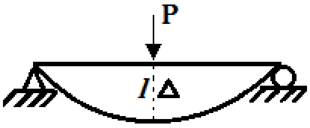
7 METHODS OF STRUCTURE ANALYSIS

7.1 Introduction

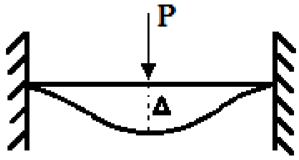
Purposes of choosing statically indeterminate structures as compared to statically determinate ones are as follows:

(a) For a given loading, the maximum stress and **deflection** of an indeterminate structure are generally smaller than that in statically determinate structure.

For example:

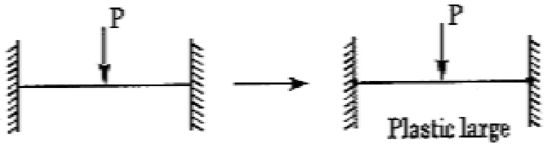


$$\text{Max BM} = \frac{Pl}{4}$$



$$\text{Max BM} = \frac{Pl}{8}$$

(b) Indeterminate structures have a tendency to redistribute its load to its redundant supports in case where faulty design or overloading occurs.



As load P is increased, plastic hinge will form at supports first arid hence it will be treated as simply supported structures. Further load can also be resisted. However, in case of simple supports, hinge will form at centre hence collapse will be early.

- Although there will be cost saving in material due to lesser stress in member, the cost of construction of supports and joint may sometime offset the saving in material.
- **<u>Differential settlement of supports, temperature variation, change in length due to</u> <u>fabrication errors</u>** in indeterminate structures will introduce internal stresses in structure.

Note:

In statically determinate structures internal stresses will not be introduced because of these factors.

In any statically indeterminate structures, it is necessary to satisfy:

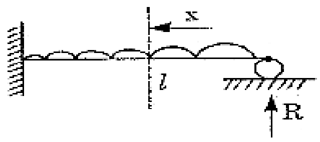
1. **Equilibrium equations**
2. **Compatibility equation**
3. **Force displacement requirement**

There are two different ways to satisfy these requirements.

7.2 Force method

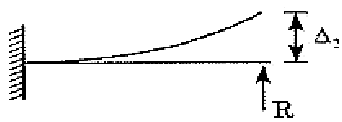
1. Also called Compatibility method or Method of consistent deformation or Flexibility method
2. Unknown in this case are Forces (reaction, BM, SF)
3. Force-displacement equation are written and solution for unknown forces is obtained from compatibility equations.

Example :



R = redundant

$$\Delta_1 = \frac{Wl}{8EI}$$



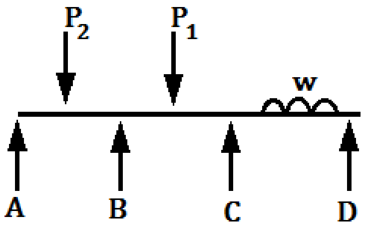
4. Once unknown forces are known, the reactive forces like BM, SF etc. are found using equilibrium equation.

$\Delta_2 = \frac{RL^3}{3EI}$: Load displacement relationship

$\frac{WL^4}{8EI} = \frac{RL^3}{3EI}$: Compatibility equation

$BM_X = Rx - \frac{Wx^2}{2}$: Equilibrium equation

5. Force method of analysis of indeterminate structure is suitable when degree of static indeterminacy is less then degree of kinematic indeterminacy.



No. of unknown reactions = 2
No. of unknown joint displacement
$= 4(\theta_A, \theta_B, \theta_C, \theta_D)$

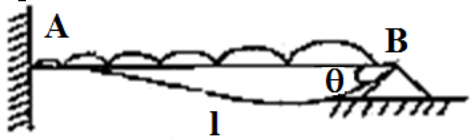
7.3 Examples of force method

a) Castigliano's theorem (method of least work)
b) Strain energy method
c) Virtual work method / Unit load method
d) Claperon.s three moment equations [used in continous beam analysis].
e) Column analogy method (used in rigid frames with fixed supports).
f) Flexibility matrix method

7.4 Displacement method

1. Also called Stifness method
2. Unknowns in this case are Displacement (Δ, 0)
3. Force-displacement equations are written and solution for unknown displacement is obtained from equilibrium equations.
4. Once unknown displacements are known, internal forces are found using compatibility and load-displacement equations.

Example :



θ = unknown displacement

$M_{AB} = M_{FAB} + \frac{2EI}{l}(-\theta_B)$: Load displacement relation

$M_{BA} = M_{FBA} + \frac{2EI}{l}(-2\theta_B)$: Load displacement relationship

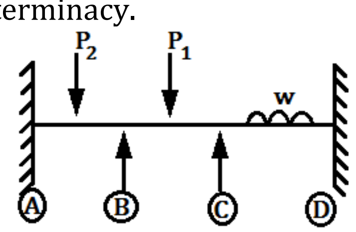
$M_{BA} = 0$: Equilibrium equation

$\Rightarrow \frac{4EI\theta_B}{l} = M_{FBA} = \frac{-Wl^2}{12}$

$\Rightarrow \theta_B = \frac{Wl^3}{48EI}$

From θ_B, M_{AB} can be found using load-displacement relationship.

5. Displacement method is suitable when degree of kinematic indeterminacy is less than the degree of static indeterminacy.



Unknown reaction = 4
unknown displacement $(\theta_B, \theta_C) = 2$

$\Rightarrow$ Displacement method is suitable

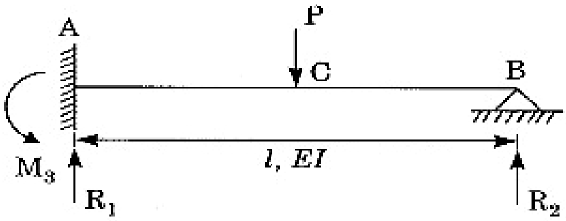
7.5 Examples of displacement methods

a) Slope deflection method
b) Moment distribution method
c) Stiffness matrix method
d) Kani's method

7.6 Various Force Methods:

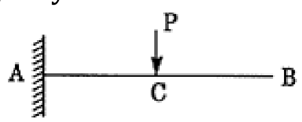
7.6.1 Method of Consistent Deformation

Example:

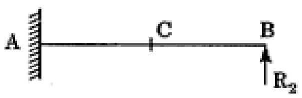


- The beam ABC is a redundant beam with degree of redundancy as one.
- Any one of the three reactions can be taken as redundant.
- Let us take R2 as redundant.
  At this stage we consider the beam to be composed of:

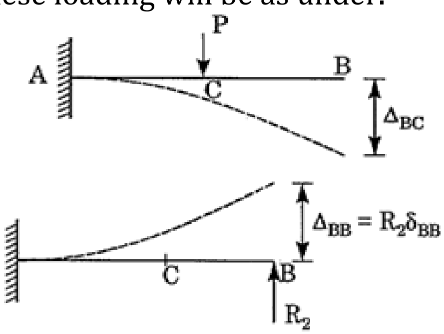
a) **Primary structure,** which is obtained by removing the redundant and loading the resulting beam with external loading only.



b) A beam with loading as redundant reaction.



Let the deformations of beam due to these loading will be as under.



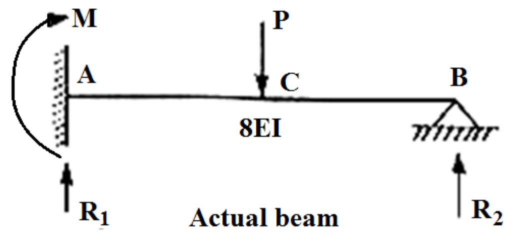
Where, δ_{BB} = deflection at B due to unit load applied at B

By compatibility condition, i.e. net deflection of end B = 0 [as it is hinged] we have,

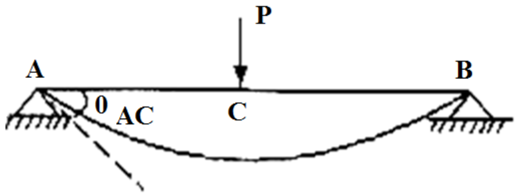
$$\Delta_{BC} = R_2\delta_{BB}$$

$$R_2 = \frac{\Delta_{BC}}{\delta_{BB}}$$

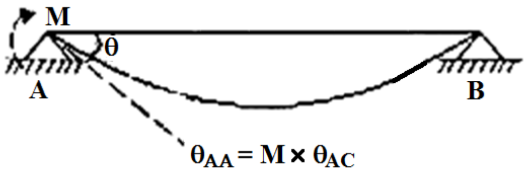
if we take M as redundant, then actual beam can be thought of as being composed of primary structure & beam with redundant loading, i.e.



Actual beam



Primary Structure



Beam with redundant loading

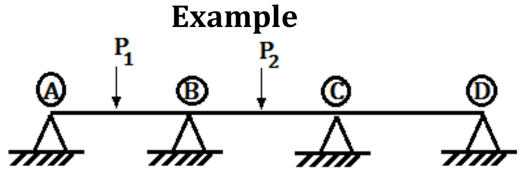
$$\theta_{AC} + M\theta_{AA} = 0$$

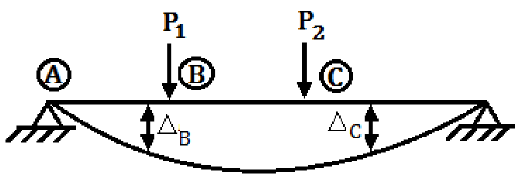
Where θ_{AC} = rotation at A due to external loading in Primary Structure.

θ_{AC} = rotation at A due to unit couple at A (unit couple).

$$M = \frac{-\theta_{AC}}{\theta_{AA}}$$

Example

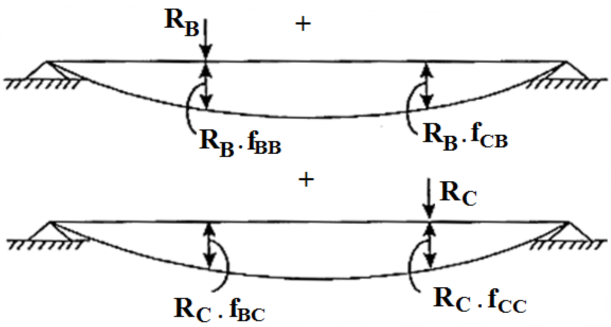




Primary Structure

f_{BB} = Deflection at B due to unit load at B

f_{CB} = Deflection at C due to unit load at B



f_{BC} = Deflection at B due to unit load at C

f_{CC} = Deflection at C due to unit load at C

From compatibility conditions net deflection at B & C is Zero.

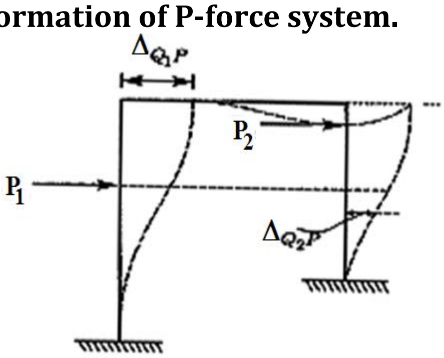
$\Rightarrow \Delta_B + R_B.f_{BB} + R_C.F_{BC} = 0 \quad ...(i)$

$\Rightarrow \Delta_C + R_B.f_{CB} + R_C.F_{CC} = 0 \quad ...(ii)$

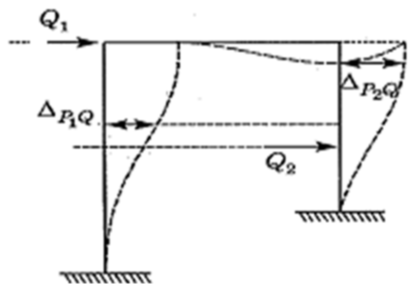
By solving (i) and (ii), R_B and R_C is calculated.

7.6.2 General Reciprocal Virtual Work Theorem (Betti's Theorem)

The virtual work done by a P-force system in going through the **deformation of Q-Force** system is equal to the virtual work done by the **Q-force system** in going through the **deformation of P-force system.**



Δ_{Q_1P} = deflection at the location of Q_1 due to P-system of forces

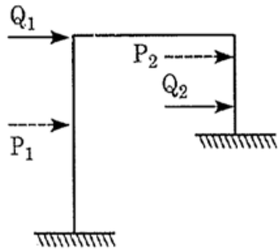


Δ_{P_2Q} = deflection at the location of P_2 due to Q-system of forces

Hence, as per Betti's theorem.

$$\boxed{P_1\Delta_{P_1Q} + P_2\Delta_{P_2Q} = Q_1\Delta_{Q_1P} + Q_2\Delta_{Q_2P}}$$

Proof of Betti's theorem



Let Q_1, Q_2 is applied 1st on the portal frame and then P_1 and P_2 are slowly applied

$$\text{Work done} = \frac{1}{2}Q_1\Delta_{Q_1Q} + \frac{1}{2}Q_2\Delta_{Q_2Q} + \frac{1}{2}P_1\Delta_{P_1P} + \frac{1}{2}P_2\Delta_{P_2P} + Q_1\Delta_{P_1P} + Q_2\Delta_{Q_2P}$$

Now if P_1 and P_2 are applied 1st on the portal frame and then Q_1 and Q_2 are applied

$$\text{Work done} = \frac{1}{2}P_1\Delta_{P_1P} + \frac{1}{2}P_2\Delta_{P_2P} + \frac{1}{2}Q_1\Delta_{Q_1Q} + \frac{1}{2}Q_2\Delta_{Q_2Q} + P_1\Delta_{P_1Q} + P_2\Delta_{P_2Q}$$

Equating the two works done (as sequence of application will not have any effect on final work done).

$$\boxed{P_1\Delta_{P_1Q} + P_2\Delta_{P_2Q} = Q_1\Delta_{Q_1P} + Q_2\Delta_{Q_2P}}$$

The works are called virtual work because Δ_{P_1Q} is not the deflection at P_1 location due to P_1 itself.

7.6.3 A Special Case of Betti's Law (Maxwell's Reciprocal Theorem)

If only two forces P and Q are acting and magnitude of P and Q are unity.

$\boxed{\Delta_{PQ} = \Delta_{QP}}$

Where

$\Delta_{PQ} =$ Deflection at the location of P due to unit load at the location of Q

$\Delta_{QP} =$ Deflection at the location of Q due to unit load at the location of P

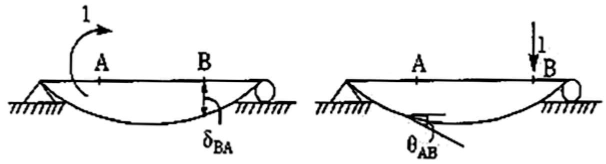
i.e., deflection at the location of Q due to unit load at 'P' is equal to deflection at the location of P due to unit load at Q.

Example

1.

$\boxed{\delta_{AB} = \delta_{BA}}$

2.

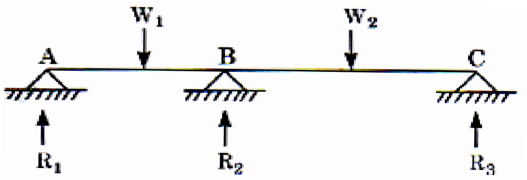


$\Rightarrow \boxed{\delta_{BA} = \delta_{AB}}$

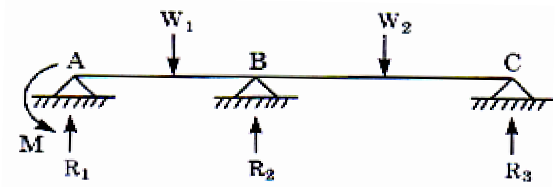
i.e., Downward deflection at B due to clockwise unit couple at A is equal to clockwise slope at A due to downward unit load at B.

7.6.4 Theorem of Least Work

For any statically indeterminate structure, the redundant should be such as to make the total internal energy within the structure a minimum.



$\frac{\partial U}{\partial R_2} = 0$ Taking R2 as redundant



$\frac{\partial U}{\partial M} = 0$ Taking M and R3 as redundant

$\frac{\partial U}{\partial R_3} = 0$

Where U = strain energy in the beam AB and BC due to combined Action of external loading and reactions.

- Thus theorem of least work is a special case of Castigliano's 2nd theorem.
- This theorem applies to beam, frame, and truss everything.

7.6.5 Castigliano's Theorem

$$\left.\begin{aligned} \frac{\partial U}{\partial \Delta_n} &= P_n \\ \frac{\partial U}{\partial \theta_n} &= M_n \end{aligned}\right\} \text{1st theorem}$$

$$\left.\begin{aligned} \frac{\partial U}{\partial P_n} &= \Delta_n \\ \frac{\partial U}{\partial M_n} &= \theta_n \end{aligned}\right\} \text{2nd theorem}$$

Note:

- Castigliano extended the principal of least work, later on, to the self-straining system.
- If $\lambda =$ Small displacement in the direction of redundant force *R* then,

  $\boxed{\frac{\partial U}{\partial R} = \lambda}$

- Self-straining may be caused by settlement of support of a redundant structure by an amount λ or by the initial misfit of a member by an amount λ too short/long.

7.6.6 Strain energy

Energy method is useful to calculate deformation of structure subjected to complicated load.

Strain energy method is based on conservation of energy principle which states that work done by all external forces acting on structure Ue, is transformed in to internal work done Ui which develops when structure deforms.

$$U_e = U_i$$

- Strain energy due to axial loading,

$$U_{axial} = \frac{P^2 L}{2AE}$$

Where, AE= Axial rigidity

- Strain energy due to bending(flexure),

$$U_{bending} = \int_0^L \frac{M_{(X)}^2}{2EI} dx$$

Where, EI= Flexural rigidity

- Strain energy due to Shear,

$$U_{shear} = \int_0^L \frac{F_{(X)}^2}{2GA} dx$$

Where, GA= Shear rigidity

- Strain energy due to Torsion,

$$U_{torsion} = \int_0^L \frac{T_{(X)}^2}{2GJ} dx$$

Where, GJ= Torsional rigidity

GATE QUESTIONS

Q.1 For a linear elastic structural system, minimization of potential energy yields
a) compatibility conditions
b) constitutive relations
c) equilibrium equations
d) strain-displacement relation
[GATE – 2004]

Q.2 The unit load method used in Structural analysis is
a) applicable only to statically indeterminate structures
b) another name for stiffness method
c) an extension of Maxwell's reciprocal theorem
d) Derived from Castigliano's theorem
[GATE – 2004]

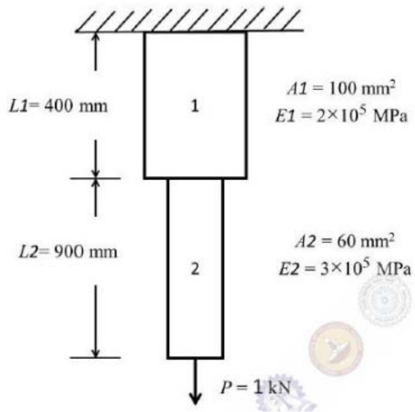
Q.3 Match List-I with List-II and select the correct answer using the codes given below the lists:
List – I
A. Slope deflection method
B. Moment distribution method
C. Method of three moments
D. Castigliano's second theorem
List – II
1. Force Method
2. Displacement

Codes:

	A	B	C	D
a)	1	2	1	2
b)	1	1	2	2
c)	2	2	1	1
d)	2	1	2	1

[GATE-2005]

Q.4 Consider the stepped bar made with linear elastic material and subjected to an axial load of 1 kN as shown in figure.



The strain energy in N.mm up to one decimal place in the bar due to an axial load is ___________
[GATE-2017-1]

ANSWER KEY:

1	2	3	4
A	D	C	35

EXPLANATIONS

Q.1 **(a)**
Compatibility conditions deals with balancing of displacements and minimization of potential energy yields compatibility conditions

Q.2 **(d)**
The Unit Load Method is derived from Castiglione's theorem-1.

Q.3 **(c)**

Q.4 **(35Nmm)**

$$U_{axial} = \frac{P^2L}{2AE}$$

$$U_{axial} = \frac{1000^2 * 400}{2*100*2*10^5} + \frac{1000^2 * 900}{2*60*3*10^5}$$

$$U_{axial} = 35Nmm$$

ASSIGNMENT

Q.1 The following methods are used for structural analysis

1.Macaulary method

2.Column analogy method

3.Kani's method

4.Method of sections

Those used for indeterminate structural analysis would include

a) 1 and 2 b) 1 and 3

c) 2 and 3 d) 2, 3 and 4

Q.2 Match List-I (Names of persons with whom the methods of analysis are associated) with List – II (Method) and selected the correct answer using the codes given below the lists

List-I

1. Clapeyron
2. Hardy cross
3. Lame
4. Euler

List – II

A. Moment distribution method

B. Method for determining crippling load on a column

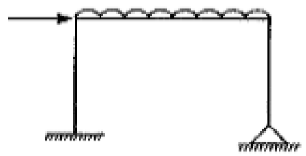
C. Theorem of Three moment

D. Thick cylinders

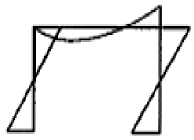
Codes:

	1	2	3	4
a)	C	A	B	D
b)	A	C	D	B
c)	C	A	D	B
d)	A	B	D	C

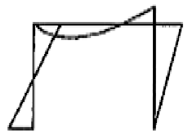
Q.3 A loaded portal frame is shown in figure. The profile of its bending moment diagram will be



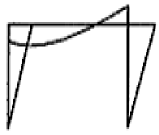
a)



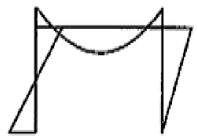
b)



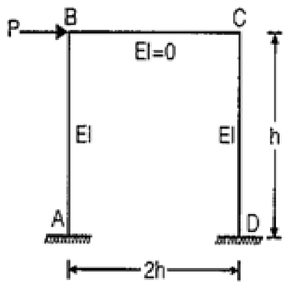
c)



d)



Q.4 If the flexural rigidity of the beam BC of the portal frame shown in the given figure is assumed to be zero, then the horizontal displacement of the beam would be



a) $\frac{Ph^3}{3EI}$ b) $\frac{Ph^3}{24EI}$

c) $\frac{Ph^3}{12EI}$ d) $\frac{Ph^3}{6EI}$

Q.5 Match List-I (Method) with List-II (Factor) and select the correct answer using the code given below the lists:

List-I

A. Moment distribution method

B. Slope deflection method

C. Kani's method

D. Force method

LIST-II

1. Rotation factor

2. Flexibility

3. Hardy Cross

4. Displacements

5. Stiffness matrices

Codes:

	A	B	C	D
a)	3	4	1	2
b)	2	1	5	3
c)	2	4	1	3
d)	3	1	5	2

Q.6 Which one of the following statements is correct?

The principle of superposition is applicable to

a) nonlinear behaviour of material and small displacement theory

b) nonlinear behaviour of material and large displacement theory

c) nonlinear elastic of material and small displacement theory

d) linear elastic behaviour of material and large displacement theory.

ANSWER KEY:

1	2	3	4	5	6
(c)	(c)	(d)	(d)	(a)	(c)

EXPLANATIONS

Q.1 **(c)**
Macaaulay's method is used for deflection and slope calculation due to point loads in prismatic beams.
Method of sections is used for statically determine trusses.

Q.2 **(c)**
1-C, 2-A, 3-D, 4-B

Q.3 **(d)**
- The horizontal force will be distributed to both the supports.
- The bending moment at hinged end will be zero and fixed end will have some B.M.
- The B.M.D for columns will be linear.
- The B.M.D for beam will be parabolic.

The choice now remain between (b) and (d).
The beam at left end in (b) shows zero B.M which is not possible
So only option D is valid.

Q.4 **(d)**
As the beam BC has no rigidity. So end B of column behaves as free end.
For a cantilever beam subjected to concentrated load at free end, the stiffness is $\frac{3EI}{l^3}$
Stiffness of given system
$= \frac{3EI}{h^3} + \frac{3EI}{h^3} = \frac{6EI}{h^3}$
So displacement of point
$B = \frac{Ph^3}{6EI}$

Q.5 **(a)**
A – 3, B – 4, C – 1, D - 2

Q.6 **(c)**
The principle of superposition states that the displacement resulting from each of a number of forces may be added to obtain the displacements resulting from the sum of forces. This method depends upon the linearity of the governing relations between the load and deflection. The linearity depends upon the two factors.

(i) The linearity between bending moment and curvature which depends upon the linear elastic materials. If non-linear materials superposition of curvatures is not possible.

(ii) The linearity between curvatures and deflection depends upon the assumption that the deflections are so small that the approximate curvature can be used in place of true curvature.

8 MATRIX METHOD OF ANALYSIS

Before starting the chapter of matrix method of analysis, we need to acquaint our self with the basics of determinant and matrix.

8.1 Definition and Notations

- Matrix is defined as an array of quantities, usually called elements, grouped together for some specific purpose arranged systematically in rows and columns.
- For demonstration purpose, these elements are enclosed by either square brackets, { }, or parentheses ( ).
- In general, the entire matrices are denoted by boldface letters A, B,C, etc. The matrix A of order m x n consisting of m rows and n columns of elements a_{ij} situated at the intersection of the i-th row and j-th column may be represented as follows :

$$A = [A]_{m\times n} = [a_{ij}]_{m\times n} = \begin{bmatrix} a_{11} & a_{12} & ---- & a_{1n} \\ a_{21} & a_{22} & ---- & a_{2n} \\ - & - & ---- & - \\ a_{m1} & a_{m2} & ---- & a_{mn} \end{bmatrix}$$

8.2 Types of Matrix

8.2.1 Square Matrix

A matrix which has equal number of rows and columns is called a square matrix i.e

m = n

For example :

$$\begin{bmatrix} 1 & 3 & -2 \\ 3 & 5 & 7 \\ 4 & 3 & 1 \end{bmatrix}$$

Is a square matrix of dimension (or order) 3. Here, the elements 1, 5 and 1 are called the diagonal elements and the diagonal is named as principal diagonal.

- If any matrix (square type) has the diagonal elements non-zero and the non-diagonal elements are all zero, it is called a diagonal matrix.
- If any square matrix has the elements symmetrical about the principal diagonal, it is called a **symmetric matrix,** i.e $a_{ij} = a_{ji}$

$$\begin{bmatrix} 1 & 4 & 3 \\ 4 & 2 & 7 \\ 3 & -6 & 1 \end{bmatrix}$$

Is a 3 X 3 symmetric matrix.

- If any diagonal matrix has the entire diagonal element same, it is called a scalar matrix.

$$\begin{bmatrix} x & 0 & 0 \\ 0 & x & 0 \\ 0 & 0 & x \end{bmatrix}_{3\times 3}$$

- If any scalar matrix has all the diagonal elements one (unit), it is called an identity or unit matrix and represented by [I], Thus,

Identity matrix of 2 dimensions

$$= I2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$ and

Identity matrix of 3 dimensions

$$= I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

8.2.2 Row and Column Matrix

- A matrix which has only one row, i.e. m = 1, is called row matrix of order **1 x n.**
- A matrix which has only one column, i.e .n = 1, is called column matrix of order **m x 1**

8.2.3 Null OR Zero Matrixes

A matrix which has all the elements zero is called null or zero matrix and is represented by [0].

Thus, $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is null matrix of 2 X 3 order

8.2.4 Singular Matrix

A singular matrix is the one when the determinant of it is zero. All rectangular matrices are singular.

8.3 Definition and Properties of Determinants

1. The determinant is a unique scalar quantity associated with each square matrix. It can be expanded and has a value. Generally, the determinant is shown by a pair of vertical lines.
So, the determinant

$$A = |A| = \begin{vmatrix} 1 & 3 & 5 \\ 3 & 1 & 1 \\ 2 & 0 & 3 \end{vmatrix}$$

$$= 3(-1)^{1+2}\begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix} + 1(-1)^{2+2}\begin{vmatrix} 1 & 5 \\ 2 & 3 \end{vmatrix} + 0(-1)^{3+2}\begin{vmatrix} 1 & 5 \\ 3 & 1 \end{vmatrix}$$

$$= -[3\times 7] + [1\times(-7)] + 0 = -28$$

8.4 Properties of Determinants

- The value of a determinant is unchanged due to interchange of all the rows and columns. So $|A| = |A^T|$
- The value of a determinant changes sign due to interchange of two rows or two columns.
- The value of a determinant is zero if all the elements in any rows or column are zero.
- The value of a determinant is zero if two rows or two columns are proportional or same. Therefore, in the case of linearly dependent rows or columns, IAI =0.
- The value of a determinant is multiplied by a factor K if all the element of a row or column of the determinant is multiplied by the same factor K.

8.5 Basic Matrix Operations

Addition and Subtraction of Matrices

Addition and subtraction of matrices can be done only if they have the same order.

Thus, $[A]_{m\times n} \pm [B]_{m\times n} = [C]_{m\times n}$

$$[A_{ij}] \pm [b_{ij}] = [(a_{ij} \pm b_{ij})]$$

- They follows commutative law, i.e. A + B = B + A and associative law, i.e.
  A + (B + C) = (A + B) + C

Example 1

Find (i) A + B, (ii) A – B, and (iii) B – A, When

$$A = \begin{bmatrix} 4 & 5 \\ 5 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix}$$

Solution

i) $A + B = \begin{bmatrix} 4 & 5 \\ 6 & 3 \end{bmatrix}_{2\times 2} + \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix}_{2\times 2}$

$$= \begin{bmatrix} (4+1) & (5-1) \\ (6+3) & (3+2) \end{bmatrix}_{2\times 2} = \begin{bmatrix} 5 & 4 \\ 9 & 2 \end{bmatrix}_{2\times 2}$$

ii) $A - B = \begin{bmatrix} 4 & 5 \\ 6 & 3 \end{bmatrix}_{2\times 2} - \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix}_{2\times 2}$

$$= \begin{bmatrix} (4-1) & (5+1) \\ (6-3) & (3-2) \end{bmatrix}_{2\times 2} = \begin{bmatrix} 3 & 6 \\ 3 & 1 \end{bmatrix}_{2\times 2}$$

iii) $B - A = \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix}_{2\times 2} - \begin{bmatrix} 4 & 5 \\ 6 & 3 \end{bmatrix}_{2\times 2}$

$$= \begin{bmatrix} (1-4) & (-1-5) \\ (3-6) & (+2-3) \end{bmatrix}_{2\times2} = \begin{bmatrix} -3 & -6 \\ -3 & -1 \end{bmatrix}_{2\times2}$$

- Here, it is to be noted that
  (B – A) = - (A – B)

8.5.1 Scalar Multiplication of a Matrix

If we consider KA = B

Then, $\left[Ka_{ij}\right] = \left[b_{ij}\right]$

Where K is a scalar constant.

8.5.2 Multiplication of Matrix

The following procedures are adopted for the multiplication of two matrices:

$[\text{Lead matrix}]_{(m \times k)} \times [\text{lag matrix}]_{(k \times n)} = [\text{product}]_{(m \times n)}$

i.e., $[A]_{m\times k}\,[B]_{k\times n}\,[C]_{m\times n}$

and $\left[C_{ij}\right] = \left[\sum_k a_{ik}\, b_{kj}\right]$

(a) The element of a row of the lead matrix A should be multiplied by the corresponding elements of a column of the lag matrix **B.**

(b) The summation of the product is put in the new product matrix C at the corresponding row of A and column of **B.**

Example 2

Find $\begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix}$

Solution

Here, we know

$$\begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix}_{2\times2} \times \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix}_{2\times3} = [C]_{2\times3}$$

Thus we get,

$$[C]_{2\times3} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & (2\times2+4) & (2\times3+4\times2) \end{bmatrix}$$

$$\therefore\ [C]_{2\times3} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 8 & 14 \end{bmatrix}$$

Here,

$C_{11} = 1\times1+0\times0=1$

$C_{12} = 1\times2+0\times1=2$

$C_{13} = 1\times3+0\times2=3$

$C_{21} = 2\times1+4\times0=2$

$C_{22} = 2\times2+4\times1=8$

$C_{23} = 2\times3+4\times2=14$

8.5.3 Properties of Matrix Multiplication

- Multiplication is not commutative, i.e. AB # BA, except in identity matrix
- Multiplication is associative, i.e. A (BC) = (AB) C; where A, B and C are three matrices of the order m X n , n X p, p X q respectively.
- Multiplication is distributive, i.e. **A (B + C) = AB + AC,** where **A, B** and **C** are three matrices of the order $m\times n$, $n\times p$, $n\times p$ respectively.

8.5.4 Transpose of Matrix

A new matrix obtained by interchanging rows and columns of the original matrix is called transposed matrix.

Thus, if we have a matrix, $A = \left[a_{ij}\right]_{m\times n}$

Then, $A^T = \left[a_{ji}\right]_{m\times n}$

Example 3

Find the transpose of the matrix

$$[A] = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 6 & 7 & 8 \end{bmatrix}$$

Solution

Transpose of the matrix A

$$A^T = \begin{bmatrix} 1 & 2 & 6 \\ 2 & 1 & 7 \\ 3 & 3 & 8 \end{bmatrix}$$

8.5.5 Properties of Transposed Matrices

- Transpose of a product of two matrices is the product of transposes of individual matrices in the reverse order.
  For example,

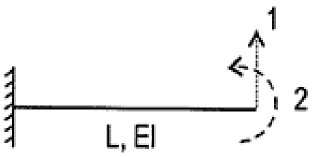
$(AB)^T = B^T . A^T$;

$(ABC)^T = C^T B^T A^T$ and so on.

- Transpose of algebraically sum of two matrices is the algebraically sum of the transpose of the individual matrices.
  For example, $(A \pm B)^T = A^T + B^T$
- Transpose of a transpose matrix is the original matrix.
  For example, $\left(A^T\right)^T = A$

8.6 Development of Flexibility Matrix

Development of Flexibility matrix can be illustrated with the following example. Let us take a cantilever beam as shown below



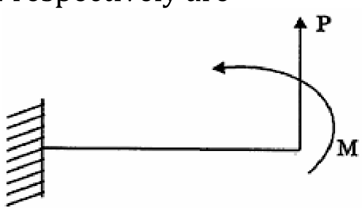
- If unit load is acting in the direction of 1 It will produce deflection along $1 = \frac{1 \times L^3}{3EI}$

  It will be proud rotation along $2 = \frac{1 \times L^2}{2EI}$

  Similarly, it will produce deflection along $1 = \frac{1 \times L^2}{2EI}$

  It will produce rotation along $2 = \frac{1 \times L}{EI}$

  Hence δ_1 and δ_2 produced along 1 and 2 due to system of force P and M along 1 and 2 respectively are



$$\delta_1 = \left(\frac{L^3}{3EI}\right)P + \left(\frac{L^3}{2EI}\right)M$$

$$\delta_2 = \left(\frac{L^2}{2EI}\right)P + \left(\frac{L}{EI}\right)M$$

If we define a quantity f_{ij}, where f_{ij} = displacement in the direction T due to unit load in the direction 'j'. Then

$$f_{11} = \frac{L^3}{3EI} \quad f_{12} = \frac{L^3}{2EI}$$

$$f_{21} = \frac{L^2}{2EI} \quad f_{22} = \frac{L}{EI}$$

$$\delta_1 = f_{11}P + f_{12}M$$

$$\delta_2 = f_{21}P + f_{22}M$$

$$\begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} \begin{bmatrix} P \\ M \end{bmatrix}$$

$\begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}$ is called flexibility matrix

$$\boxed{[\delta] = [f][R]}$$

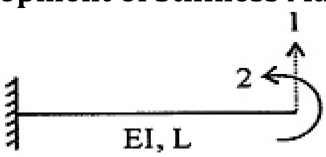
In the flexibility matrix f_{ij} = element of i^th^ row and j^th^ column.

Important Point

- Order of matrix is the no. of co-ordinate chosen for solution of problem.
- Elements of flexibility matrix are displacements.
- Flexibility Matrix will always be a square matrix.
- Elements along the diagonal will always be positive. Other elements can be zero or negative.
- Matrix will always be a symmetric matrix about its main diagonal.
  [This follows from Maxwell reciprocal theorem, $f_{ij} = f_{ji}$]

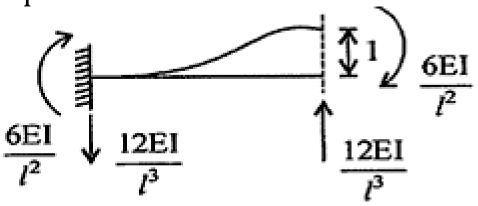
  Flexibility matrix can be calculated only for stable structure in which there are no rigid body displacements.

8.7 Development of Stiffness Matrix

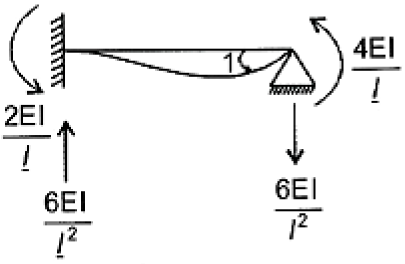


Let 1 and 2 is two Co-ordinate directions in which force are P and M.

- When unit displacement is given in the direction of '1' without any displacement in the direction '2'.



- Force developed in the direction $1 = \frac{12EI}{l^2}$
- Force developed in the direction $2 = -\frac{6EI}{l^2}$
- When unit displacement is given in the direction of '2' without any displacement in the direction 1
  Hence forces developed along 1 and 2 due to displacements A and 0 along 1 and 2 respectively are



- Force developed in the direction $1 = -\frac{6EI}{l^2}$
- Force developed in the direction $2 = -\frac{4EI}{l}$

  Stiffness of BA = $\frac{4 \times E \times 2I}{4} = 2EI$

  $$P = \left(\frac{12EI}{l^3}\right)\Delta + \left(-\frac{6EI}{l^2}\right)\theta$$

  $$M = \left(-\frac{6EI}{l^2}\right)\Delta + \left(\frac{4EI}{l}\right)\theta$$

If we define a quantity 'K_{ij}', where K_{ij} = Force developed in the direction of *i* due to unit displacement in the direction of j without any displacement in other direction.

{[k] is stiffness matrix in which element} K_{ij} = element of ith row and jth column.

$$K_{11} = \frac{12EI}{l^3} \quad K_{12} = -\frac{6EI}{l^2}$$

$$K_{21} = -\frac{6EI}{l^2} \quad K_{22} = \frac{4EI}{l}$$

Hence

$$P = K_{11}\Delta + K_{12}\theta$$

$$M = K_{21}\Delta + K_{22}\theta$$

$$\begin{bmatrix} P \\ M \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} \Delta \\ \theta \end{bmatrix}$$

$$\boxed{[P] = [K][\Delta]}$$

Important Point

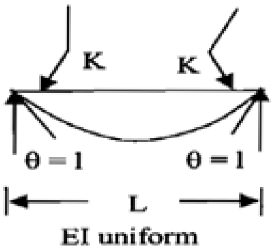
- All the properties discussed for flexibility matrix holds for Stiffness matrix also except that elements of Stiffness matrix are forces.
- Thus, to find out j^{th} column of the Stiffness matrix, unit displacement is given in the direction of 7 without any displacement at other co-ordinates and forces developed at all the co-ordinates are determined. These forces constitute the elements of j^{th} column.

GATE QUESTIONS

Q.1 The stiffness K. of a beam deflecting in a symmetric mode, as shown in the figure, is

a) EI /L b) 2El/ L

c) 4El/ L d) 6El/ L



[GATE – 2003]

Q.2 The stiffness coefficient kij indicates

a) force at i due to a unit deformation at j

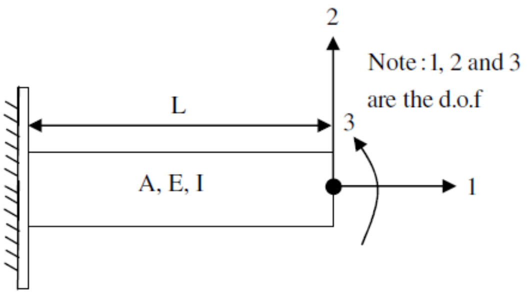
b) deformation at j due to a unit force at i

c) deformation at i due to a unit force at j

d) force at j due to a unit deformation at i

[GATE-2007]

Q.3 For the beam shown below, the stiffness coefficient K_{22} can be written as



a) $\frac{6EI}{L^2}$ b) $\frac{12E1}{L^3}$

c) $\frac{3E1}{L}$ d) $\frac{EI}{6L^2}$

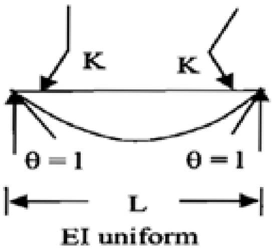
[GATE-2015]

ANSWER KEY:

1	2	3
(b)	(a)	(b)

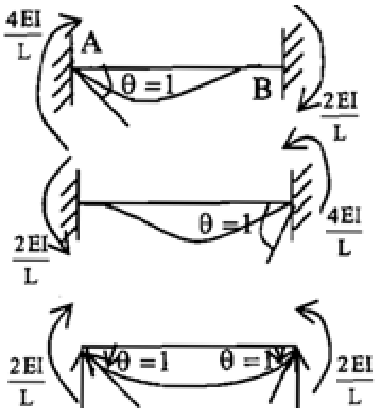
EXPLANATIONS

Q.1 (b)



We know that moment required to produce a unit rotation is called stiffness.

∵ slope θ = 1 at both ends



Initially for $\theta = 1$ (clockwise) At A,

Keeping 'B' fixed.

$$M_{AB} = \frac{4EI}{L} \text{ (clockwise)}$$

$$M_{BA} = \frac{2EI}{L} \text{ (clockwise)}$$

The allow $\theta = 1$ (anticlockwise)

At B, keeping 'A' as fixed.

Now,

$$M_{BA} = \frac{4EI}{L} \text{ (anticlockwise)}$$

$$M_{AB} = \frac{2EI}{L} \text{ (anticlockwise)}$$

If unit rotation at both ends, as shown

$$M_{AB} = \frac{4EI}{L} - \frac{2EI}{L}$$

$$= \frac{2EI}{L} \text{ (clockwise)}$$

$$M_{BA} = \frac{4EI}{L} - \frac{2EI}{L}$$

$$= \frac{2EI}{L} \text{ (anticlockwise)}$$

$$\text{Hence, } K = \frac{2EI}{L} = M$$

Q.2 (a)

Q.3 (b)