GATEFLIX

HEAT TRANSFER

For

CHEMICAL ENGINEERING

HEAT TRANSFER

SYLLABUS

Steady and unsteady heat conduction, convection and radiation, thermal boundary layer and heat transfer coefficients, boiling, condensation and evaporation; types of heat exchangers and evaporators and their process calculations. Design of double pipe, shell and tube heat exchangers, and single and multiple effect evaporators.

ANALYSIS OF GATE PAPERS

Exam Year	Total
2001	10
2002	7
2003	10
2004	7
2005	13
2006	5
2007	8
2008	5
2009	11
2010	9
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1 INTRODUCTION

INTRODUCTION

From thermodynamics we learned that energy can be exchanged between systems and surrounding. This energy in transit is called work and heat. The thermodynamics deals with the end state of process during which interaction but does not explain the way heat of interactions. The objective of **heat transfer** is to extend thermodynamic analysis through study of the modes of heat transfer and development of relations to calculate heat transfer rates.

Heat is the energy in transit due to difference in temperature. Heat always flows from higher temperature to lower temperature. The various modes by which the heat transfer can take place are

1. Conduction
2. Convection
3. Radiation

1.1 CONDUCTION

Conduction is the transfer of energy from more energetic particle to adjacent less energetic particle. Conduction takes place in solid, liquid and gases. In solids conduction takes by transport of energy from higher temperature region to lower temperature region by free electrons or by vibration of molecules. In liquids and gases, the energy is transported due to collision or diffusion of molecules during their random motion. The rate of heat transfer by conduction can be determined by **Fourier's law** which states that

$$Q_{cond} = -kA\frac{\Delta T}{\Delta x}$$

For Lim $\Delta x \to 0$, equation can be written as

$$Q_{cond} = -kA\frac{dT}{dx}$$

The minus sign is to show that the flow of Heat is from hotter to colder. If the temperature decreases with x, Q will be positive and will flow in the direction of x. If the temperature increases with x, Q will be negative, and will flow opposite to the direction of x.

1.2 CONVECTION

Convection heat transfer mainly occurs between a fluid in motion and a surface when the two are at different temperatures.The convection heat transfer mode is comprised of two mechanisms

1. Energy transfer due to random molecular motion (diffusion),
2. Energy transferred by the bulk or macroscopic, motion of the fluid.

This fluid motion is associated with the fact that, at any instant, large numbers of molecules are moving collectively as a bunch. Such motion, in the presence of a temperature difference leads to heat transfer. The molecules in the bunch also have random motion; the total heat transfer is then due to energy transport by the random motion of the molecules and by the bulk motion of the fluid.

When surface comes in contact with moving fluid, a region is developed near to the surface in which velocity gradient exists. This region is called **hydrodynamic boundary layer**.similary, if the surface and flow temperatures differ, there will be a region of the fluid through which the temperature varies .This region, called the **thermal boundary layer**.Convection heat transfer may be classified according to the nature of the flow. **Forced convection** is when flow is caused by external means, such as by a fan, a pump, or atmospheric winds. **Free (or natural) convection**, the flow is induced by buoyancy forces, which are due to density differences caused by temperature variations in the fluid. An example is the free convection heat

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transfer that occurs from hot components on a vertical array of circuit boards in air.

The heat transfer rate between fluid and surface is determined by Newton's Law of cooling

$$Q_{conv} = hA\left(T_S - T_\infty\right)$$

Where,

Q_{conv} Convective heat flow rate, W

A Surface area in fluid contact, m^2

T_S Temperature of surface, K

T_∞ Temperature of fluid, K

The constant of proportionality 'h' is known as heat transfer coefficient. Convection heat transfer coefficient can be defined as the rate of heat transfer between a solid surface and a fluid per unit surface area per unit temperature difference. Its unit is W/m^2K. **The value of heat transfer coefficient depends mainly on type of flow, surface condition, velocity of fluid and thermo physical properties of fluid**.

Typical values of Heat transfer coefficient

1. Free convection
   Gases 2–25 W/m^2K
   Liquids 50–1000 W/m^2K
2. Forced convection
   Gases 25–250 W/m^2K
   Liquids 100–20,000 W/m^2K
3. Convection with phase change Boiling or condensation 2500–100,000 W/m^2K

1.3 THERMAL RADIATIONS

Thermal radiation is energy emitted by matter that is at a nonzero temperature (i.e. >0K). Thermal radiation wave is emitted by solids, liquids and gases. Regardless of the form of matter, the emission is due changes in the electron configurations of the constituent atoms or molecules. The radiation energy is transported by electromagnetic waves (or alternatively, photons). While the transfer of energy by conduction or convection requires the presence of a material medium, radiation does not. In fact, radition transfer occurs most efficiently in a vacuum.

Consider radiation transfer processes for the surface shown in figure. Radiation that is emitted by the surface originates from the thermal energy of matter bounded by the surface, and the rate at which energy is released per unit area (W/m^2) is termed the **surface emissive power (E)**. This is maximum emissive power a surface can emit at given temperature. Such surface emitting maximum emissive power is known as **black body**.

The radiations emitted by blackbody is given by the **Stefan–Boltzmann law**

$$E_b = \sigma\left(T_S^4\right)$$

Where, Stefan –Boltzmann constant $\sigma = 5.67\times10^{-8} w/m^2k^4$, T_s is surface temperature (K)

The heat flux emitted by a real surface is less than that of a blackbody at the same temperature and is given by

$$E = \varepsilon\sigma\left(T_S^4\right)$$

Where, ε is emissivity

The heat interaction between surface and surrounding is shown in figure.

The rate at which all radiations is incident on a unit area of the surface is called irradiations G (W/m^2).

Net rate of heat transfer from surface is

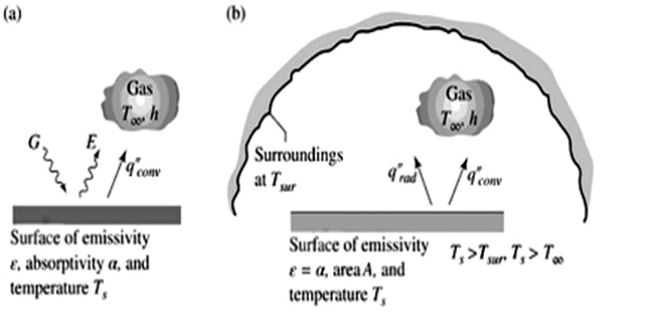

Figure 1. Radiation exchange: (a) at a surface and (b) between a surface and large surroundings.

give by

Q = Total radiations emitted– Total radiations absorbed

$$Q = EA - \alpha GA = \varepsilon\sigma A\left(T_S^4\right) - \alpha\sigma A\left(T_{Sur}^4\right)$$

According to Kirchhoff's Law ($\varepsilon=\alpha$) , where, ε is emissivity & α is absorbity

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$$Q_{rad} = \varepsilon\sigma A\left(T_S^4 - T_{sur}^4\right) \text{ ------ (1)}$$

This expression provides the difference between thermal energy that is released due to radiation emission and that which is gained due to radiation absorption.

NOTE: Kirchhoff's law, which states that the emissivity of a body which is in thermal equilibrium with its surrounding is equal to its absorptivity of the body. It should be noted that the source temperature is equal to the temperature of the irradiated surface. However, in practical purposes it is assumed that emissivity and absorptivity of a system are equal even if it is not in thermal equilibrium with the surrounding. The reason being the absorptivity of most real surfaces is relatively insensitive to temperature and wavelength. This particular assumption leads to the concept of grey body. The emissivity is considered to be independent of the wavelength of radiation for grey body.

There are many applications for which it is convenient to express the net radiation heat exchange emission and that which is gained due to radiation absorption.

$$Q_{rad} = h_{rad}A\left(T_s - T_{sur}\right) \text{ --------(2)}$$

where, from Equation(1),the radiation heat transfer coefficient h_{rad} is

$$h_{rad} = \varepsilon\sigma\left(T_s + T_{sur}\right)\left(T_S^2 + T_{sur}^2\right)$$

Here we have modeled the radiation mode in a manner similar to convection. In this sense we have linearized the radiation rate equation, making the heat rate proportional to a temperature difference rather than to the difference between two temperatures to the fourth power. Note, however, that h_{rad} depends strongly on temperature, while the temperature dependence of the convection heat transfer coefficient h is generally weak. The surfaces of Figure1 may also simultaneously transfer heat by convection to an adjoining gas. For the conditions of Figure1, the total rate of heat transfer from the surface is then

$$Q = Q_{conv} + Q_{rad} = hA(T_s - T_\infty) + \varepsilon\sigma A\left(T_S^4 - T_{sur}^4\right)$$

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2 CONDUCTION

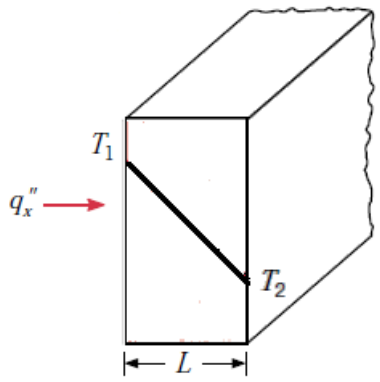
2.1 BASICS OF CONDUCTION

2.1.1 CONDUCTION RATE EQUATION

Consider a slab of known material is insulated on its lateral surface, while its end faces are maintained at different temperatures. The temperature difference causes conduction heat transfer in the positive *x* direction. It is observed that heat transfer rate *Q* is directly proportional to cross sectional area '*A*'. Similarly, holding the temperature difference and '*A*' constant, we observe that 'Q' varies inversely with thickness. Finally, holding *A* and thickness constant, we find that 'Q' is directly proportional to temperature difference. The collective effect is then

$$Q \propto -A\frac{\Delta T}{\Delta x}$$

Q is Rate of heat flow (kcal/hr or kJ/hr or W)
A is cross sectional area (perpendicular to the direction of heat flow)
Δx is thickness of material along the path of heat flow.
ΔT is temperature difference.
$\frac{\Delta T}{\Delta x}$ is temperature gradient



$$Q = -kA\frac{\Delta T}{\Delta x}$$

For Lim $\Delta x \to 0$, equation can be written as

$$Q = -kA\frac{dT}{dx}$$

-ve sign Indicates that the heat flow is in the direction of negative temperature gradient.
'k' is the constant of proportionality known as thermal conductivity , a property of material

Thermal conductivity may be defined as the amount of heat conducted per unit time across unit area and through unit thickness. When temp different of unit is maintained across the bounding surface.

unit of thermal conductivity k is

$$\frac{kcal}{m.hr.dg} / \frac{kcal}{m.hr.k} / \frac{kJ.}{m.hr.l} / \frac{watt}{m.k} / \frac{watt}{m.C}$$

Heat Flux

Heat flux is the heat transfer rate per unit area given by

$$Q'' = \frac{Q}{A} = -k\frac{dT}{dx}$$

The heat flux is a directional quantity. The direction of heat flux is normal to the cross-sectional area A or, more generally, the direction of heat flow will always be normal to a surface of constant temperature, called an isothermal surface.

The heat flux is a vector quantity, we can write a more general statement of the conduction rate equation (Fourier's law) as follows

$$Q'' = -\left(k_x\frac{dT}{dx}\hat{i} + k_y\frac{dT}{dy}\hat{j} + k_z\frac{dT}{dz}\hat{k}\right)$$

For Isotropic Material thermal conductivity is independent of direction

$$k_x = k_y = k_z = k$$

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$$Q'' = -kT = -k\left(\frac{dT}{dx}\hat{i}+\frac{dT}{dy}\hat{j}+\frac{dT}{dz}\hat{k}\right)$$

2.1.2 THERMAL CONDUCTIVITY

Thermal conductivity refers to the amount/speed of heat transmitted through a material. Heat transfer occurs at a higher rate across materials of high thermal conductivity than those of low **thermal conductivity**. Materials of high **thermal conductivity** are widely used in heat sink applications and materials of low thermal conductivity are used as thermal insulation. **Thermal conductivity** of materials is temperature dependent. The reciprocal of **thermal conductivity** is called thermal resistivity. Metals with high **thermal conductivity**, e.g. copper, exhibit high electrical conductivity. The heat generated in high **thermal conductivity** materials is rapidly conducted away from the region of the weld. For metallic materials, the electrical and **thermal conductivity** correlate positively, i.e. materials with high electrical conductivity (low electrical resistance) exhibit high **thermal conductivity.**

In general, the thermal conductivity of a solid is larger than that of a liquid, which is larger than that of a gas.

$$k_{solid} > k_{liquid} > k_{gas}$$

It is because of because inter molecular spacing.

(a) Solid State

In solid transport of thermal energy may be due to two effects: the migration of free electrons and lattice vibration waves. When viewed as a particle-like phenomenon, the lattice vibration quanta are termed **phonons**. In pure metals, the electron contribution to conduction heat transfer dominates, while in non-conductors and semiconductors, the phonon contribution is dominant. When electrons and phonons carry thermal energy leading to conduction heat transfer in a solid, the thermal conductivity may be expressed as

$$K = K_{e-} + K_{ph}$$

- Metals are best conductors while liquids are generally poor conductor.
- Thermal conductivity is always higher in the purest form of a metal.

  Pure metals>Alloys>Non-metallic
- Thermal conductivity depends on the regularity of the lattice arrangement. Crystalline (well ordered) materials like quartz having a higher thermal conductivity than amorphous materials like glass.
- Non-metallic solids such as diamond and beryllium oxide, k_{ph} can be quite large, exceeding values of k associated with good conductors, such as aluminium.
- Mechanical forming or heat treatment of metal cause considerable variation in thermal conductivity.
- For most materials, the dependence of thermal conductivity on temperature is almost linear.

  $k = k_o(1+\beta T)$

  β is +ve for nonmetals, insulators

  β is –ve for metal [except Al & certain nonferrous alloys].
- Density is another parameter that affects the thermal conductivity.it increases with density growth.

  At densities 400 & 8000 kg/m^3, thermal conductivity values of asbestos are .105 & 0.248 W/m-k.

(b) Fluid state

The fluid state includes both liquids and gases. Since the intermolecular spacing is much larger and the motion of the molecules is more random for the fluid state than for the solid state, thermal energy transport is less effective. The thermal conductivity of gases and liquids is

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therefore generally smaller than that of solids.

(i) Liquid state

The thermal conductivity of non-metallic liquids generally decreases with increasing temperature (water, glycerine, and engine oil are notable exceptions). The thermal conductivity of liquids is usually insensitive to pressure except near the critical point. Also, it is generally true that thermal conductivity decreases with increasing molecular weight.

(ii) Gaseous state

The thermal conductivity of gases increases with increase in temperature. The thermal conductivity of gases is insensitive to pressure except, when conditions approach that of a perfect vacuum. Thermal conductivity decreases with increasing molecular weight.

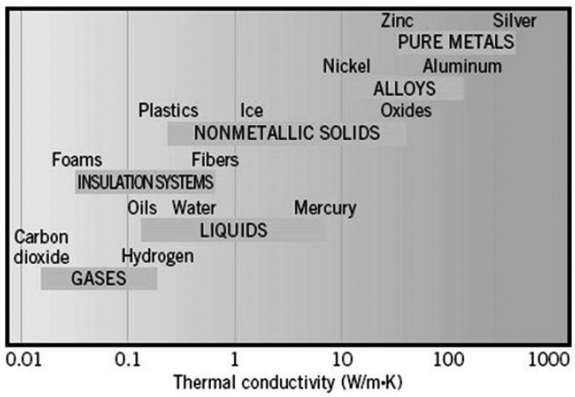


FIGURE Range of thermal conductivity for various states of matter at normal temperatures and pressure.

2.1.3 MULTIDIMENSIONAL HEAT CONDUCTION EQUATION

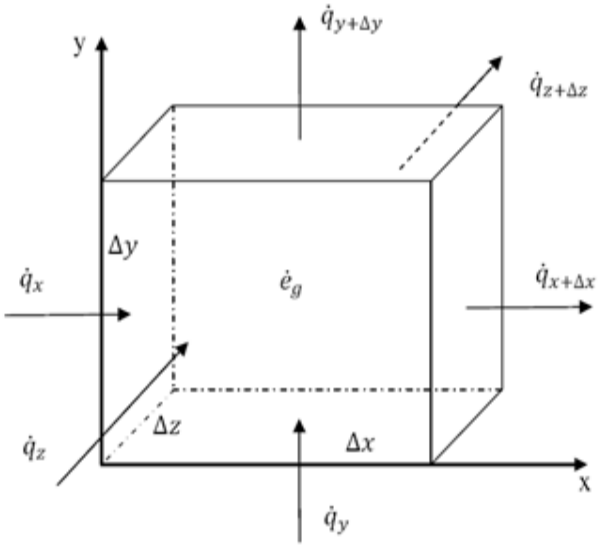
A major objective in a conduction analysis is to determine the temperature field in a medium resulting from conditions imposed on its boundaries.

Temperature distribution can be used for following:

1. To evaluate heat flux by Fourier's law at any point in the medium or on its surface.
2. To ascertain structural integrity through determination of thermal stresses, expansions, and deflections.
3. To optimize the thickness of an insulating material or to determine the compatibility of special coatings or adhesives used with the material.

(a) Heat conduction Equation in Cartesian Co-ordinates

We now consider the manner in which the temperature distribution can be evaluated



By energy balance

$$E_{in} - E_{out} = E_{stored}$$

$$(Q_{in} - Q_{out})_x + (Q_{in} - Q_{out})_y + (Q_{in} - Q_{out})_z + Q_{gen} = Q_{stored}$$

There may be heat sources inside the control volume due to nuclear fission, flow of electric current etc. Total heat generated in control volume is Q_{gen} . it is defines per unit volume. Its unit is w/m^3

The heat flow rate entering the control volume is given by Fourier's law

$$Q_x = -k_x (dy.dz)\left(\frac{\partial T}{\partial x}\right)$$

The heat flow rate going out from the control volume is given by Taylor series. By neglecting the higher order terms we get,

$$Q_{x+dx} = Q_x + \frac{\partial}{\partial x}(Q_x)dx$$

Rate of heat accumulation in x direction is given by

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$$dQ_x = (Q_{in} - Q_{out})_x = Q_x - \left[Q_x + \frac{\partial}{\partial x}(Q_x)dx \right]$$

Similarly, Rate of heat accumulation in Y & Z direction is given by

$$dQ_y = (Q_{in} - Q_{out})_y = Q_y - \left[Q_y + \frac{\partial}{\partial y}(Q_y)dy \right]$$

$$dQ_z = (Q_{in} - Q_{out})_z = Q_z - \left[Q_z + \frac{\partial}{\partial z}(Q_z)dz \right]$$

$$dQ_x = \frac{-\partial}{\partial x}(Q_x).dx$$

$$dQ_x = \frac{-\partial}{\partial x}\left[(-k_x dy.dz)\left(\frac{\partial T}{\partial x}\right) \right]$$

$$dQ_x = \frac{\partial}{\partial x}\left(k_x \frac{\partial T}{\partial x} \right).(dy.dz.dx)$$

Similarly,

$$dQ_y = \frac{\partial}{\partial y}\left(k_y \frac{\partial T}{\partial y} \right).(dy.dz.dx).$$

$$dQ_z = \frac{\partial}{\partial z}\left(k_z \frac{\partial T}{\partial z} \right)(dy\ dz\ dx).$$

Sum of heat accumulation gives the total heat stored in the elemental volume. Total accumulation of heat is equal to

$$\left[\frac{\partial}{\partial x}\left(k_x \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z}\left(k_z \frac{\partial t}{\partial z} \right) \right] dx.dy.dz.$$

Heat generated rate in volume is given by $Q_{gen} dy\ dx\ dz$

Where,

Q_{gen} is heat generated per unit volume per unit time. Its unit is W/m^3

The increase in thermal energy is reflected by the time rate of change in the heat capacity of the control volume.

$\frac{\partial T}{\partial t}$ is rate of change of temperature with time.

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z}\left(k_z \frac{\partial T}{\partial z} \right) + Q_{gen} = \rho C \frac{\partial T}{\partial t}$$

The above equation is general **heat diffusion equation**. It is valid for following conditions

I. Volumetric heat generation
II. Unsteady state
III. Three dimensional heat conduction
IV. Anisotropic material (Materials for which the properties varies along the direction)

Isotropic: - Properties are no directional characteristic of the material.

i.e. $k_x = k_y = k_z = k$

For isotropic material

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{Q_{gen}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Thermal diffusivity (α)

The ratio of the thermal conductivity to the heat capacity per unit volume (**k/ρc**) is an important property termed as **Thermal diffusivity'α'** which has units of m^2/s.It measures the ability of a material to conduct thermal energy relative to its ability to store thermal energy. Materials of large 'α'will respond quickly to changes in their thermal environment, while materials of small 'α' will respond more sluggishly, taking longer time to reach a new equilibrium condition.

I. **Steady State**

In steady state the temperature remains constant w.r.to time.

$$\frac{\partial T}{\partial t} = 0$$

Multidimensional heat equation for steady state is given by

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{Q}_{gen}}{k} = 0$$

(This equation is known as **Poisson's Eq**.)

II. Unsteady heat flow with no internal heat generation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\left(\frac{\partial T}{\partial t} \right)$$

This equation is known as **Heat diffusion equation**

III. Steady, no heat generation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0$$

This equation is known as **Laplace equation**

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(b) Heat conduction Equation in Cylindrical Co-ordinates

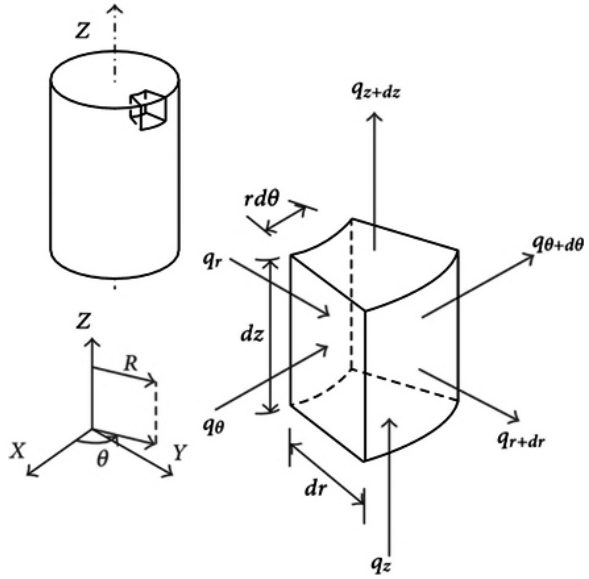
The cylindrical co-ordinates are expressed as (r,θ, z)

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\left(\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\left(\frac{\partial^2 T}{\partial^2 \theta}\right)+\left(\frac{\partial^2 T}{\partial z^2}\right)+\frac{Q_{gen}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Case-I

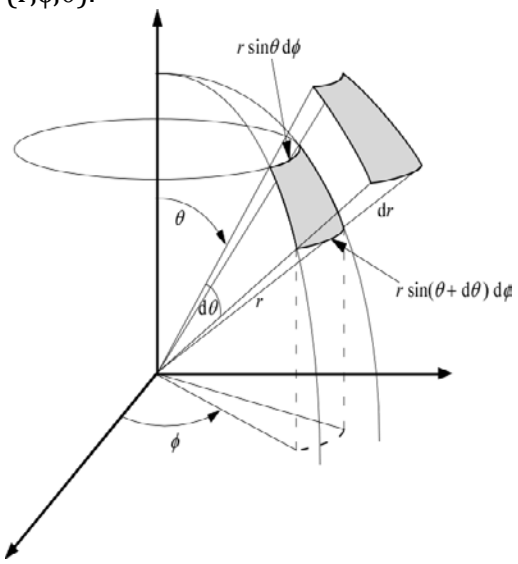
When temperature varies in r-direction only

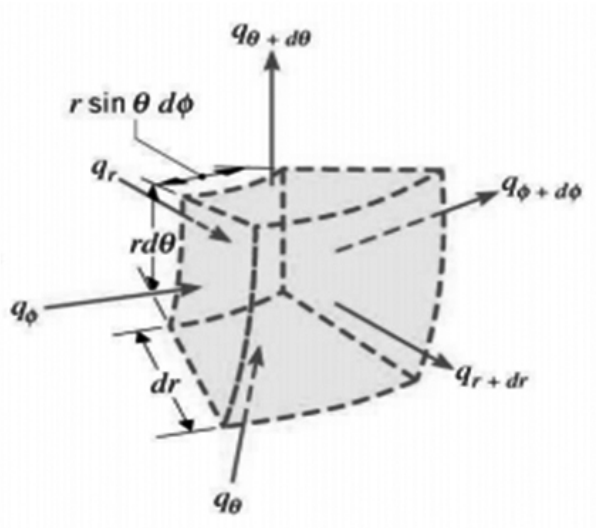
$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r\partial T}{\partial r}\right)+\frac{Q_{gen}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$



(c) Heat conduction Equation in spherical Co-ordinates

The spherical coordinates are expressed as (r,ϕ,θ).





General Heat conduction equation in spherical co-ordinate is given by

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial\phi}\left(k\frac{\partial T}{\partial\phi}\right)+$$

$$\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial\theta}\left(k\sin\theta\frac{\partial T}{\partial\theta}\right)+Q_{gen}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$

For temperature variation only in radial direction.

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right)=0$$

Combined one dimensional Heat conduction Equation

$$\frac{1}{r^n}\frac{\partial}{\partial r}\left(\frac{r^n k\partial T}{\partial r}\right)+\frac{Q_{gen}}{k}=\rho C\frac{\partial T}{\partial t}$$

Where,

n = 0 for plane wall

n = 1 for cylinder

n = 2 for sphere

Note: - Heat generation is a volumetric phenomenon. That is, it occurs throughout the body of a medium. Therefore, the rate of heat generation in a medium is usually specified per unit volume

2.1.4 BOUNDARY CONDITION

To provide Unique solution to a problem it is required a specific governing equation & Boundary/ Initial condition. The multidimensional heat conduction equation is second order in the spatial coordinates. Two boundary conditions must be

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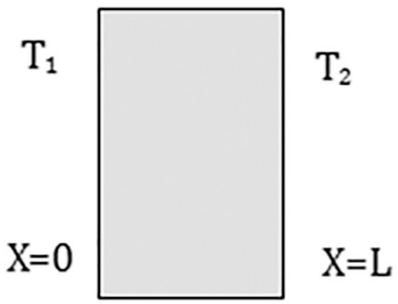
expressed for each coordinate needed to describe the system. Because the equation is first order in time only one condition is required.The temperature at any point on the wall at a specified time also depends on the condition of the wall at beginning of heat conduction process. Such a condition, which is usually specified at time (t = 0), is called the initial condition.

(a) Specified Temperature Boundary Condition

Specified boundary Condition corresponds to a situation for which the surface is maintained at a fixed temperature. It is commonly termed a **Dirichlet condition**, or a boundary condition of the **first kind**. It is closely approximated, for example, when the surface is in contact with a melting solid or a boiling liquid. In both cases there is heat transfer at the surface, while the surface remains at the temperature of the phase change process.

$T(0) = T_1$

$T(L) = T_2$

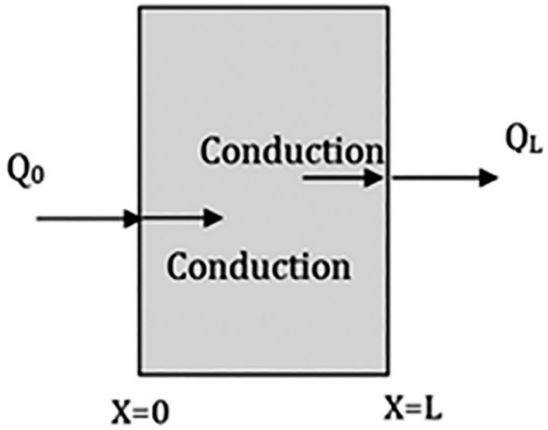


(b) Specified heat flux boundary condition

The second condition corresponds to the existence of a fixed or constant heat flux at the surface. This heat flux is related to the temperature gradient at the surface by Fourier's law, this condition is termed as **Neumann condition**, or a boundary condition of the **second kind**.

$$Q_o = -k\frac{\partial T(0,t)}{\partial x}$$

$$Q_L = -k\frac{\partial T(L,t)}{\partial x}$$



The sign of heat flux is determined by inspection, positive if the heat flux is in positive direction negative if it is opposite direction.

For e.g.

For a plate of thickness L subjected to heat flux 50 w/m^2 into the medium from both side.

$$\frac{-k\partial T(0,t)}{\partial x} = 50$$

$$\frac{-k\partial T(L,t)}{\partial x} = -50$$

Note: - The heat flux at the surface at x = L is in negative direction.

Insulated Boundary

It is special case of specified heat flux boundary condition. Some surfaces are commonly insulated to minimize heat loss. At insulated boundary the heat flux is zero. The thermal boundary condition at insulation is given by

$$-k\frac{\partial T(L,t)}{\partial x} = 0$$

$$\text{or}\ \frac{\partial T(L,t)}{\partial x} = 0$$

Thermal Symmetry

Thermal symmetry is a result of symmetry in imposed thermal conditions. Two surface of a large hot plate of thickness L suspended vertically in air will be subjected to same thermal conditions, and thus the temperature distribution in one half of the plate will be the same as that in other half.

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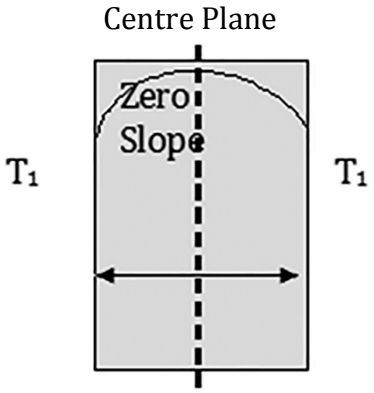


Due to symmetry about center plane the slope at this point will be zero.

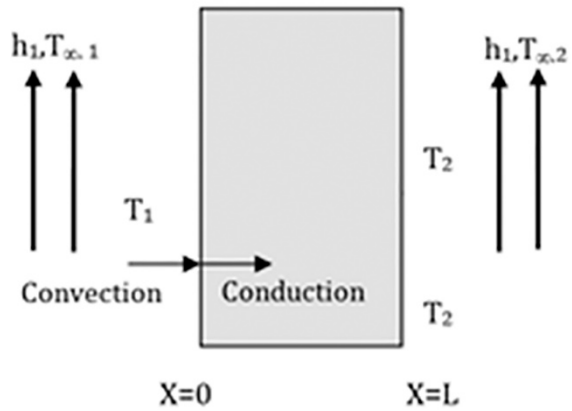
$$\frac{\partial T(L/2,t)}{\partial x}=0$$

The center plane can be viewed as an insulated plane surface and the thermal condition which resembles the insulation or zero flux boundary conditions.

(c) Convection Boundary Condition

The boundary condition of the **third kind** corresponds to the existence of convection heating (or cooling) at the surface and is obtained from the surface energy balance.

$$\begin{pmatrix}\text{Heat conduction}\\ \text{at the surface}\\ \text{in a direction}\end{pmatrix}=\begin{pmatrix}\text{Heat convection}\\ \text{at the surface}\\ \text{in same direction}\end{pmatrix}$$



Boundary condition as x= 0

$$-k\frac{\partial T(0,t)}{\partial x}=h_1\left(T_{\infty,1}-T_1\right)$$

Bounddary conditio at x= L

$$-k\frac{\partial T(0,t)}{\partial x}=h_1\left(T_{\infty,1}-T_1\right)$$

(d) Interface Boundary

Some bodies are made up of layers of different materials. The boundary condition is based on assumptions that

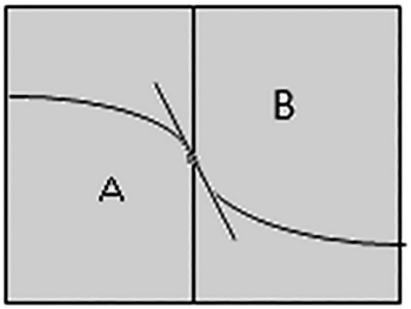
1) Two bodies in contact should have same temperature at contact area.
2) Interface cannot store energy.

The temperature distributions in both bodies are different.

Thermal boundary condition at the interface of two bodies A and B in perfect contact are

$$T_A\left(x_o,t\right)=T_B(x_o,t)$$

$$\frac{-k_A\partial T_A(x_o,t)}{\partial x}=\frac{-k_B\partial T_B(x_o,t)}{\partial x}$$



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2.2 ONE DIMENSIONAL, STEADY STATE CONDUCTION WITHOUT HEAT GENERATION

In one-dimensional system, temperature gradients exist along only a single coordinate direction, and heat transfer occurs exclusively in that direction. The system is characterized by steady-state conditions if the temperature at each point is independent of time. The objective is to determine expressions for the temperature distribution and heat transfer rate in common (planar, cylindrical, and spherical) geometries.

2.2.1 PLANE WALL

For one-dimensional conduction in a plane wall, temperature is a function of the *x* coordinate only and heat is transferred exclusively in this direction.

General heat conduction in plane wall is given by

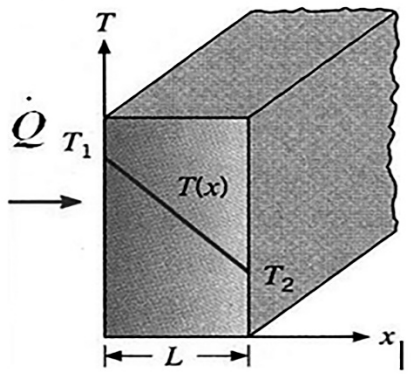
$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{Q_g}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$

For Steady, One Dimensional, and No heat generation the conduction equation is given by

$$\frac{d^2 T}{dx^2}=0$$

General solution of above differential equation is

$$T(x)=c_1 x+c_2$$



To evaluate C_1 & C_2, the specified temperature Boundary Condition is applied

$$T(0)=T_1 \text{ at } x=0$$

$$T(L)=T_2 \text{ at } x=L$$

$$c_2=T_1 \;\&\; c_1=(T_2-T_1)/L$$

Substituting the values into the general solution, then the temperature distribution in plane wall obtained is

$$T(x)=T_1+\frac{(T_2-T_1).x}{L}$$

For one-dimensional, steady-state conduction in a plane wall with no heat generation and constant thermal conductivity, the temperature varies linearly with x.

Heat conduction rate is determined by Fourier's law

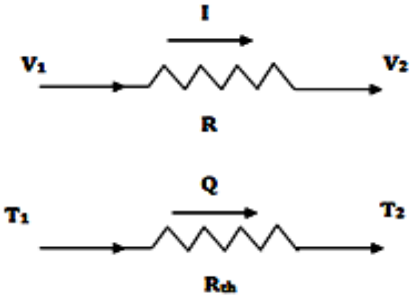
$$Q=-kA\frac{dT}{dx}=kA\frac{(T_1-T_2)}{L}$$

(a) Network Analysis

There exists an analogy between the conduction of heat and conduction of electricity. An electrical resistance is associated with the conduction of electricity; a thermal resistance may be associated with the conduction of heat. Resistance is defined as the ratio of a driving potential to the corresponding energy transfer rate.

Electrical resistance is given by

$$R=\frac{V_1-V_2}{I}$$



Where,

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ΔV is potential deifference
I is current
Thermal resistance is given by

$$R_{th} = \frac{T_1 - T_2}{Q}$$

Thermal resistance of plane wall is

$$\boxed{R_{th} = \frac{\Delta T}{Q} = \frac{L}{kA}}$$

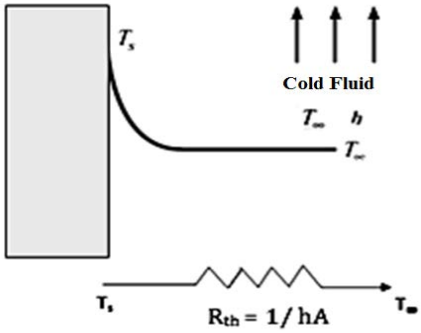
Thermal resistance due to convection

A thermal resistance may also be associated with heat transfer by convection at a surface.
From Newton's law of cooling,

$$Q = hA(T_S - T_\infty)$$

Rearranging the equation

$$\boxed{R_{th} = \frac{T_s - T_\infty}{Q} = \frac{1}{hA}}$$



Thermal resistance due to Radiation

A thermal resistance may also be associated with heat transfer by Radiation at a surface. From Stefan's law, Heat transfer rate is expressed by

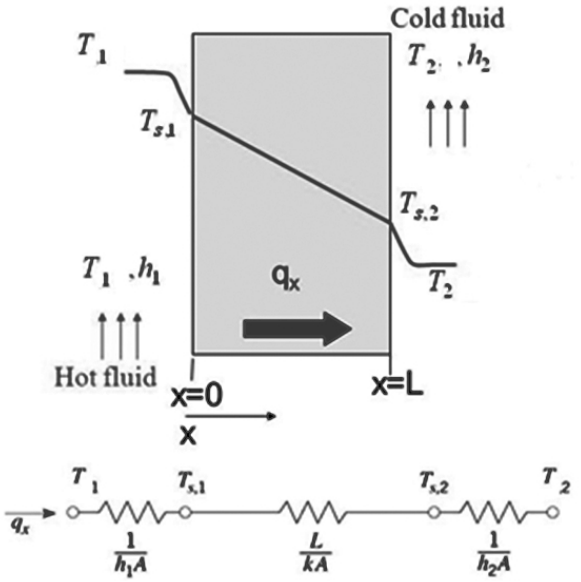
$$Q = \in \sigma A\left(T_S^4 - T_\infty^4\right)$$

Rearranging the equation

$$R_{th} = \frac{T_s - T_\infty}{Q} = \frac{1}{\in \sigma A(T_s + T_\infty)(T_S^2 + T_\infty^2)}$$

(b) Network analysis of plan wall

The equivalent thermal circuit for the plane wall with convection surface conditions is shown in Figure. The heat transfer rate may be determined from separate consideration of each element in the network.



Since Q is constant throughout the network, it follows that

$$Q = \frac{T_{,1} - T_{s,1}}{1/h_1A} = \frac{T_{s,1} - T_{s,2}}{L/kA} = \frac{T_{s,2} - T_{,2}}{1/h_2A} = \frac{T_{,1} - T_{,2}}{\frac{1}{h_1A} + \frac{L}{kA} + \frac{1}{h_2A}}$$

(c) Overall heat transfer coefficient

For composite systems it is often convenient to work with an overall heat transfer coefficient, U, which is defined by an expression analogous to Newton's law of cooling.

$$Q = UA\Delta T$$

Where, ΔT is the overall temperature difference. The overall heat transfer coefficient is related to the total thermal resistance

$$UA = \frac{1}{\sum R}$$

(d) Composite Walls

Equivalent thermal circuits may also be used for more complex systems, such as composite walls. Such walls may involve any number of series and parallel thermal

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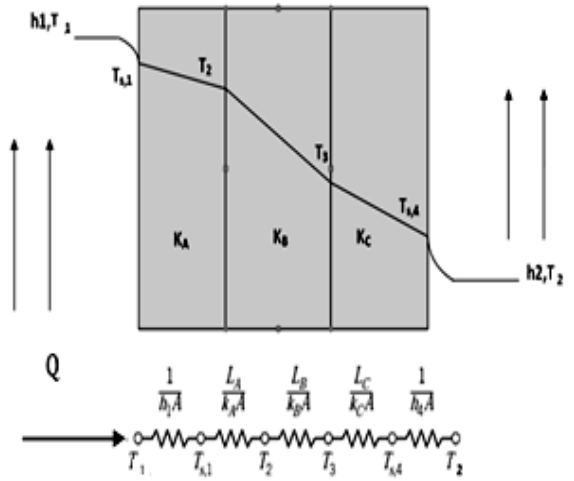
resistances due to layers of different materials.

Series

Let us consider three slabs in series. Hot fluid and cold fluid are flowing on the sides of the slab as shown in figure



The one-dimensional heat transfer rate for this system may be expressed as

$$Q = \frac{T_{,1} - T_{,2}}{\sum R}$$

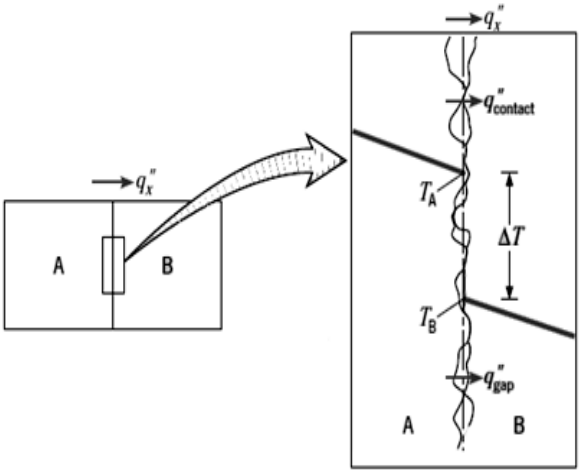
$$Q = h_c A$$

$$Q = \frac{T_{,1} - T_{s,1}}{1/h_1 A} = \frac{T_{s,1} - T_{s,2}}{L_A / k_A A} = \frac{T_{s,2} - T_{s,3}}{L_B / k_B A}$$

$$= \frac{T_{s,3} - T_{s,4}}{L_C / k_C A} = \frac{T_{s,4} - T_2}{1/h_2 A}$$

(e) Thermal contact Resistance

In the analysis of heat conduction through multilayer solids, we assumed "perfect condition" at the interface of two layers, and thus no temperature drop at the interface. In reality , however ,even flat surfaces that appear smooth to the eye turn out to be rather rough .When two such surfaces are pressed against each other ,the peaks will form good material contact but the valleys will form voids filled with air. As result interface will contain numerous air gaps of varying sizes that act as insulation because of low thermal conductivity of air. This resistance per unit interface area is called the thermal contact resistance, R_c



$$Q = Q_{contact} + Q_{gap}$$

It can also be expressed in an analogous manner to Newton's law of cooling as

$$Q = h_c A \, \Delta T_{interface}$$

Where A is the apparent interface area (which is same as the cross sectional area of the rods) and $\Delta T_{interface}$ is the effective temperature difference at the interface. The quantity h_c which corresponds to the convection heat transfer coefficient, is called the thermal contact conductance and is expressed as

$$R_c = \frac{1}{h_c} = \frac{\Delta T_{interface}}{Q/A}$$

2.2.1 RADIAL SYSTEM

(a) Radial heat conduction through cylindrical system

For Steady, One Dimensional, No heat generation the conduction equation in cylindrical coordinate system is given by

$$\frac{d}{dr}\left[r\frac{dT}{dr}\right] = 0$$

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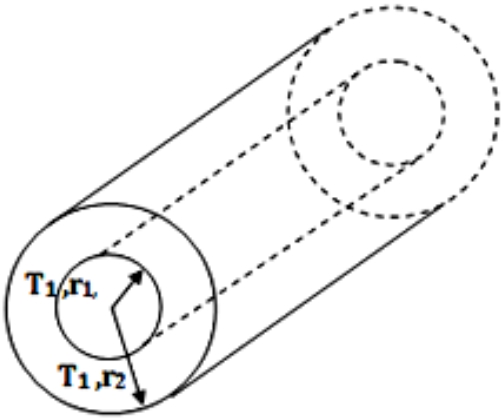

By solving the above differential equation

$T(r) = C_1 \ln r + C_2$

To evaluate C_1 & C_2, the specified temperature Boundary Condition is applied

$T(r_1) = T_1$ & $T(r_2) = T_2$

Applying the boundary condition to general solution

$T(r_1) = T_1 = C_1 \ln r_1 + C_2$

$T(r_2) = T_2 = C_1 \ln r_2 + C_2$

Solving the above equations & substituting the value of C_1& C_2 in general solution temperature distribution obtained is

$$T(r) = T_2 + (T_1 - T_2)\frac{\ln(\frac{r}{r_2})}{\ln\left(\frac{r_1}{r_2}\right)}$$

Note that the temperature distribution associated with radial conduction through a cylindrical wall is logarithmic, not linear, as it is for the plane wall under the same conditions

Heat conduction rate is determined by Fourier's law

$$Q = -k\,A\frac{dT}{dr} = \frac{2\pi kL(T_2 - T_1)}{\ln\left(\frac{r_2}{r_1}\right)}$$

From this result it is evident that, for radial conduction in a cylindrical wall, the thermal resistance is of the form

$$R_{cond,cyl} = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi kL}$$

Let us rearrange this result

$$R_{cond,cyl} = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi kL} = \left(\frac{r_2 - r_1}{r_2 - r_1}\right)\frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi kL}$$

$$= \left(\frac{r_2 - r_1}{\frac{k(2\pi r_2 L - 2\pi r_1 L)}{\ln\left(\frac{2\pi r_2 L}{2\pi r_1 L}\right)}}\right) = \frac{r_2 - r_1}{kA_{lm}}$$

Where, we have introduced a new symbol A_{lm}, which stands for log mean area. It is defined as

$$A_{lm} = \frac{(A_2 - A_1)}{\ln\left(\frac{A_2}{A_1}\right)}$$ Where, $A_1 = 2\pi r_1 L$ is the inner surface area of pipe wall and $A_2 = 2\pi r_2 L$ is the outer surface area of pipe wall. The log mean always lies between two values being averaged. Try comparing it with the more common arithmetic average, which is $\frac{A_1 + A_2}{2}$. You will find that the log mean is always smaller than the arithmetic average. For $(A_2 / A_1) \le 2$, the difference between two averages is less than 4%

(b) Radial Heat conduction through spherical system

For Steady, One Dimensional, No heat generation the conduction equation in spherical coordinate system is given by

$$\frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2 k\,dT}{dr}\right) = 0$$

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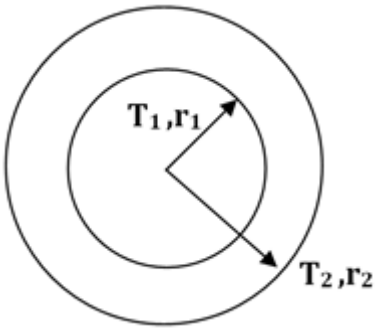
$$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)=0$$

By solving the above differential equation

$$T(r)=-\frac{C_1}{r}+C_2$$



To evaluate C_1& C_2, the specified temperature Boundary Condition is applied

$$T(r_1)=T_1 \;\&\; T(r_2)=T_2$$

Applying the condition to general solution

$$T(r)=-\frac{C_1}{r_1}+C_2$$

$$T(r)=-\frac{C_1}{r_2}+C_2$$

As Solving the above equations & substituting the value of C_1& C_2 in general solution Temperature distribution obtained is

$$T(r)=T_1-(T_1-T_2)\left[\frac{1-(\frac{r}{r_1})}{1-\left(\frac{r_1}{r_2}\right)}\right]$$

Note that the temperature distribution associated with radial conduction through a spherical wall is not linear, as it is for the plane wall under the same conditions.

Heat conduction rate is determined by Fourier's law

$$Q=-k\,A\frac{dT}{dr}=\frac{4\pi\, r_1r_2\; k\left(T_1-T_2\right)}{\left(r_2-r_1\right)}$$

From this result it is evident that, for radial conduction in a spherical wall, the thermal resistance is of the form

$$R_{cond,sphere}=\frac{\left(r_2-r_1\right)}{4\pi\, r_2r_1kL}$$

Let us rearrange this result,

$$R_{cond,sphere}=\frac{\left(r_2-r_1\right)}{4\pi\, r_2r_1k}=\frac{\left(r_2-r_1\right)}{4\pi\, r_{GM}{}^2k}=\frac{r_2-r_1}{kA_{GM}}$$

Where, A_{GM} is a geometric mean area, which is given by, $A_{GM}=4\pi\, r_1r_2$

And r_{GM} is a geometric mean radius, which is given by $r_{GM}=\sqrt{r_1r_2}$.

Summary of one dimensional temperature distribution, heat conduction rate & thermal resistance

	Plane Wall	Cylindrical Wall	Spherical Wall
Heat Equation	$\frac{d^2T}{dx^2}=0$	$\frac{d}{dr}[r\frac{dT}{dr}]=0$	$\frac{d}{dr}[r^2\frac{dT}{dr}]=0$
Temperature Distribution	$T_1+(T_1-T_2)\frac{x}{L}$	$T_2+(T_1-T_2)\frac{\ln(\frac{r}{r_2})}{\ln(\frac{r_1}{r_2})}$	$T_1-(T_1-T_2)\frac{1-(\frac{r}{r1})}{1-(\frac{r1}{r2})}$
Heat Rate (Q)	$kA\frac{(T_1-T_2)}{L}$	$\frac{2\pi kl(T_1-T_2)}{\ln\frac{r_2}{r_1}}$	$\frac{4\pi r_1r_2\,k\,(T_1-T_2)}{(r_2-r_1)}$
Thermal Resistance	$\frac{L}{kA}$	$\frac{\ln(r_2/r_1)}{2\pi kL}$	$\frac{r_2-r_1}{4\pi(r_1r_2)k}$

(c) Composite radial systems

Steady heat transfer through multilayered cylindrical or spherical shells can be handled just like multilayered plane walls.

Simply adding an additional resistance in series for each additional layer. For example, the steady heat transfer rate through the three – layered composite cylinder of length L

$$R_{total}=R_{conv,1}+R_{cyl,1}+R_{cyl,2}+R_{cyl,3}+R_{conv,2}$$

$$R_{total}=\frac{1}{h_1A_1}+\frac{1n(r_2/r_1)}{2\pi LK_1}+\frac{1n(r_3/r_2)}{2\pi LK_2}+\frac{1n(r_4/r_3)}{2\pi LK_3}+\frac{1}{h_2A_4}$$

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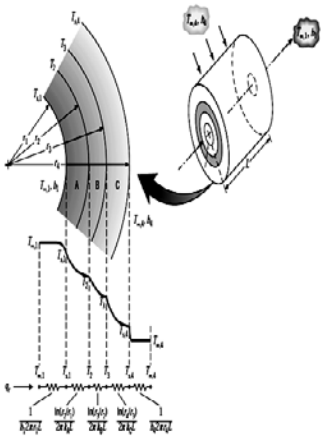

The overall heat transfer coefficient can be defined as

$$U_1A_1 = U_2A_2 = U_3A_3 = 1/R_{total}$$

Above equation can also be used for a three layered spherical shell by replacing the thermal resistance of cylindrical layers by the corresponding spherical ones. Again note from the thermal resistance network that the resistances are in series, and thus the total thermal resistance is simply the arithmetic sum of the individual thermal resistances in the path of heat flow.

2.2.3 INSULATIONS

Insulation is a material of lower thermal conductivity which reduces heat loss from the system. For a plane wall, heat transfer area A is constant and adding insulation always increases the thermal resistance of the wall without affecting the convection resistance.

Adding insulation to a cylindrical pipe or spherical shells however is a different matter. The additional insulation increases the conduction resistance of the insulation layer but decreases the convection resistance of the surface because of the increase in the outer surface area for convection. The Heat transfer from the pipe may increase or decrease, depending on which effect dominates.

Consider a cylindrical pipe of outer radius r_1 whose outer surface temperature T_1 is maintained constant. The pipe is now insulated with a material whose thermal conductivity is 'K_{ins}' and outer radius is 'r'. Heat is lost from the pipe to the surrounding medium to temperature 'T_∞' with a convection heat transfer coefficient 'h' .The rate of heat transfer from the insulate pipe to the surrounding air can be expressed as

$$Q = \frac{T_1 - T_\infty}{R_{ins} + R_{conv}} = \frac{T_1 - T_\infty}{\frac{\ln(r/r_1)}{2\pi LK_{ins}} + \frac{1}{h(2\pi rL)}}$$

Total thermal resistance is given by

$$R_{total} = R_{ins} + R_{conv}$$

An optimum insulation thickness would be associated with the value of r_2 that minimized Q or maximized R_{total} . Such a value could be obtained from the requirement that

$$\frac{dR_{total}}{dr} = 0$$

By differentiation and equating to zero, we obtain

$$\frac{1}{2\pi K_{ins} r} - \frac{1}{2\pi r^2 h} = 0$$

By rearranging

$$r = \frac{K_{ins}}{h}$$

By differentiating again w.r.t r

$$\frac{d^2R_{total}}{dr^2} = -\frac{1}{2\pi K_{ins} r_2^2} + \frac{1}{\pi r_2^3 h}$$

At $r = \frac{K_{ins}}{h}$

$$\frac{d^2R_{total}}{dr_2^2} > 0$$

Since this result is always positive, it follows that r is the insulation radius for which the total resistance is a minimum, not a maximum. Hence an optimum insulation thickness does not exist.

The radius $r = \frac{K_{ins}}{h}$ is known as critical radius of insulation. The rate of heat transfer from the cylinder increases with the addition of insulation for $r < r_{cr}$, reaches a maximum when $r = r_{cr}$, and starts to

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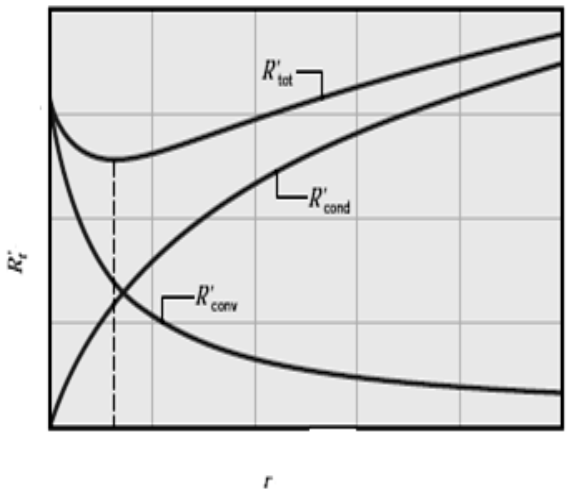
decrease for $r > r_{cr}$ .Thus, insulating the pipe may actually increase the rate of heat transfer from the pipe instead of decreasing it when $r < r_{cr}$.



This value would be even smaller when the radiation effects are considered. The critical radius would be much less in forced convection, often less than 1 mm, because of much larger *h* values associated with forced convection. Therefore, we can insulate hot water or steam pipes freely without worrying about the possibility of increasing the heat transfer by insulating the pipes.

The radius of electric wires may be smaller than the critical radius. Therefore, the plastic electrical insulation may actually enhance the heat transfer from electric wires and thus keep their steady operating temperatures at lower and thus safer levels.

The discussions above can be repeated for a sphere, and it can be shown in a similar manner that the critical radius of insulation for a spherical shell is $r = \frac{2K_{ins}}{h}$

where k_{ins} is the thermal conductivity of the insulation and *h* is the convection heat transfer coefficient on the outer surface

2.2.4 SYSTEMS WITH VARIABLE THERMAL CONDUCTIVITY

When the variation of thermal conductivity with a temperature in a specified temperature interval is large, it may be necessary to account for this variation. The variation in thermal conductivity of a material with temperature in the temperature range of interest can often be approximated as a linear function and is expressed

$$k = k_o(1+BT)$$

Heat transfer rate is given by Fourier's Law

$$Q = -kA\left(\frac{dT}{dx}\right)$$

$$Q = -k_o(1+BT)\frac{dT}{dx}A$$

$$Q\int_0^L dx = -\int_{T_1}^{T_2} k_o(1+BT)\,dT.A$$

$$QL = -k_o\left[\left(T_2+\frac{\beta T_2^{\,2}}{2}\right)-\left(T_1+\frac{\beta T_1^{\,2}}{2}\right)\right]A$$

$$Q = \frac{-k_o}{L}\left[(T_2-T_1)+\beta\frac{(T_2^{\,2}-T_1^{\,2})}{2}\right]A$$

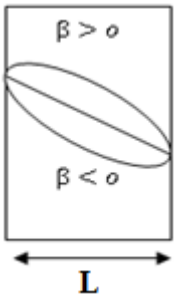
$$Q = \frac{-k_o}{L}(T_2-T_1)\left[1+\beta\frac{(T_2+T_1)}{2}\right]A$$

$$Q = k_{mean}\frac{(T_1-T_2)}{L}A$$

Where,

$$k_{mean} = k_o\left[1+B\frac{(T_2+T_1)}{2}\right]$$

Temperature distribution in plane wall is dependent on the β.



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Hollow cylinder with variable thermal conductivity.

$$k_m = k_o(1+BT)$$

$$Q = \frac{2\pi k_m L(T_i - T_o)}{\ln \frac{r_o}{r_i}}$$

2.3 ONE DIMENSIONAL STEADY STATE CONDUCTION WITH HEAT GENERATION

In the previous section, with no heat generation the temperature distribution is only because of boundary conditions. The temperature distribution is changed due to heat generation.

Energy generation in system may occur due to

1. Conversion of electrical to thermal energy in a current-carrying medium
2. Deceleration and absorption of neutrons in the fuel element of a nuclear reactor
3. Exothermic chemical reactions occurring within a medium.
4. Endothermic reactions have the inverse effect (a thermal energy sink) of converting thermal energy to chemical bonding energy.

It is important to note that energy stored is different from energy generation.

2.3.1 PLAN WALL

For one-dimensional conduction in a plane wall, temperature is a function of the *x* coordinate only and heat is transferred exclusively in this direction.

General heat conduction in plane wall is given by

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{Q_g}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

For Steady, One Dimensional, and with heat generation the conduction equation is given by

$$\frac{d^2 T}{dx^2} + \frac{Q_g}{k} = 0$$

The solution of above differential equation is

$$T(x) = -\frac{Q_g}{2k}x^2 + C_1 x + C_2 \qquad \text{------(1)}$$

To evaluate C_1 & C_2, the specified temperature Boundary Condition is applied

$$T(0) = T_1 \text{ at } x = 0$$

$$T(L) = T_2 \text{ at } x = L$$

$$C_2 = T_1 \quad ; C_1 = \frac{(T_2 - T_1)}{L} + \frac{Q_g}{2k}L$$

Substituting the value of C_1 & C_2 in equation (1), then the Temperature distribution in plane wall obtained is

$$T(x) = -\frac{Q_g}{2k}x^2 + \left(\frac{(T_2 - T_1)}{L} + \frac{Q_g}{2k}L\right)x + T_1$$

For $T_2 = T_1$

$$T(x) = -\frac{Q_g}{2k}x^2 + \left(\frac{Q_g}{2k}L\right)x + T_1$$

Evaluation of surface temperature

Consider a body is inserted in medium of temperature T_∞ & convection heat transfer coefficient 'h', Surface temperature is given by convective boundary condition

$$h(T_1 - T_\infty) = -k\left(\frac{dT}{dx}\right)_{x=L}$$

By solving the above equation

$$T_1 = T_2 = T_\infty + \frac{Q_g L}{2h}$$

2.3.2 RADIAL SYSTEMS

Consider solid cylinder generating heat. Solid cylinder can represent a current carrying conductor or fuel element in nuclear reactor. For steady-state conditions the rate at which heat is generated within the cylinder must equal the rate at which heat is convicted from the surface of the cylinder to a moving fluid. This condition allows the surface temperature to be maintained at a fixed value of T_s.

For Steady, One Dimensional, and with heat generation the conduction equation is given by

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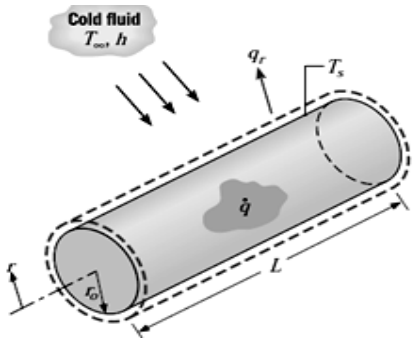


$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\frac{Q_g}{k}=0$$

The solution of above differential equation is

$$T(r)=-\frac{Q_g}{4k}r^2+C_1\ln r+C_2$$

To evaluate C_1 & C_2, the Boundary conditions are applied. For the solid cylinder the centerline is a line of symmetry for the temperature distribution and the temperature gradient must be zero.

$$\text{At } r=0,\ \frac{dT}{dr}=0 \text{ and } T(r_0)=T_s$$

From boundary conditions

$$C_1=0 \text{ and } C_2=Ts+\frac{Q_g}{4k}(r_0)^2$$

The temperature distribution is given by

$$T(r)=\frac{Q_g}{4k}r_0^2\left(1-\frac{r^2}{r_0^2}\right)+T_s$$

To relate the surface temperature T_s, to the temperature of the cold fluid, T_∞, either a surface energy balance or an overall energy balance may be used.

$$Q_g\left(\pi r_0^2 l\right)=h(2\pi r_0 l)(T_s-T_\infty)$$

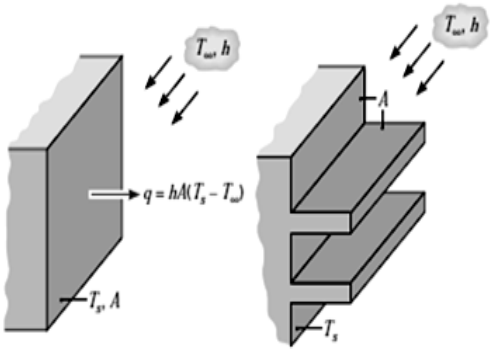
$$T_s=T_\infty+\frac{Q_g r_0}{2h}$$

2.4 EXTENDED SURFACES

The rate of heat transfer from a surface at a temperature T_S to the surrounding medium at T_∞ is given by Newton Law of Cooling.

$$Q_{conv}=hA\left(T_s-T_\infty\right)$$

Q_{conv} can be increased by increasing A or h. Increasing 'h' requires more efforts (installing pump, fan etc.).Therefore, alternative is to install extended surface called **fins** made up of highly conductive & low density material like Aluminum Alloy.



2.4.1 ANALYSIS OF HEAT TRANSFER THROUGH EXTEND SURFACE

The following assumptions are made for fin analysis

1. Steady-state condition
2. The temperature is only a function of *x*.
3. Thermal conductivity is constant
4. Radiation from the surface is negligible,
5. Heat generation effects are absent,
6. Convection heat transfer coefficient is uniform over the surface.

By Energy balance

$$Q_{cond_x}=Q_{cond_{x+\Delta x}}+dQ_{conv}$$

Form Newton's law for convection

$$dQ_{conv}=h\,P(dx)(T-T_\infty)$$

Where,

'h' is convective heat transfer coefficient

'P' is the perimeter

'T' is the temperature at distance x from base

T_∞ is the temperature of cool fluid/ambient

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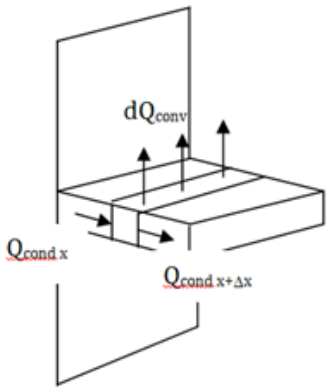


By Fourier's Law

$$Q_{cond_x} = -kA_c \frac{dT}{dx}$$

where, **A_c is cross section** of fin at distance x

$$Q_{cond_{x+\Delta x}} = Q_{cond_x} + \frac{d}{dx}\left(-kA_c \frac{dT}{dx}\right)\Delta x$$

By combining above equations

$$\frac{d}{dx}\left(-kA_c \frac{dT}{dx}\right)\Delta x + h\,P\Delta x\left(T - T_\infty\right) = 0$$

$$k\frac{d}{dx}\left(\frac{A_c dT}{dx}\right) - h\,P\left(T - T_\infty\right) = 0$$

General equation is **applicable to variable area also**,

if A_c is constant

$$kA_c \frac{d^2T}{dx^2} - h\,P\left(T_s - T_\infty\right) = 0$$

$$\text{Assuming } \left(T_s - T_\infty\right) = \theta$$

$$\frac{d^2T_s}{dx^2} = \frac{d^2\theta}{dx^2}$$

$$\frac{d^2\theta}{dx^2} - \left(\sqrt{\frac{h\,P}{kA_c}}\right)^2 \theta = 0$$

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0, \text{ Where } m = \sqrt{\frac{h\,P}{kA_c}}$$

The above equation is Linear, homogenous; second order differential equation with constant coefficients.

General solution of above differential equation is given by

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$$

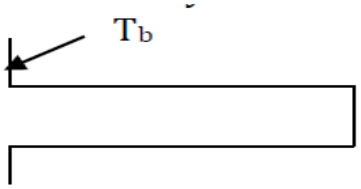
To determine the unique solution boundary conditions are required.

2.4.2 BOUNDARY CONDITION FOR EXTENDED SURFACES

Boundary condition at fin base

At the fin base we have a specified temperature boundary condition, expressed as

$$\theta(0) = \theta_b = T_b - T_\infty$$



(a) Boundary condition at Tip

At the fin tip we have several possibilities, including specified temperature, negligible heat loss (idealized as an insulated tip), convection and combined convection and radiation.

(i) Infinitely Long Fin

For a sufficiently long fin of uniform cross section (A_c=constant), the temperature of the fin at fin tip will approach the environment temperature T_∞ and thus θ will approach zero.

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$$

Boundary condition at base

$$x = 0, \quad \theta = \theta_b$$

$$\theta(0) = \theta_b = C_1 + C_2$$

Boundary condition at tip

$$x \to \infty, \quad T_{tip} \to T_\infty$$

Using the above boundary conditions, the temperature distribution in fin is given by

$$T(x) - T_\infty = \left(T_b - T_\infty\right) e^{-\sqrt{\frac{h\,P}{k\,A_c}}\,x}$$

$$T(x) - T_\infty = \left(T_b - T_\infty\right) e^{-m\,x}$$

$$\text{where, } m = \sqrt{\frac{h\,P}{k\,A_c}}$$

Total heat lost by Fin is given by adding the heat loss from small elements of length 'dx'.

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$$dQ = h\,P(dx)\left(T - T_\infty\right)$$

By integrating the above equation

$$\int_o^Q dQ = \int_o^\infty h\,P\left(T_b - T_\infty\right)e^{-\sqrt{h\,P/KA_c}.x}.dx$$

Total heat lost by infinite fin is given by

$$Q_{fin} = \sqrt{hPKA_c}\left(T_b - T_\infty\right)$$

(b) Negligible heat loss from fin tip (Insulated Tip)

Fins are not likely to be so long that their temperature approaches the surrounding temperature at the tip. A more realistic situation is for heat transfer from the fin tip to be negligible since the heat transfer from the fin is proportional to its surface area, and the surface area of the fin tip is usually a negligible fraction of the total fin area. Then the fin tip is assumed to be insulated.

Boundary Condition at tip is

$$\left.\frac{d\theta}{dx}\right|_{x=L} = 0$$

Boundary condition at base

$x = 0,\ \theta = \theta_b$

Solving the general equation using above boundary condition,

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\cosh\, m(L - x)}{\cosh\, mL},$$

Where $m = \sqrt{\dfrac{h\,P}{k\,A_c}}$

Heat lost by fin is equal to the heat entering the fin. The heat entering the fin is equal to

$$Q_{fin} = -k\,A_c\left.\frac{dT}{dx}\right|_{x=o}$$

$$Q_{fin} = \sqrt{hPKA_c}\left(T_b - T_\infty\right)\tanh\, mL.$$

Note: The heat transfer relation for very long fin and the fin with negligible heat loss at the tip differ by the factor tan h (mL), which approaches 1 as L becomes very large.

(c) Actual fin with Convection at tip

The fin tips are exposed to the surroundings, and thus the proper boundary condition for the fin tip is convection. The fin equation can still be solved in this case using the convection at the fin tip as the second boundary condition

Boundary Condition at tip is

$$-k\,A_c\left.\frac{dT}{dx}\right|_{x=L} = hA\left(T - T_\infty\right)$$

Boundary condition at base

$x = 0,\ \theta = \theta_b$

Solving the general equation using above boundary conditions, the temperature distribution in fin is

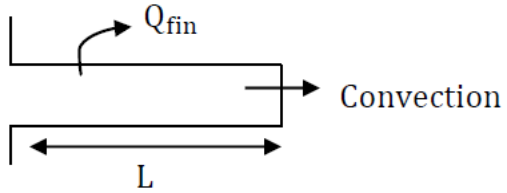
$$\frac{T - T_\infty}{T_b - T_\infty} = \frac{\cosh m(l - x) + \dfrac{h}{mk}\sinh m(l - x)}{\cosh ml + h/mk\,\sinh ml}$$

Heat lost by fin is given by

$$Q_{fin} = \sqrt{h\,P\,KA_c}\left(T_b - T_\infty\right)\frac{\tanh\, ml + h/mk}{1 + \dfrac{h}{mk}\tanh(ml)}$$

Alternate method

The above analysis become more involved, and it results in rather lengthy expressions for the temperature distribution and the heat transfer. A practical way of accounting for the heat loss from the fin tip is to replace the fin length 'L' in relation for the insulated tip case by a corrected length 'L_C'.



The additional area at tip is A_c.the total area or corrected are through which heat loss takes place is given by

$$A_{corrected} = A_{fin} + A_{tip}$$

$$A_{fin} = LP, A_{tip} = A_c, A_{corrected} = L_cP$$

Or

$$L_c.P = LP + A_c$$

$$L_c = L + A_c/p.$$

The length of Insulated Tip to be increased to account for A_c is $\left(L_c = A_c/p\right)$

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Corrected length of rectangular fin is

$$L_{c,\text{rectangular fin}} = L + \frac{t}{2}$$

Corrected length of cylindrical fin is

$$L_{c,\text{cylindrical fin}} = L + \frac{D}{4}$$

2.4.3 FIN PERFORMANCE

(a) Fin Efficiency (η_{fin})

Fin efficiency is defined as ratio of heat lost by actual fin to maximum heat lost by fin. **The maximum rate at which a fin could dissipate energy is the rate that would exist if the entire fin surface were at the base temperature.** However, the actual fin has finite thermal conductivity and a temperature gradient exist along the fin. Ideal Fin is assumed to have infinite thermal conductivity ($k \to \infty$),and temperature of the fin will be uniform at the base value of T_b.

$$\eta_{fin} = \frac{Q_{finactual}}{Q_{fin}(Ideal/max)}$$

$$\eta_{fin} = \frac{Q_{fin}actual}{h\,A_{fin}(T_b - T_\infty)}$$

$$Q_{fin\,act} = \eta_{fin}\; h\,A_{fin}(T_b - T_\infty)$$

Assuming the convection coefficient of the finned surface to be equivalent to that of the without fin base

1. Efficiency of **infinite long fin** is given by

$$\eta_f = \frac{\sqrt{h\,P\,KA_c}\,(T_b - T_\infty)}{h\,A_{fin}(T_b - T_\infty)} = \frac{1}{L}\sqrt{\frac{KA_c}{h\,P}} = \frac{1}{mL}$$

$$\eta_f = \frac{1}{mL} \;(\text{ for } \infty \text{ long fin})$$

2. Efficiency of **Insulated Tip**

$$\eta_f = \frac{\sqrt{h\,P\,K\,A_c}\,(T_b - T_\infty)\tanh mL}{h\,A_{fin}(T_b - T_\infty)} = \frac{1}{mL}(\tanh mL)$$

$$\eta_f = \frac{1}{mL}(\tanh mL) \quad \text{(for insulated tip fin)}$$

3. Efficiency of **actual fin with tip convection**

$$\eta_f = \frac{\sqrt{h\,P\,K\,A_c}\,(T_b - T_\infty)\tanh mL_c}{h\,A_{fin}(T_b - T_\infty)}$$

$$= \frac{1}{mL_c}(\tanh mL_c)$$

$$\eta_f = \frac{1}{mL}(\tanh mL) \quad \text{(for insulated tip fin)}$$

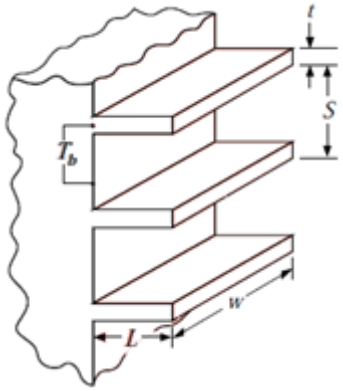
(b) Overall surface efficiency

The fin efficiency (η_f) characterizes only one fin. To find the efficiency of an array of fins as shown in figure, overall efficiency (η_O) is defined.

$$\eta_O = \frac{Q_{fin\,total}}{Q_{fin}(ideal/max)total}$$

$Q_{fin\,total}$ is the total heat lost by fin and the exposed portion of base also known as **prime surface**.

$$Q_{fin\,total} = \eta_f NA_f h(T_b - T_\infty) + hA_b(T_b - T_\infty)$$



Where the convection coefficient 'h' is assumed to be equivalent for the finned and prime surfaces and η_f is the efficiency of a single fin. A_f is the area of single fin, N is the number of fins and A_b is the area of prime surface.

A_t is the total area including area of fins and area of prime surface given by

$$A_t = NA_f + A_b$$

Overall efficiency is given by

$$\eta_O = \frac{\eta_f NA_f h(T_b - T_\infty) + hA_b(T_b - T_\infty)}{hA_t(T_b - T_\infty)}$$

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(c) Fin Effectiveness (ε_{fin})

The performance of the fin is judged on the basis of the enhancement in heat transfer relative to no fin condition. The use of fin cannot be justified till enhancement in heat loss justifies the added cost and complexities associated with the fins. To evaluate enhancement in heat transfer, **fin effectiveness**(ε_{fin}) is determined. It is defined as **the ratio of the fin heat lost with fin to the heat lost without the fin**.
Effectivness is given by

$$\varepsilon_{fin} = \frac{Q_{fin}}{h\, A_b (T_b - T_\infty)}$$

Where Q_{fin} is the actual heat lost by fin and A_b is the area of base without fin which is equal to the area of cross section of fin.

I. $\varepsilon_{fin} = 1$
Indicates that addition of fin will not enhance the heat flow.

II. $\varepsilon_{fin} < 1$
Indicates that fin actually acts as insulation, slowing down the heat transfer rate. This occur when thermal conductivity is very low.

III. $\varepsilon_{fin} > 1$
Enhancing the heat transfer from the surface.

(d) Relation between fin efficiency (η_{fin}) and fin effectiveness (ε_{fin})

$$\varepsilon_{fin} = \frac{Q_{fin}}{h\, A_b (T_b - T_\infty)}$$

From the definition of fin efficiency, heat lost by single fin is given by

$$Q_{fin} = \eta_{fin} . h\, A_{fin} (T_b - T_\infty) .$$

$$\varepsilon_{fin} = \eta_{fin} \frac{A_{fin}}{A_b}$$

Effectiveness of Long Fin
Assuming the convection coefficient of the finned surface to be equivalent to that of the without fin base

$$\varepsilon_{long.fin} = \frac{\sqrt{h\, P\, K\, A_c}\,(T_b - T_\infty)}{h\, A_b (T_b - T_\infty)}$$

$$\varepsilon_{long.fin} = \sqrt{\frac{k\, P}{h\, A_c}}$$

NOTE:-
Several important trends may be inferred from this result.

1. Fin effectiveness is enhanced by the choice of a material of high thermal conductivity.
2. Fin effectiveness is also enhanced by increasing the ratio of the perimeter to the cross-sectional area. For this reason, the use of thin, but closely spaced fins is preferred, with the provision that the fin gap not be reduced to a value for which flow between the fins is severely impeded, thereby reducing the convection coefficient.
3. Fin with triangular & parabolic profiles contain less material and are more efficient than the one with rectangular profile.
   Fins are to be used on a surface separating a gas and a liquid; they are generally placed on the gas side, which is the side of lower convection coefficient.

2.5 TRANSIENT (UNSTEADY STATE) HEAT CONDUCTION

In most of the practical problems, the temperature of system changes with time. The surface temperature of a system is altered; the temperature at each point in the system will also begin to change. The changes will continue to occur until a steady-state temperature distribution is reached. Consider hot metal billet that is removed from a furnace and exposed to a cool airstream. Energy is transferred by convection and radiation from its surface to the surroundings. Energy transfer by conduction also occurs from the interior of the metal to the surface, and the temperature at each point in the billet

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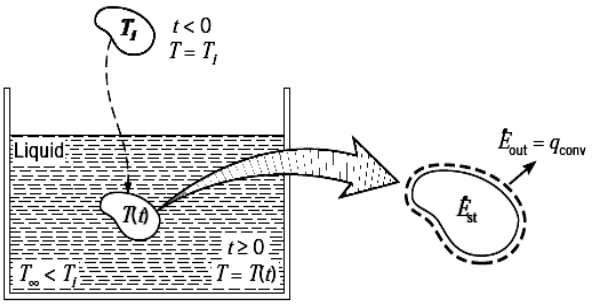
decreases until a steady-state condition is reached.

2.5.1 LUMPED PARAMETER ANALYSIS

In lumped analysis it is assumed that thermal conductivity of body is very high($k \to \infty$), thermal Resistance is so small that heat flow of or from the solid is controlled primarily by the Convective resistance.



Consider a Solid body is heated to a temperature T_i and is immersed in liquid to cool.

During cooling process temperature reduces to the ambient temperature T_∞.

According to Energy balance, decrease in stored energy is equal to the heat convicted to the ambient.

$$\dot{E}_{st} = Q_{conv}$$

$$\dot{E}_{st} = -\frac{d}{dt}(mC\,T) = hA(T - T_\infty)$$

Where, 'T' is the instantaneous temperature of solid body. Negative sign indicates the reduction in stored energy.

$$-\rho VC\frac{dT}{dt} = hA(T - T_\infty)$$

By integrating the above equation

$$-\int_{t_i}^{t}\frac{dt}{(T - T)} = \int_{o}^{\tau}\left(\frac{h\,A}{\rho VC}\right)d\tau$$

Or

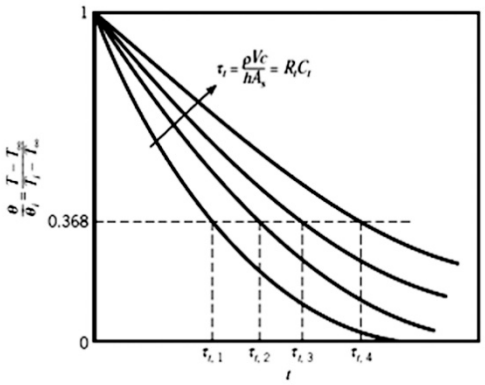
$$\ln\left(\frac{T - T_\infty}{T_i - T_\infty}\right) = \frac{-h\,A}{\rho VC}\,\tau$$

$$\left(\frac{T - T_\infty}{T_i - T_\infty}\right) = \exp\left[\left(\frac{-h\,A}{\rho VC}\right)\tau\right]$$

$\frac{\rho VC}{hA}$ Has dimensions of Time and is called thermal time cost.

$$\frac{\rho VC}{hA} = \frac{1}{hA}(\rho VC) = R_t C_t$$

Where, R_t is the resistance to convection heat transfer and C_t is the lumped thermal capacitance of the solid. Any increase in R_t or C_t will cause a solid to respond more slowly to changes in its thermal environment. This behavior is analogous to the voltage decay that occurs when a capacitor is discharged through a resistor in an electrical RC circuit.



The term $\frac{h\,A}{\rho VC}\,\tau$ can be expressed as

$$\left(\frac{hV}{KA}\right)\left(\frac{A^2}{V^2}\frac{k\,\tau}{\rho C}\right) = \frac{h\,L}{k}\left(\frac{\alpha\tau}{l^2}\right)$$

$\frac{V}{A} = L$ is characteristic length

$\&\frac{k}{\rho C} =$ is thermal diffusivity

$\left(\frac{hL}{k}\right)$ is biot number, B_i

$\left(\frac{\alpha\tau}{l^2}\right)$ is Fourier Number, F_o

Significance of Biot Number (B_i) & Fourier number (F_o)

B_i No. is a non-dimensional number.B_i no. can be expressed as

$$B_i = \left(\frac{h\,L}{k}\right) = \frac{L}{kA} / \frac{1}{hA}$$

It gives indication of internal conduction to the external convection resistance. A small

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value of B_i No. indicates the system has a small conduction resistance. Small conduction resistance will result in less temperature gradient in the body.
For lumped analysis $B_i \leq 0.1$

Fourier number (F_o)
It signifies the degree of penetration of heating or cooling effect through solid. For Small α/l^2, large time 'τ' will be required to obtain significant changes of temperature.

Response Time of a Temperature Measuring System

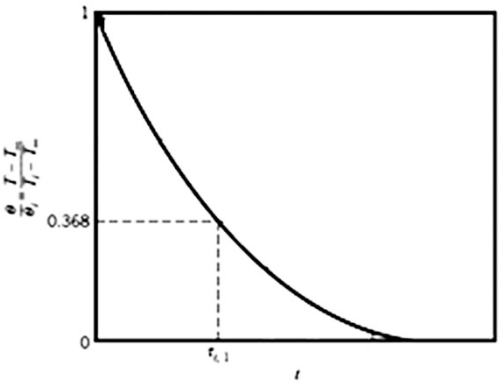
The lumped parameter analysis finds its application in thermocouple and thermometer. It is expected for measuring instrument attains source temperature as soon as possible.

For quick response $\left(\frac{h\,A}{\rho VC}\right)$ should be as large as possible.

$$\text{for } \tau = \tau^*, \text{where, } \tau^* = \frac{\rho VC}{hA} \text{ is time const.}$$

$$\frac{T - T_\infty}{T_i - T_\infty} = 0.368$$

In practical application temperature of thermocouple is to be recorded after $4\tau^*$.



EXAMPLES

Example: 1
A plane slab of thickness δ = 60cm is made of a material of thermal conductivity k=17.5 W/m ^oC . The left side of the slab absorbs a net amount of radiant energy from a radiant source at the rate of $q = 530$ watt / m^2. If the right hand face of the slab is at a constant temperature $t_2 = 38^oC$, set up an expression for temperature distribution within the slab as a function of relevant space co-ordinates. There from work out the temperature at the mid-plane of the slab and the maximum temperature within the slab. It may be presumed that the temperature distribution is steady and there is no heat generation.

Solution:
Under stipulations of steady state and no heat generation, the energy absorbed from the radiant source equals the rate at which it is conducted through the slab.

Heat flux $q = \frac{k(t_1 - t_2)}{\delta}$

$$530 = \frac{17.5(t_1 - 38)}{0.6}$$

$\therefore$ Temperature at the left side of slab,

$$t_1 = \frac{530 \times 0.6}{17.5} + 38 = 56.17^oC$$

This also represents the maximum temperature within the slab.
From the expression for steady state temperature distribution,

$$t = t_1 + \frac{(t_2 - t_1)}{\delta} x$$

At the mid-point x=30cm

$$\therefore t = 56.17 + \frac{(38 - 56.17)}{0.6} \times 0.3 = 47.08^oC$$

Example:2
Establish a relation for the time taken to form a layer of ice on the surface of a pond. How much time it will take for a layer of ice of thickness 20cm to increase by 1mm on

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the surface of a pond when the temperature of surroundings is $-20^{o}C$?
Thermal conductivity of ice=7.5kJ/m-hr-^{o}c
Latent heat of ice =335kJ/kg
Density of ice at $0^{o}C = 1000 kg/m^3$

Solution:

An ice layer of thickness x has been formed on a pond where the temperature of air over the ice is $-t^{o}C$, land that of water below the ice is $0^{o}C$.

Consider that the thickness of ice layer increases by dx in time dt. If A is the surface area of the pond, ρ its mass density and LH the latent heat of fusion of ice, then

Mass of ice formed = Adx ρ

Heat lost by water = Adx ρ LH ...(i)

This heat has to be conducted across a layer of ice of thickness dx upwards to the surroundings at $-t^{o}C$.

$$\text{Heat conducted} = kA\frac{dt}{x}d\tau$$

$$= kA\frac{\left[0-\left(-t\right)\right]}{x}d\tau = kA\frac{t}{x}d\tau \quad ...(ii)$$

Equating expressions (i) and (ii),

$$kA\frac{t}{x}d\tau = Adx\rho LH$$

$$\frac{dx}{d\tau} = \frac{kt}{\rho LHx}$$

Here dx/ dτ represents the rate of growth of thickness of ice

$$d\tau = \frac{\rho LH}{kt}x\,dx$$

Total time taken by the layer of ice to increase in thickness by x

$$\int d\tau = \frac{\rho LH}{kt}\int x\,dx$$

$$\tau = \frac{\rho LH}{kt}\frac{x^2}{2} + \text{constant}$$

When τ=0, x=0. That gives constant as zero

$$\therefore \tau = \frac{\rho LH}{2kt}x^2 \quad ...(iii)$$

If the thickness of ice layer is to increase from x_1 to x_2

$$\tau = \frac{\rho LH}{2kt}\left(x_2^2 - x_1^2\right) \quad ...(iv)$$

Substituting the given data in the above expression,

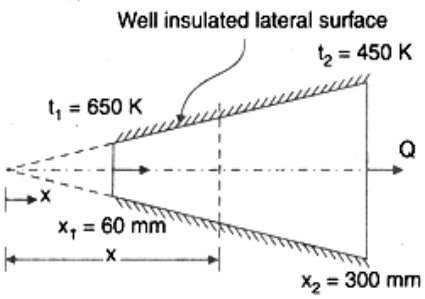
$$\tau = \frac{1000\times 335}{2\times 7.5\times 20}\left[\left(0.201\right)^2 - \left(0.20\right)^2\right]$$

=0.446 hours = 1605.6 seconds

Example:3

Derive an expression for temperature distribution and conduction heat flow in a circular conical rod with diameter at any section given by D=ax where x is the distance measured from the apex of the cone and 'a' is a certain numerical constant. It may be presumed that there is no internal heat generation, steady state conditions prevail and the lateral surface is well insulated. Further proceed to obtain the numerical value for heat flow rate if the smaller end located at x_1 =60mm has a temperature 650K and the larger end at x_2 =300mm has a temperature of 450K. The parameter 'a' equals 0.20 and the material of the conical rod has an average thermal conductivity k=3.45 W/m-^{o}c.

Solution:



The well insulated lateral surface implies that the situation corresponds to one-dimensional conduction in the x-direction for which

$$Q = -kA\frac{dT}{dx}$$

Substituting for $A = \left(\pi/4\right)D^2 = \left(\pi/4\right)a^2x^2$ and separating the variables

$$-k\,dT = \frac{4Q}{\pi a^2}\frac{dx}{x^2}$$

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Since k and Q are constant, integration from x_1 to x within the conical rod yields

$$-k\int_{T_2}^{T} dT = \frac{4Q}{\pi a^2}\int_{x_1}^{x}\frac{dx}{x^2}$$

$$\text{Or } -k\left(T-T_1\right) = \frac{4Q}{\pi a^2}\left(\frac{1}{x_1}-\frac{1}{x}\right)$$

And solving for T, one obtains

$$T = T_1 - \frac{4Q}{\pi a^2 k}\left(\frac{1}{x_1}-\frac{1}{x}\right) \qquad ...(i)$$

Evaluation of the above expression at $x = x_2$ where $T = T_2$ gives,

$$T_2 = T_1 - \frac{4Q}{\pi a^2 k}\left(\frac{1}{x_1}-\frac{1}{x_2}\right) \qquad ...(ii)$$

The heat flow rate then works out as

$$Q = \frac{\pi a^2 k\left(T_1-T_2\right)}{4\left(1/x_1 - 1/x_2\right)} \qquad ...(iii)$$

When substitution for Q is made in expression (i), we get the temperature distribution as

$$T = T_1 + \left(T_1-T_2\right)\left[\frac{1/x - 1/x_1}{1/x_1 - 1/x_2}\right] \qquad ...(iv)$$

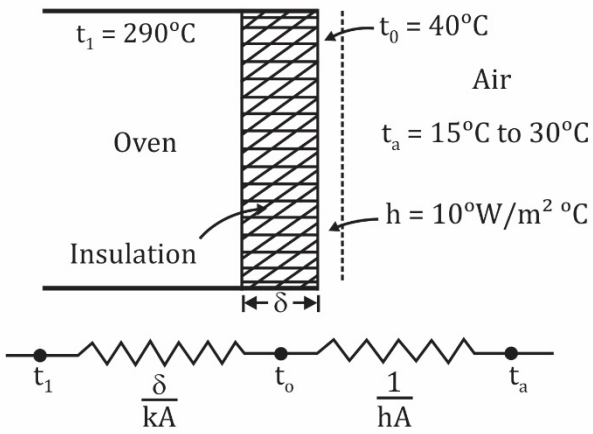
Substituting the given data in expression (iii) for the flow rate,

$$Q = \frac{\pi\times\left(0.20\right)^2\times 3.45\left(650-450\right)}{4\left(1/0.06 - 1/0.3\right)} = 1.62\text{W}$$

Example: 4

A kitchen oven has its maximum operating temperature set at 290°C, whereas the temperature in the kitchen may vary from 15°C to 30°C due to seasonal variations. Workout the necessary thickness of fiber glass (K=0.035W/m-^{0}C) insulation to ensure that the outside surface temperature of oven does not exceed 40°C. The average heat transfer coefficient between the outside oven surface and the kitchen air is $10\text{W}/\text{m}^2-^{0}\text{C}$. Neglect thermal resistance of metal wall and presume that steady conditions prevail.

Solution :



The thermal resistance to heat flow is essentially provided by the fiber glass insulation and the convective heat transfer coefficient between the outside oven surface and the kitchen air. Therefore for unit area of the wall,

$$\frac{Q}{A} = \frac{t_i - t_a}{\delta/k + 1/h}$$

Further for steady state conditions, heat flowing through each section is same.

$$\therefore \quad \frac{t_i - t_a}{\delta/k + 1/h} = \frac{t_o - t_a}{1/h}$$

$$\text{Or} \qquad \left(t_i - t_a\right)\frac{1}{h} = \left(t_o - t_a\right)\frac{\delta}{k} + \left(t_o - t_a\right)\frac{1}{h}$$

$$\text{Or} \qquad \delta = \frac{t_i - t_o}{t_o - t_a}\times\frac{k}{h}$$

The thickness of fiber glass insulation will Larger for $t_a = 30^{\circ}\text{C}$.

$$\therefore \delta = \frac{290-40}{40-30}\times\frac{0.035}{10} = 0.0875\text{m Or } 8.75\text{cm}$$

Example: 5

The hot combustion gases at 150°C flow through a hollow cylindrical pipe of 10 cm inner diameter and 12 cm outer diameter. The pipe is located in a space at 30°C and the thermal conductivity of the pipe material is 200 W/mK. Neglecting surface heat transfer coefficients, calculate the heat loss through the pipe per unit length and the temperature at a point halfway between the

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inner and the outer surface. What should be the surface area normal to the direction of heat flow so that the heat transfer through the pipe can be determined by considering material of the pipe as a plane wall of the same thickness?

Solution:

In terms of geometrical parameters, thermal resistance of a cylindrical pipe is

$$R_t = \frac{1}{2\pi kl}\log_e\left(r_2/r_1\right)$$

$$= \frac{1}{2\pi\times 200\times 1}\log_e\left(12/10\right)$$

$$= 1.4516\times 10^{-4}\ ^\circ C/W$$

$$\text{Heat loss } Q = \frac{\Delta T}{R_t} = \frac{150-30}{1.4516\times 10^{-4}} = 826674 W$$

(ii) Radius at halfway through the pipe wall,

$$r = \frac{r_1 + r_2}{2} = \frac{10+12}{2} = 11cm$$

Thermal resistance of cylindrical pipe up to its mid-plane

$$= \frac{1}{2\pi kl}\log_e\left(r/r_1\right)$$

$$= \frac{1}{2\pi\times 200\times 1}\log_e\left(11/10\right)$$

$$= 7.5884\times 10^{-5}\ ^\circ C/W$$

Since heat flow through each section is same;

$$826674 = \frac{T_1 - T}{7.5884\times 10^{-5}}$$

∴ Temperature at the mid plane,

$$T = 150 - 826674\times 7.5884\times 10^{-5} = 87.27^\circ C$$

Alternately from the expression for temperature distribution

$$\frac{T-T_1}{T_2-T_1} = \frac{\log_e(r/r_1)}{\log_e(r_2/r_1)}$$

$$T = T_1 - (T_1 - T_2)\frac{\log_e(r/r_1)}{\log_e(r_2/r_1)}$$

$$= 150 - (150-30)\frac{\log_e(11/10)}{\log_e(12/10)} = 87.18^\circ C$$

(iii) The equivalent logarithmic mean area is

$$A_m = \frac{A_2 - A_1}{\log_e(A_2/A_1)}$$

$$= \frac{2\pi(r_2-r_1)l}{\log_e(r_2/r_1)} = \frac{2\pi(0.12-0.10)\times 1}{\log_e(12/10)} = 0.689 m^2$$

Check: $Q = \frac{kA_m(t_1-t_2)}{r_2-r_1}$

$$= \frac{200\times 0.689\times(150-30)}{0.12-0.10} = 826800 W$$

This is approximately same as calculated above.

Example: 6

A steam pipe 10cm outside diameter is covered with two layers of insulation, each having a thickness of 2.5 cm. The average thermal conductivity of one material is 3 times that of other and the surface temperatures of the insulated steam pipe are fixed. Examine the position of better insulating layer relative to the steam pipe if heat dissipation from steam is to be Minimum. What % saving in heat dissipation results from that arrangement?

Solution:

The geometrical dimensions of the lagged steam pipe are:

$$r_1 = 5cm;\ r_2 = 7.5cm;\ r_3 = 10cm$$

Let k and 3k be the thermal conductivities of the two insulating materials. Obviously the material with low thermal conductivity is a better insulator.

(i) Better insulator (material with low thermal conductivity) is placed inside, i.e., next to the steam pipe.

Thermal resistance

$$= \frac{1}{2\pi k_1 l}\log_e\left(\frac{r_2}{r_1}\right) + \frac{1}{2\pi k_2 l}\log_e\left(\frac{r_3}{r_2}\right)$$

$$= \frac{1}{2\pi kl}\log_e\left(\frac{r_2}{r_1}\right) + \frac{1}{2\pi(3k)l}\log_e\left(\frac{r_3}{r_2}\right)$$

$$= \frac{1}{2\pi kl}\left[\log_e\left(\frac{7.5}{5.0}\right) + \frac{1}{3}\log_e\left(\frac{10}{7.5}\right)\right] = \frac{0.2506}{\pi kl}$$

Heat dissipation

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$$Q_1 = \frac{\Delta T}{R_t} = \frac{\Delta T \times \pi kl}{0.2506} = 3.99\ \pi kl\Delta T$$

(ii) Better insulator is outside

Thermal resistance

$$= \frac{1}{2\pi k_2 l}\log_e\left(\frac{r_2}{r_1}\right) + \frac{1}{2\pi k_1 l}\log_e\left(\frac{r_3}{r_2}\right)$$

$$= \frac{1}{2\pi(3k)l}\log_e\left(\frac{r_2}{r_1}\right) + \frac{1}{2\pi kl}\log_e\left(\frac{r_3}{r_2}\right)$$

$$= \frac{1}{2\pi kl}\left[\frac{1}{3}\log_e\left(\frac{r_2}{r_1}\right) + \log_e\left(\frac{r_3}{r_2}\right)\right]$$

$$= \frac{1}{2\pi kl}\left[\frac{1}{3}\log_e\left(\frac{7.5}{5.0}\right) + \log_e\left(\frac{10}{7.5}\right)\right] = \frac{0.2114}{\pi kl}$$

Heat dissipation

$$Q_2 = \frac{\Delta T}{R_t} = \frac{\Delta T}{0.2114} \times \pi kl = 4.7303\ \pi kl\Delta T$$

$$\therefore \frac{Q_1}{Q_2} = \frac{3.99}{4.7303} = 0.843$$

Obviously the heat dissipation is small when the material with low thermal conductivity is placed next to the steam pipe

Saving in heat dissipation

$$= \left(\frac{4.7303 - 3.99}{3.99}\right) \times 100 = 18.55\%$$

Example: 7

The following data pertains to a hollow Cylinder and a hollow sphere made of the same material and having the same temperature drop over the wall thickness. Inside radius=0.1m and outside surface area= $1m^2$

If the outside radius for both the geometrics is same, calculate the ratio of heat flow in the cylinder to that in the sphere.

Solution:

Let suffix 1 and 2 refer to inside and outside surface respectively.

For **sphere**: $A_2 = 4\pi r_2^2;\ 1 = 4\pi r_2^2$

Or $r_2 = \sqrt{\frac{1}{4\pi}} = 0.282m$

For **cylinder**: $A_2 = 2\pi r_2 l$

The outside radius of the cylinder is same as that of sphere, i.e., r_2 =0.282m

$$\therefore \quad l = \frac{A_2}{2\pi r_2} = \frac{1}{2\pi \times 0.282} = 0.565m$$

Now, $$\frac{Q_{cyl}}{Q_{sph}} = \frac{2\pi kl\dfrac{(T_1 - T_2)}{\log_e(r_2/r_1)}}{\dfrac{4\pi k\, r_1 r_2 (T_1 - T_2)}{(r_2 - r_1)}}$$

$$\frac{Q_{cyl}}{Q_{sph}} = \frac{l(r_2 - r_1)}{2r_1 r_2 \log_e(r_2/r_1)}$$

$$= \frac{0.565}{2 \times 0.1 \times 0.282 \log_e(0.282/0.1)} = 1.756$$

Example: 8

An infinite slab of 50mm thickness and $0.1m^2$ Cross-sectional area has its sides maintained at temperatures of 300^oC and 30^oC respectively. Measurements indicate that when 1kW of energy as heat flows through it, the temperature at its centre plane is 125^oC. set up an expression for thermal conductivity of the slab material if it varies linearly with temperature.

Solution:

Consider a slab of thickness δ and temperatures at the boundary surfaces T_1 and T_2.

From Fourier's law of heat conduction,

$$Q = -kA\frac{dT}{dx}$$

Assuming linear relationship between k and T as k = mT + c, we can write

$$Q = -(mT + c)A\frac{dT}{dx}$$

Separating the variables and integrating between the prescribed limits, we have

$$\frac{Q}{A}\int_0^{\delta} dx = -\int_{T_1}^{T_2}(mT + c)dT$$

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$$\text{Or } \frac{Q}{A}\delta = \frac{m}{2}\left(T_1^2 - T_2^2\right) + c\left(T_1 - T_2\right)$$

$$\text{Or} \qquad Q = \frac{A}{\delta}\left[\frac{m}{2}\left(T_1^2 - T_2^2\right) + c\left(T_1 - T_2\right)\right]$$

For left half part of slab:

$T_1 = 300^oC,\ T_2 = 125^oC$ and δ=25mm

Then:

$$1000 = \frac{0.1}{0.025}\left[\frac{m}{2}\left(300^2 - 125^2\right) + c\left(300 - 125\right)\right]$$

Or

$$250 = 37187.5m + 175c \qquad \ldots(i)$$

For right half part of slab:

$T_1 = 125^oC,\ T_2 = 30^oC$ and δ=25mm

Then:

$$1000 = \frac{0.1}{0.025}\left[\frac{m}{2}\left(125^2 - 30^2\right) + c\left(125 - 30\right)\right]$$

$$\text{Or} \qquad 250 = 15175m + 95c \quad \ldots(ii)$$

Simultaneous solution of identities (i) and (ii) gives

m= -0.0228 and c=6.273

The expression for thermal conductivity may then be written as

K = −0.022T + 6.273

Example: 9

A flat plate fuel element for a nuclear reactor is 6 mm thick and is clad on each face with aluminum 2 mm thick. The rate of heat generation is uniform within the element and has a magnitude of $5.6\times10^8\ W/m^3$. If the coolant temperature is 140^oC, make calculations for the temperature at the free surface of the aluminum, the aluminum/ uranium interface, and at the centre of the fuel element.

Thermal conductivity for uranium and aluminum are $24.5\ W/mK$ and $200\ W/mK$ respectively, and the heat transfer coefficient at the aluminum/coolant interface is $2.85\times10^4\ W/m^2K$

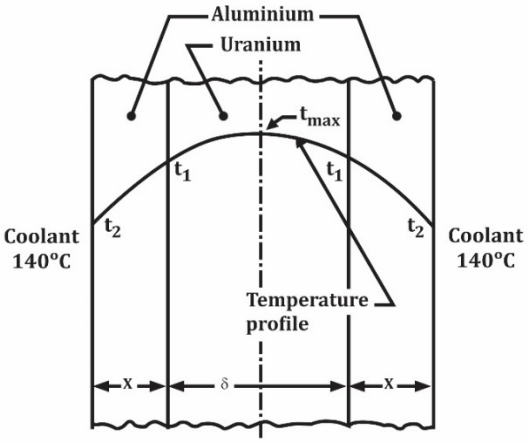
Solution:

A sketch of the arrangement of the fuel element is shown in fig. Let Q_g be the total heat generated within the uranium per unit area of the plate.

$$Q_g = 5.6\times10^8\times(1\times0.006)$$

$$= 33.6\times10^5\ W \text{ per unit area}$$

Then heat flow in each normal direction is $16.8\times10^5\ W/m^2$. under steady state conditions, this heat is first conducted through the aluminum cladding and then convicted to the coolant.



$$16.85\times10^5 = \frac{kA}{x}(T_1 - T_2) = hA(T_2 - 140)$$

Where T_2 is the aluminum/coolant interface temperature, T_1 is the aluminum/ uranium interface temperature and x is the thickness of aluminum cladding.

Taking $A = 1m^2$, we obtain,

$$T_2 = \frac{16.85\times10^5}{2.85\times10^4\times1} + 140 = 199.12^oC$$

$$T_1 = \frac{16.85\times10^5\times0.2\times10^{-2}}{200\times1} + 199.12 = 215.97^oC$$

Since the uranium fuel is subject to uniform internal heat generation, the maximum temperature within this material will be described by the equation:

$$T_{max} = \frac{q_g}{8k}\delta^2 + T$$

Here $\delta = 6mm = 6\times10^{-3}m$ and T is the common temperature on both surfaces of the uranium fuel $= 215.97^oC$

$$\therefore T_{max} = \frac{5.6\times10^8}{8\times24.5}(6\times10^{-3})^2 + 215.97$$

$$= 318.82^oC$$

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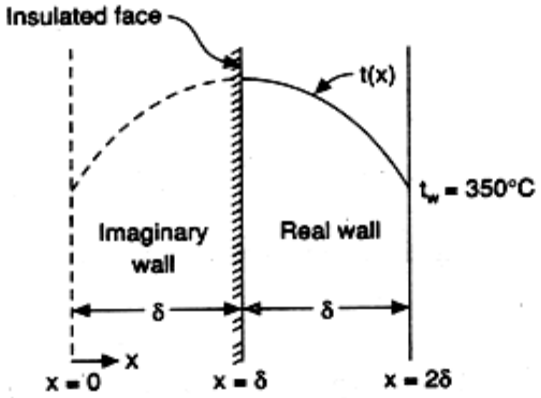
Example: 9

A plane wall 0.5m thick $(k = 20W/m-^{o}c)$ is insulated on one side while the other side is maintained at a constant temperature at $350^{o}C$. The rate of heat generation within the wall is $500W/m^3$. Determine the maximum temperature to which the wall will be subjected and location of the plane where it occurs.

Solution:



With uniform heat generation and both sides of plane wall maintained at the same temperature, the temperature distribution is prescribed by the relation

$$t = t_w + \frac{q_g}{2k}(\delta - x)x$$

The above relation applies equally well as to a plane wall which is perfectly insulated at one face and maintained at a fixed temperature on the other face. A full hypothetical wall with thickness 2δ can be considered with the same bounding conditions:

$t = t_w$ at $x = 0$ and $x = 2\delta$

The location $x = \delta$ corresponds to mid plane of the hypothetical wall or insulated face of the given wall. Thus for an insulated wall of thickness δ, the temperature distribution conforms to the relation

$$t = t_w + \frac{q_g}{2k}(2\delta - x)x$$

The maximum temperature occurs at mid plane (insulated face of the given wall) where $x = \delta$

$$t_{max} = t_w + \frac{q_g}{2k}(2\delta - \delta)\delta$$

$$= t_w + \frac{q_g}{2k}\delta^2$$

$$= 350 + \frac{500}{2\times 20}(0.5)^2 = 353.125^{o}C$$

Example: 10

The internal heat generation in a cylindrical fuel rod of nuclear reactor has been prescribed by the relation

$$q_g = q_o\left[1 - \left(\frac{r}{R}\right)^2\right]$$

Where q_g is the heat generating rate at any radius r and q_o is the heat generating rate at the centre and R is the outside radius of the fuel element. Establish a relation for the temperature drop from the centre line to the outside surface of the rod. Work out this temperature drop for 3 cm outside diameter cylindrical rod of thermal conductivity $25W/m-^{o}c$, if heat is removed from the outside surface at the rate of $2.25\times 10^6 W/m^2$. Also determine the surface heat transfer coefficient if the maximum wall surface temperature is limited to $200^{o}C$ and fluid surrounding the rod is at $75^{o}C$.

Solution:

The heat flow at the outside surface of the rod is

$$Q = \frac{q_o AR}{4} \Rightarrow \frac{Q}{A} = \frac{q_o R}{4}$$

$$\therefore q_o = \left(\frac{Q}{A}\right)\times\frac{4}{R} = 2.25\times 10^6 \times \frac{4}{0.015}$$

$$= 6\times 10^8 W/m^3$$

Temperature drop from the centre line $(T = T_{max})$ to the outside surface $(T = T_w)$ of the rod is

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$$T_{max} - T_w = \frac{3q_o R^2}{16R} = \frac{3\times 6\times 10^8 \times (0.015)^2}{16\times 25}$$

$$= 1102.5^oC$$

(b) Invoking relation for convective heat flux (heat flow per unit area),

$$Q = hA(T_w - T_a)$$

$\therefore$ Surface heat transfer coefficient

$$h = \left(\frac{Q}{A}\right)\times\frac{1}{(T_w - T_a)}$$

$$= (2.25\times 10^6)\times\frac{1}{(200-75)}$$

$$= 18000 W/m^2 - ^o c$$

Example:11

Two long rods of the same diameter, one made of brass (k=85 $W/m-^o c$) & the other of copper (k=375 $W/m-^o c$), have one of their ends inserted into a furnace. At a section 10.5 cm away from the furnace, the temperature of the brass rod is 120^oC. At what distance from the furnace end, the same temperature would be reached to the copper rod. Both rods are exposed to the same environment.

Solution:

Treating the rods as infinitely long fins, the temperature distribution is prescribed by the relation,

$$\frac{\theta}{\theta_0} = \frac{t - t_a}{t_0 - t_a} = e^{-mx}$$

Or

$$t = t_a + (t_0 - t_a)e^{-mx}$$

For brass rod :

$$120 = t_a + (t_0 - t_a)e^{-m_1 l_1} \quad ...(i)$$

For copper rod :

$$120 = t_a + (t_0 - t_a)e^{-m_2 l_2} \quad ...(ii)$$

Where l_1 and l_2 are the lengths up to which same temperature is reached in both the rods.

Obviously from expression (i) and (ii) , we get

$$m_1 l_1 = m_1 l_2$$

Or $$\sqrt{\frac{h_1 p_1}{k_1 A_1}}\, l_1 = \sqrt{\frac{h_2 p_2}{k_2 A_2}}\, l_2$$

Since the rods are exposed to the same environment $(h_1 = h_2)$ and are of the same diameter $(P_1 = P_2$ And $A_1 = A_2)$ we get

$$\frac{l_1}{\sqrt{k_1}} = \frac{l_2}{\sqrt{k_2}}$$

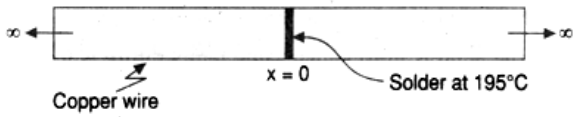
Or $$l_2 = l_1\sqrt{\frac{k_2}{k_1}} = 10.5\sqrt{\frac{375}{85}} = 22.05 cm$$

Example:12

Estimate the energy input required to solder together two very long pieces of bare copper wire 0.1625 cm in diameter with a solder that melts at 195^oC. The wires are positioned vertically in air at 24^oC and the heat transfer coefficient on the wire surface is $17 W/m^2-^o c$. For the wire alloy, take the thermal conductivity $335 W/m-^o c$.

Solution:

The physical situation approximates as two infinite fins with a base temperature of 195^oC in an environment at 24^oC with the given value of surface coefficient.



Cross-sectional area,

$$A_c = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.001625)^2 = 2.073\times 10^{-6} m^2$$

Perimeter, P= $\pi d = \pi\times 0.001625 = 0.0051 m$

$$m = \sqrt{\frac{hP}{kA_c}} = \sqrt{\frac{17\times 0.0051}{335\times 2.073\times 10^{-6}}} = 11.17 m^{-1}$$

Heat dissipation from an infinitely long fin is

$$Q_{fin} = kA_c m(t_o - t_a)$$

$$= 335\times(2.073\times 10^{-6})\times 11.17(195-24)$$

$$= 1.326 W$$

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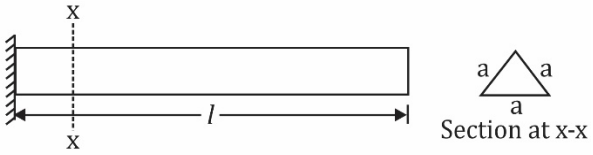
Therefore, the energy input required for two wires is 2.652 W

Example:13

A carbon steel rod (k=55 $W/m-^{o}c$) has been attached to a plane wall which is maintained at a temperature of 350°C. The rod is 8cm long and has the cross-section of an equilateral triangle with each side 5mm. Determine the heat dissipation from the rod if it is exposed to a convection environment at 25°C with unit surface conductance 100 $W/m^2-^{o}c$. Consider end surface loss to be negligible.

Solution:



For a fin of triangular cross-section

P=3a

$$A_c = \frac{1}{2}a\left(\sqrt{3}\frac{a}{2}\right)$$

$$= \frac{\sqrt{3}}{4}a^2$$

$$m = \sqrt{\frac{hP}{k\,A_c}}$$

$$= \left[\frac{100\times(3\times0.005)}{55\times\frac{\sqrt{3}}{4}\times(0.005)^2}\right]^{1/2} = 50.19\ m^{-1}$$

For a fin end loss negligible (tip insulated)

$$Q = k\,A_c\,m\,(t_0 - t_a)\tanh\,ml$$

$$= 55\times\frac{\sqrt{3}}{4}(0.005)^2\times50.19\times(350-40)\tanh(50.19\times0.08)$$

$$= 9.26W$$

Example:13

An electronic semiconductor device generates 0.16kJ/hr of heat. To keep the surface temperature at the upper safe limit of 75°C, it is desired that the generated heat should be dissipated to the surrounding environment which is at 30°C. The task is accomplished by attaching aluminum square fins of 0.5 mm side and 10mm long to the surface. Work out the number of fins if thermal conductivity of fin material is 690 kJ/m-hr-°c and the heat transfer coefficient is 45 $kJ/m^2-hr-^{o}C$. Neglect the heat loss from the tip of the fin.

Solution:

For a fin of rectangular cross-section,

$$P = 2(b+d) = 2(0.5+0.5) = 2mm = 2\times10^{-3}m$$

$$A_c = b\times d = 0.5\times0.5 = 0.25mm^2 = 0.25\times10^{-6}m^2$$

$$m = \sqrt{\frac{hp}{kA_c}} = \sqrt{\frac{45\times2\times10^{-3}}{690\times0.25\times10^{-6}}} = 22.84m^{-1}$$

For a fin with insulated tip,

$$Q_{fin} = kA_c m(t_o - t_a)\tanh ml$$

$$= 690\times\left(0.25\times10^{-6}\right)\times22.84\times(75-30)\tanh(22.84\times0.01)$$

$$= 39.77\times10^{-3}\,kJ/hr\ per\ fin$$

$$\therefore\ \text{Number of fins} = \frac{0.16}{39.77\times10^{-3}} = 4.02$$

Thus, 4 fins are needed to dissipate the required amount of heat.

Example:14

A turbine blade 5cm long, 4.5 cm^2 cross-sectional area and 10cm perimeter is made of stainless steel of thermal conductivity 100 kJ/m-hr-°C. The temperature at the root of blade is 500°C and it is exposed to products of combustion passing through the turbine at 850°C. Determine the temperature at the middle of blade and the rate of heat flow from it. The film coefficient between the blade and the combustion gases is 1100 $kJ/m^2-hr-^{o}c$. The blade may be treated as a fin losing heat at the tip.

Solution:

For a fin losing heat at the tip, the temperature distribution is prescribed by the relation:

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$$\frac{\theta_x}{\theta_o}=\frac{(t_x-t_a)}{(t_o-t_a)}=\frac{\cosh m(l-x)+\frac{h}{km}\sinh m(l-x)}{\cosh ml+\frac{h}{km}\sinh ml}$$

Where,

$$m=\sqrt{\frac{hP}{kA_c}}=\sqrt{\frac{1100\times0.10}{100\times4.5\times10^{-4}}}=49.44m^{-1}$$

$ml = 49.44\times0.05=2.472$

$$\frac{h}{km}=\frac{1100}{100\times49.44}=0.222$$

$\sinh ml = 5.881$; $\cosh ml = 5.965$; $\tanh ml = 0.986$

$$\therefore \quad \frac{t_x-850}{500-850}=\frac{\cosh m(l-x)+0.222\sinh m(l-x)}{5.965+0.222\times5.881}$$

From this expression, temperature at any distance x from the root of the blade can be worked out. At the mid of turbine blade x=0.025m.

$$\cosh m(l-x)=\cosh 49.44(0.05-0.025)$$
$$=1.236$$
$$\sinh m(l-x)=\sinh 49.44(0.05-0.025)$$
$$=1.576$$
$$\therefore \frac{t_x-850}{500-850}=\frac{1.236+0.222\times1.576}{5.965+0.222\times5.881}=0.218$$
$$t_x=850+0.218(500-850)=773.7^{\circ}C$$

(b) The heat flow through the blade is given by

$$Q=kA_cm(t_o-t_a)\frac{\tan ml+\frac{h}{km}}{1+\frac{h}{km}\tanh ml}$$

$$=100\times(4.5\times10^{-4})(49.44)(500-850)\left(\frac{0.986+0.222}{1+0.222\times0.986}\right)$$

=-771.64 kJ

The –ve sign indicates that the heat flows from the combustion gases to the blade.

Example: 15

A 2cm thick steel slab heated to 525°C is held in air stream having a mean temperature of 25°C. Estimate the time interval when the slab temperature would not depart from the mean value of 25°C by more than 0.5°C at any point in the slab. The steel plate has the following thermo-physical properties:

$\rho=7950kg/m^3 : c_p=455J/kg-^oc$

$: k=46W/m-^oc$

h= (heat transfer coefficient on plate surface)

$=36\ W/m^2-^oc$

Solution:

For a flat plate (thickness δ, breadth b, height h), the heat exchange occurs from both the sides; the area exposed for heat transfer is 2bh. The characteristic length then equals

$$l=\frac{\text{volume of plate}}{\text{surface area}}=\frac{\delta bh}{2bh}=\frac{\delta}{2}=\frac{0.02}{2}=0.01m$$

Biot number

$$B_i=\frac{hl}{k}=\frac{36\times0.01}{46}=0.0078<0.1$$

Since the Biot number is less than 0.1, the internal temperature gradients are small. Consideration of the steel slab as a lumped system would be quite accurate; it will introduce an error of no more than 5 percent. The lumped-parameter solution for transient conduction states as,

$$\frac{t-t_a}{t_i-t_a}=\exp\left(-\frac{hA}{\rho Vc}\tau\right)$$

Now:

$$\frac{A}{V}=\frac{2bh}{\delta bh}=\frac{2}{\delta}=\frac{2}{0.02}=100m^{-1}$$

$$\frac{h}{\rho c}\left(\frac{A}{V}\right)=\frac{36}{7950\times455}\times100=9.952\times10^{-4}$$

$$\therefore \frac{0.5}{525-25}=\exp[-9.952\times10^{-4}\tau]$$

Or $\exp[-9.952\times10^{-4}\tau]=\frac{525-25}{0.5}=1000$

Or $9.952\times10^{-4}\tau=\log_e(1000)=6.9077$

$$\therefore \quad \tau=\frac{6.9077}{9.952\times10^{-4}}=6941\ s$$

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GATE QUESTIONS

Q.1 The heat flux (from outside to inside) across an insulating wall with thermal conductivity k = 0.04 W/mK and thickness 0.16 m is 10 W/m^2. The temperature of the inside wall is -5 °C. The outside wall temperature is

(A) 25 °C (B) 30 °C
(C) 35 °C (D) 40 °C

[GATE-2001]

Q.2 A 10 cm diameter steam pipe, carrying steam at 180 °C, is covered with an insulation (conductivity=0.6 W/m °C). It loses heat to the surroundings at 30 °C. Assume a heat transfer coefficient of 8.0 W/m^2 °C for heat transfer from surface to the surroundings. Neglect wall resistance of the pipe and film resistance of steam. If the insulation thickness is 2 cm, the rate of heat loss from this insulated pipe will be

(A) Greater than that of the un-insulated steam pipe
(B) Less than that of the un-insulated steam pipe
(C) Equal to that of the un-insulated steam pipe
(D) Less than the steam pipe with 5 cm insulation

[GATE-2002]

Q.3 A composite wall consists of two plates A and B placed in series normal to the flow of heat. The thermal conductivities are K_A and K_B and the specific heat capacities are $C_{p\text{-}A}$ and $C_{p\text{-}B}$ for plates A and B respectively. Plate B has twice the thickness of plate A. At steady state, the temperature difference across plate A is greater than that across plate B, when

(A) $C_{p\text{-}A} > C_{p\text{-}B}$ (B) $C_{p\text{-}A} < C_{p\text{-}B}$
(C) $K_A < 0.5K_B$ (D) $K_A > 2K_B$

[GATE-2002]

Q.4 The inner wall of a furnace is at a temperature of 700 °C. The composite wall is made of two substances, 10 and 20 cm thick with thermal conductivities of 0.05 and 0.1 W/m °C respectively. The ambient air is at 30 °C and the heat transfer coefficient between the outer surface of wall and air is 20W/m^2 °C. The rate of heat loss from the outer surface in W/m$_2$ is

(A) 165.4 (B) 167.5
(C) 172.5 (D) 175

[GATE-2003]

Q.5 The unit of resistance to heat transfer is

(A) $\frac{J}{m^2K}$ (B) $\frac{J}{m\,K}$

(C) $\frac{W}{m^2\,K}$ (D) $\frac{m^2K}{W}$

[GATE-2003]

Q.6 For a given ambient air temperature with increase in the thickness of insulation of a hot cylindrical pipe, the rate of heat loss from the surface would

(A) Decrease
(B) First decrease and then increase
(C) Increase
(D) First increase and then decrease

[GATE-2003]

Q.7 The left face of a one dimensional slab of thickness 0.2 m is maintained at 80 °C and the right face is exposed to air at 30 °C. The thermal conductivity of the

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slab is 1.2 W/m K and the heat transfer coefficient from the right faced is $10 W/m^2K$. At steady state, the temperature of the right face in °C is

(A) 77.2 (B) 71.2
(C) 63.8 (D) 48.7

[GATE-2004]

Q.8 A circular tube of outer diameter 5 cm and inner diameter 4 cm is used to convey hot fluid. The inner surface of the wall of the tube is at a temperature of 80 °C while the outer surface of the wall of the tube is at 25 °C. What is the rate of heat transport across the tube wall per metre length of the tube at steady state, if the thermal conductivity of the tube wall is 10 W/m-K?

(A) 13823 W/m (B) 15487 W/m
(C) 17279 W/m (D) 27646 W/m

[GATE-2005]

Q.9 A semi-infinite slab occupying the region x = 0 and $x = \infty$ is at an initial temperature T_0. At time t = 0, the surface of the slab at x = 0 is brought into contact with a heat bath at a temperature T_H. The temperature T(x,t) of the slab rises according to the equation

$$\frac{T_H - T(x,t)}{T_H - T_0} = \frac{2}{\sqrt{\pi}} \int_0^{\frac{x}{\sqrt{4\pi t}}} e^{-\eta^2} d\eta$$

where, x is position and t is time. The heat flux at the surface x=0 is proportional to

(A) $t^{-1/2}$ (B) $t^{1/2}$
(C) t (D) $t^{3/2}$

[GATE-2005]

Q.10 A stagnant liquid film of 0.4 mm thickness is held between two parallel plates. The top plate is maintained at 40 °C and the bottom plate is maintained at 30 °C. If the thermal conductivity of the liquid is 0.14 W/(m-K), then the steady state heat flux (in W/m^2) assuming one-dimensional heat transfer, is

(A) 3.5 (B) 350
(C) 3500 (D) 7000

[GATE-2006]

Q.11 One dimensional steady state heat transfer occurs from, a flat vertical wall of length 0.1 m into the adjacent fluid. The heat flux into this fluid is $21 W/m^2$. The wall thermal conductivity is 1.73W/m-K. If the heat transfer coefficient is $30 W/m^2$-K and the Nusselt number based on the wall length is 20, then the magnitude of the temperature gradient at the wall on the fluid side (in K/m) is

(A) 0.7 (B) 12.14
(C) 120 (D) 140

[GATE-2006]

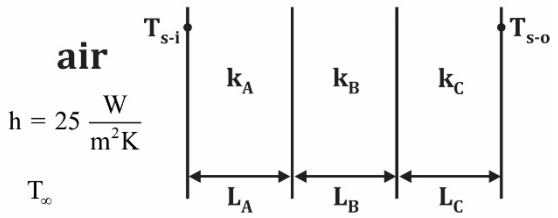
Q.12 The composite wall of an oven consists of three materials A, B and C. Under steady state operating conditions, the outer surface temperature T_{so} is 20 °C. The inner surface temperature T_{si} is 600 °C and the oven air temperature is T_∞ =800 °C.

For the following data:

Thermal conductivities K_A =20 W/m–K and K_C = 50 W/m–K;

Thickness LA = 0.3 m, L_B = 0.15 m and L_C= 0.15 m.

Inner-wall heat transfer coefficient h = $25 W/m^2$–K



The thermal conductivity K_B in W/m–K of the material B, is calculated as

(A) 35 (B) 1.53
(C) 0.66 (D) 0.03

[GATE-2007]

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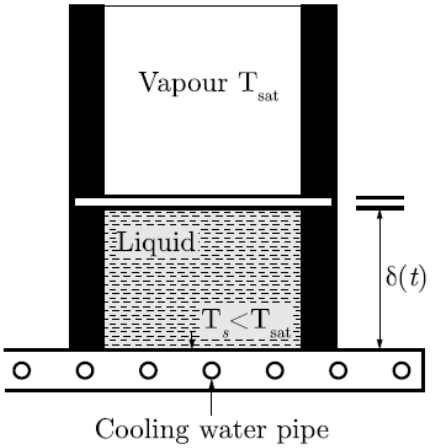
Q.13 Consider a liquid stored in a container exposed to its saturated vapour at constant temperature Tsat. The bottom surface of the container is maintained at a constant temperature Ts<Tsat, while its side walls are insulated. The thermal conductivity is K_1 of the liquid, its latent heat of vaporisation (λ) and density (ρ_1) are known. Assuming a linear temperature distribution in the liquid, the expression for the growth of the liquid layer (δ) as a function of time t is given by



(A) $\delta(t)=\left[\dfrac{4k_1\left(T_{sat}-T_S\right)}{\rho_1\lambda}t\right]^{\frac{1}{2}}$

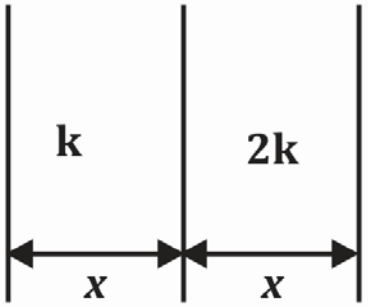
(B) $\delta(t)=\left[\dfrac{k_1\left(T_{sat}-T_S\right)}{2\rho_1\lambda}t\right]^{\frac{1}{2}}$

(C) $\delta(t)=\left[\dfrac{2k_1\left(T_{sat}-T_S\right)}{\rho_1\lambda}t\right]^{\frac{1}{2}}$

(D) $\delta(t)=\left[\dfrac{k_1\left(T_{sat}-T_S\right)}{\rho_1\lambda}t\right]^{\frac{1}{2}}$

[GATE-2007]

Q.14 Two plates of equal thickness (t) and cross-sectional area are joined together to form a composite as shown in the figure. If the thermal conductivities of the plates are k and 2k, then the effective thermal conductivity of the composite is



(A) 3k/2 (B) 4k/3

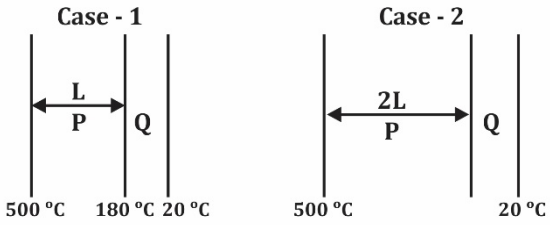
(C) 3k/4 (D) 2k/3

[GATE-2008]

Q.15 For the composite wall shown below (Case 1), the steady state interface temperature is 180cC. If the thickness of layer P is doubled (Case 2), then the rate of heat transfer (Assuming 1-dimensional conduction) is reduced by

(A) 20 % (B) 40 %

(C) 50 % (D) 70 %



[GATE-2009]

So, from (1)

$$\frac{Q_1}{A}=\frac{480}{3R_2} \quad .....(2)$$

In case - 2

$$\frac{Q_2}{A}=\frac{500-20}{2R_1+R_2}=\frac{480}{5R_2} \quad .....(3)$$

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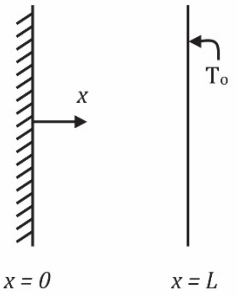
Common Data Question

A slab of thickness L with one side (x = 0) insulated and the other side (x=L) maintained at a constant temperature T_0 is shown below



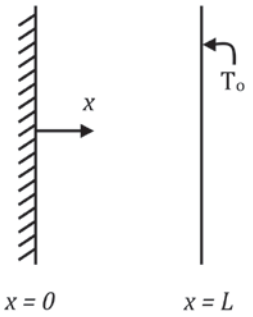
A uniformly distributed internal heat source produces heat in the slab at the rate of S W/m^3. Assume the heat conduction to be steady and 1-dimensional along the x -direction.

Q.16 The maximum temperature in the slab occurs at x equal to

(A) 0 (B) L/4
(C) L/2 (D) L

[GATE-2009]

Sol: (A)



$$\frac{dT}{dx} = -\frac{S}{k}x$$

For maximum Temp., $\frac{dT}{dx} = 0$,

Q.17 The heat flux at x = L is

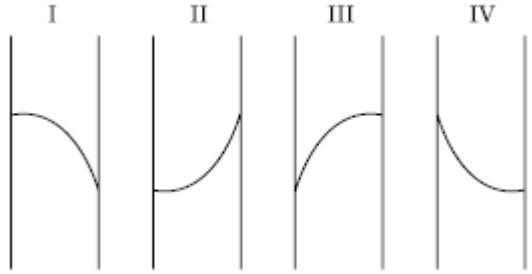
(A) 0 (B) SL/4
(C) SL/2 (D) SL

[GATE-2009]

Q.18 The figure below shows steady state temperature profiles for one dimensional heat transfer within a solid slab for the following cases:

P : Uniform heat generation with left surface perfectly insulated
Q : Uniform heat generation with right surface perfectly insulated
R : Uniform heat consumption with left surface perfectly insulated
S : Uniform heat consumption with right surface perfectly insulated

Match the profiles with appropriate cases.



(A) P-I, Q-III, R-II, S-IV
(B) P-II, Q-III, R-I, S-IV
(C) P-I, Q-IV, R-II, S-III
(D) P-II, Q-IV, R-I, S-III

[GATE-2010]

Q.19 Heat is generated at a steady rate of 100 W due to resistance heating in a long wire (length=5 m, diameter =2 mm). This wire is wrapped with an insulation of thickness 1 mm that has a thermal conductivity of 0.1 W/m-K. The insulated wire is exposed to air at 30 °C. The convective heat transfer between the wire and surrounding air is characterized by a heat transfer coefficient of 10 W/m^2. The temperature (in °C) at the interface between the wire and the insulation is

(A) 211.2 (B) 242.1
(C) 311.2 (D) 484.2

[GATE-2012]

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Q.20 A brick wall of 20 cm thickness has thermal conductivity of $0.7\,Wm^{-1}K^{-1}$. An insulation of thermal conductivity $0.2\,Wm^{-1}K^{-1}$ is to be applied on one side of the wall so that the heat transfer through the wall is reduced by 75%. The same temperature difference is maintained across the wall before and after applying the insulation. The required thickness (in cm) of the insulation is

(A) 2.67 (B) 17.15

(C) 49.63 (D) 124.47

[GATE-2014]

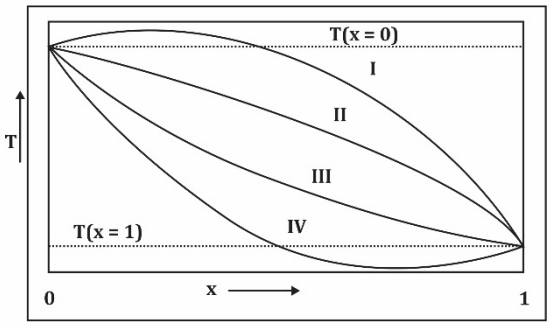
Q.21 The bottom face of a horizontal slab of thickness 6 mm is maintained at $300^{\circ}C$. The top face is exposed to a flowing gas at $30^{\circ}C$. The thermal conductivity of the slab is $5\,Wm^{-2}K^{-1}$ and the convective heat transfer coefficient is $30Wm^{-2}K^{-1}$. At steady state, the temperature (in $^{\circ}C$) of the top face is

(A) 71.07 (B) 171.07

(C) 217.07 (D) 291.07

[GATE-2014]

Q.22 Consider a solid block of unit thickness for which the thermal conductivity decreases with an increase in temperature. The opposite faces of the block are maintained at constant but different temperatures: T(x = 0) > T(x = 1). Heat transfer is by steady state conduction in x-direction only. There is no source or sink of heat inside the block. In the figure below, identify the correct temperature profile in the block.

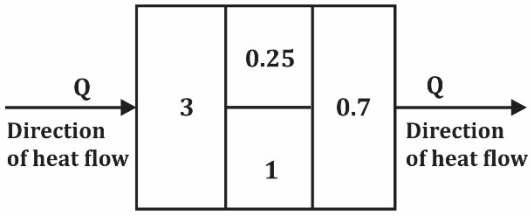


(A) I (B) II

(C) III (D) IV

[GATE-2015]

Q.23 A composite wall is made of four different materials of construction in the fashion shown below. The resistance (in K/W) of each of the section of the wall is indicated in the diagram.



The overall resistance (in K/W, rounded off to the first decimal place) of the composite wall, in the direction of heat flow, is _____

[GATE-2016]

Q.24 In an experimental setup, mineral oil is filled in between the narrow gap of two horizontal smooth plates. The setup has arrangements to maintain the plates at desired uniform temperature. At these temperatures, ONLY the radiative heat flux is negligible. The thermal conductivity of the oil does not vary perceptibly in this temperature range. Consider four experiments at steady state under different experimental conditions, as shown in the figure below.

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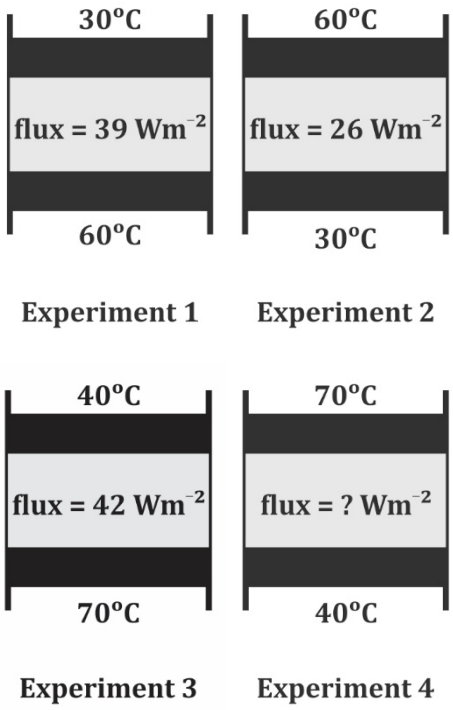
The figure shows plate temperatures and the heat fluxes in the vertical direction.

What is the steady state heat flux (in W/m^2) with the top plate at 70°C and the bottom plate at 40°C?



(A) 26 (B) 39

(C) 42 (D) 63

[GATE-2016]

Q.25 The wall of a pipe of radius 1 m is at a uniform temperature of 200°C, and is covered by insulation of thickness 0.1 m. The ambient air outside the insulated pipe is at 20°C and has heat transfer coefficient of $10\ W\,m^{-2}\,K^{-1}$. The thermal conductivity of the insulation material is $0.05\ W\,m^{-1}\,K^{-1}$. If the heat transfer occurs at steady state, the temperature (in °C) of the outer surface of insulation is _____ (rounded off to second decimal place).

[GATE-2018]

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ANSWER KEYS

1	2	3	4	5	6	7	8	9	10
(C)	(A)	(C)	(A)	(D)	(D)	(D)	(B)	(A)	(C)
11	12	13	14	15	16	17	18	19	20
(12.54)	(B)	(D)	(B)	(B)	(A)	(D)	(A)	(A)	(B)
21	22	23	24	25					
(C)	(C)	(3.9)	(A)	(28.20)					

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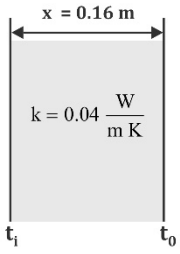
EXPLANATIONS

Q.1 (C)



Given $\frac{Q}{A}=10\frac{W}{m^2}$

$k=0.04\frac{W}{mK}$

$t=0.16\ m$

$t_i=-5^\circ C$

Now,

$\frac{Q}{A}=k\frac{\Delta T}{\Delta x}$

$10=0.04\frac{(t_o-t_i)}{0.16}$

$\frac{(0.16)(10)}{(0.04)}=[t_o+5]$

$t_o=35^\circ C$

Q.2 (A)

Given:

$d=10cm,\ r=5cm$

$k=0.6\frac{W}{m^\circ C}$

$T_\infty=30^\circ C$

$h=8\frac{W}{m^{2\circ}C}$

$t=2\ cm$

$r_{cr}=\frac{k}{h_0}=\frac{0.6}{8}=0.075m=7.5cm$

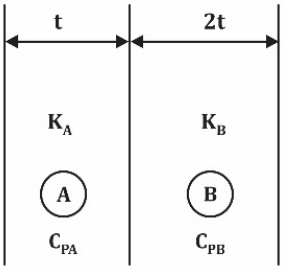
$r_0=7\,cm$ (given)

So, $r_c>r_0$. So, HT is more than uninsulated pipe.

When thickness is 5 cm, $r_0=10cm$

Here, $r_0>r_c$, So heat loss will be greater than r_c

Q.3 (C)



At steady state

$\frac{q_A}{A}=\frac{q_B}{A}$

$-k_A\frac{\Delta T_A}{t}=-k_B\frac{\Delta T_B}{2t}$

$\frac{2k_A}{k_B}=\frac{\Delta T_B}{\Delta T_A}.....(1)$

Now, $\Delta T_A>\Delta T_B\,(Given)$

So, $\frac{\Delta T_B}{\Delta T_A}<1.....(2)$

So,Equation (1) becomes,

$\frac{2k_A}{k_B}<1$

$k_A<\frac{1}{2}k_B$

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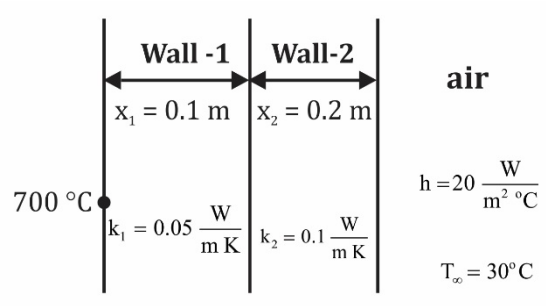
Q.4 (A)



$$\frac{Q}{A} = \frac{\Delta T}{\left[\frac{x_1}{k_1} + \frac{x_2}{k_2} + \frac{1}{h}\right]}$$

$$= \frac{700 - 30}{\left(\frac{0.1}{0.05}\right) + \left(\frac{0.2}{0.1}\right) + \left(\frac{1}{20}\right)}$$

$$\frac{Q}{A} = 165.4 \frac{W}{m^2}$$

Q.5 (D)

$$\frac{q}{A} = \frac{\text{Driving fource}}{\text{Resistance}}$$

$$\frac{q}{A} = \frac{\Delta T}{R}$$

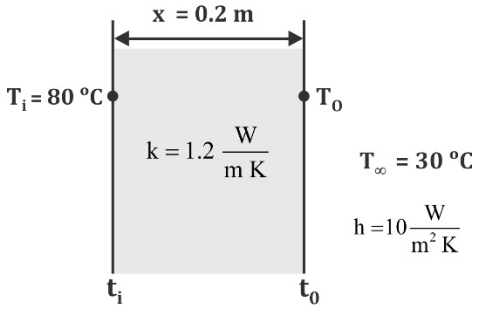
$$R = \frac{\Delta T(A)}{q}$$

$$R = \frac{m^2\,K}{W}$$

Q.6 (D)

First Increase and then decrease

Q.7 (D)



At steady state,

The flux Remains constant

$$\frac{Q_1}{A} = \frac{Q_2}{A}$$

$$\frac{T_i - T_\infty}{\left(\frac{x}{k}\right) + \left(\frac{1}{h}\right)} = \frac{T_i - T_o}{\left(\frac{x}{k}\right)}$$

$$\frac{80 - 30}{\left(\frac{0.2}{1.2}\right) + \left(\frac{1}{10}\right)} = \frac{80 - T_0}{\left(\frac{0.2}{1.2}\right)}$$

$$T_0 = 48.75\ °C$$

Q.8 (B)

$d_O = 5$ cm $\quad r_O = 2.5$ cm

$d_i = 4$ cm $\quad r_i = 2$ cm

$T_i = 80\ °C \quad T_O = 25\,°C$

$$k = 10 \frac{W}{m\,K}$$

$$Q = \frac{\Delta T}{\left(\frac{\ln\left(\frac{r_O}{r_i}\right)}{2\pi lk}\right)} = \frac{80 - 25}{\frac{\ln\left(\frac{2.5}{2}\right)}{2\pi(1)10}} = 15487\left(\frac{W}{m}\right)$$

Q.9 (A)

$$\frac{T_H - T(x,t)}{T_H - T_0} = \frac{2}{\sqrt{\pi}} \int_0^{\frac{x}{\sqrt{4\pi t}}} e^{-\eta^2} d\eta$$

$$\frac{Q}{A} = -k\frac{dt}{dx} \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

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$$\left.\frac{dT}{dT}\right|_{x=0} = \frac{1}{(T_H - T_o)} \frac{2}{\sqrt{\pi}} \frac{d}{dx}\left[\int_0^{\eta} e^{-\eta^2} d\eta\right]_{x=0}$$

$$\frac{dT}{dx} \propto \left(e^{-\eta^2}\right) \left.\frac{d\eta}{dx}\right|_{x=0}$$

$$\frac{dT}{dx} \propto (1) \frac{d}{dx}\left(\frac{x}{\sqrt{4\pi t}}\right)$$

$$\frac{dT}{dx} \propto \frac{1}{\sqrt{4\pi t}}$$

So, $\frac{Q}{A} \propto \frac{1}{\sqrt{4\pi t}}$

So, $\frac{Q}{A} \propto t^{-\frac{1}{2}}$

Q.10 (C)

$T_1 = 40\ ^{o}C$

$T_2 = 30\ ^{o}C$

$k = 0.14\ \frac{W}{m\,K}$

$x = 0.4\ mm = 0.4 \times 10^{-3} m$

$$\frac{Q}{A} = k \frac{dT}{dx}$$

$$= (0.14) \frac{(40-30)}{(0.4 \times 10^{-3})}$$

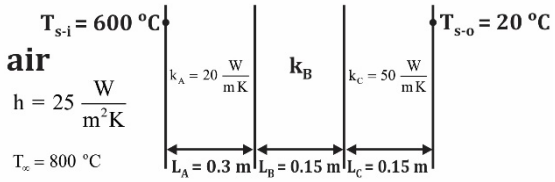
$$= 3500\ \frac{W}{m^2}$$

Q.11 (12.54)

$$q = -k \frac{dT}{dx}$$

$$-\frac{dT}{dx} = \frac{q}{k} = \frac{21}{1.73} = 12.54\ \frac{K}{m}$$

Q.12 (B)



$Q_1 = Q_2$

$$hA\ (T_\infty - T_{si}) = \frac{A(T_{si} - T_{so})}{\frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_c}{k_c}}$$

$$25(800-600) = \frac{(600-20)}{\left(\frac{0.3}{20}\right) + \left(\frac{0.15}{k_B}\right) + \left(\frac{0.15}{50}\right)}$$

$$k_B = 1.53\ \frac{W}{m\,K}$$

Q.13 (D)

Q.14 (B)

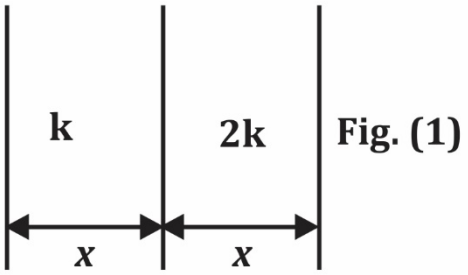


Fig. (1)

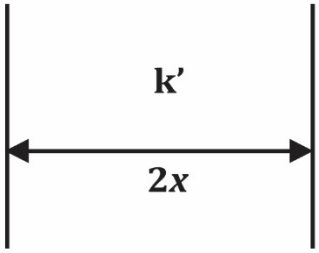


Fig. (2)

for fig (1)

$$R = \frac{x}{kA} + \frac{x}{2kA}$$

$$= \frac{2KAx + KAx}{2K^2A^2}$$

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$$R = \frac{3x}{2kA} \quad .....(1)$$

for fig. (2)

$$R = \frac{2x}{k'A} \quad .....(2)$$

From (1) &(2)

$$\frac{3x}{2kA} = \frac{2x}{k'A}$$

$$\text{So, } k' = \frac{4k}{3}$$

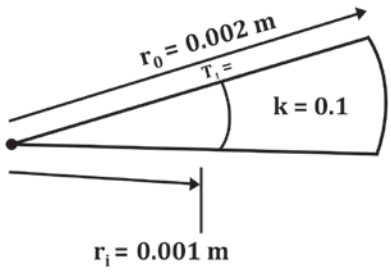
Q.15 (D)

$$q_x\Big|_{x=L} = -k\frac{dT}{dx}\Big|_{x=L} = -k\left(-\frac{S}{k}x\right)\Big|_{x=L}$$

$$q_x\Big|_{x=L} = SL$$

Q.18 (A)

Q.19 (A)



$Q = 100\,W$

$L = 5\,m$

$d_i = 0.002\,m, \qquad r_i = 0.001m$

$d_o = 0.002 + 0.002 = 0.004\ m$

$r_o = 0.002\ m$

$$k = 0.1\,\frac{W}{m\,K}$$

$Tair = 30\,^oC$

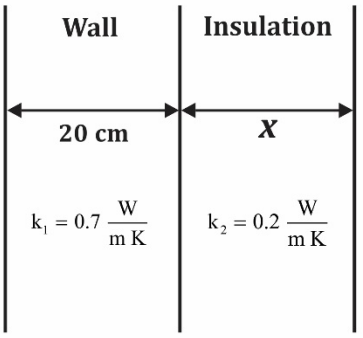
$$h = 10\,\frac{W}{m^2\,K}$$

$$Q = \frac{\Delta T(2\pi L)}{\left[\frac{\ln\left(\frac{r_2}{r_1}\right)}{k_1}\right] + \frac{1}{h_o r_o}}$$

$$\Rightarrow 100 = \frac{(T_i - 30)(2\pi\times 5)}{\left[\frac{\ln\left(\frac{0.002}{0.001}\right)}{0.1}\right] + \frac{1}{(10\times 0.002)}}$$

$$T = 211.2°C$$

Q.20 (B)



Given: $Q_1 = Q$

$Q_2 = 0.25Q$

So, $Q_1 = 4Q_2$

$$\frac{(T_1 - T_2)}{\frac{x_1}{k_1}} = \frac{4(T_1 - T_2)}{\frac{x_1}{k_1} + \frac{x}{k_2}}$$

$$\left[\frac{20}{0.7} + \frac{x}{0.2}\right] = 4\left(\frac{20}{0.7}\right)$$

$$\frac{x}{0.2} = 114.28 - 28.57$$

$\therefore x = 17.15\,cm$

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Q.21 (C)

At steady state,

$$\frac{(T_1 - T_{air})A}{\frac{x}{k} + \frac{1}{30}} = \frac{(T_1 - T_2)A}{\frac{x}{k}}$$

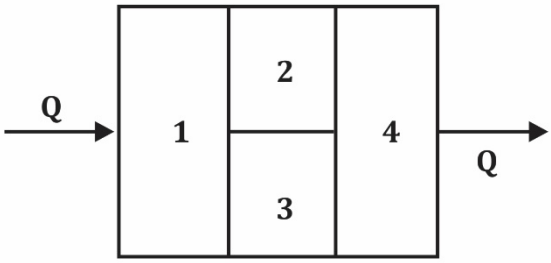
$$\frac{300 - 30}{\left(\frac{0.006}{1.5} + \frac{1}{30}\right)} = \frac{300 - T_2}{\left(\frac{0.006}{1.5}\right)}$$

$$\frac{270}{0.037} \times \frac{0.006}{1.5} = 300 - T_2$$

$$T_2 = 217.07\,°C$$

Q.22 (C)

Q.23 (3.9)



$$R_{overall} = R_1 + R_{23} + R_3$$

$$\frac{1}{R_{23}} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{0.25} + \frac{1}{1}$$

$$R_{23} = 0.2$$

$$R_{overall} = 3 + 0.2 + 0.7$$

$$= 3.9\frac{K}{W}$$

Q.24 (A)

In experiment-1and experiment-3 the heat flux takes place in positive y-direction so it is done by both conduction and convection. Where as in experiment-2 and experiment-4 the heat flux takes place in negative y-direction and by conduction only. The temperature difference between both the plates in same in both so heat flux in expermint-4 is 26 W/m².

Q.25 (28.20)

$$r_i = 1\text{m},\ r_o = 1.1\text{m},\ h = 10\frac{W}{m\text{-}K},\ k = 0.05\frac{W}{m^2\text{-}K}$$

At steady state: $q = \frac{T_i - T_o}{R_i + R_o}$

R_i =resistance due to insulation

R_o = Ambient air heat transfer resistance

Taking basis of 1 m length of pipe,

$$R_i = \frac{r_o - r_i}{kA_{lm}}$$

$$A_{lm} = \frac{2\pi L(r_o - r_i)}{\ln\left(\frac{r_o}{r_i}\right)} = \frac{2\pi(1)(1.1-1)}{\ln\left(\frac{1.1}{1}\right)}$$

$$= 2\pi(1.0493)\ m^2$$

$$R_i = \frac{(1.1-1)}{0.05 \times 2\pi(1.0493)} = \frac{1}{2\pi}(1.906)\frac{K}{W}$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{10 \times 2\pi(1.1)(1)} = \frac{1}{2\pi}(0.09)$$

$$q = \frac{(200 - 20)}{\frac{1}{2\pi}(1.906 + 0.09)} = \frac{180}{1.996} \times 2\pi$$

At steady state,

$$\frac{180}{1.996} \times 2\pi = h_o A_o (T_s - T_\infty) = (10)(2\pi \times 1.1 \times 1)(T_s - 20)$$

$$(T_s - 20) = 8.198$$

$$\therefore T_s = 28.198\,°C$$

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3 CONVECTION

3.1 INTRODUCTION

Convection heat transfer involves fluid motion as well as heat conduction. The process of heat conducting in fluid is due to random molecular motion (diffusion). The fluid motion enhances heat transfer; since it brings warmer & cooler chunks of fluid in contact causing higher rate of conduction at large sites. Heat transfer takes place by convection in the presence of bulk motion and by conduction in the absence of it. Conduction is the limiting case of convection. Heat transfer by conduction occurs near the surface where there is no bulk motion and convection occurs above that region.

The convection process can be further differentiated on the basis of fluid motion.

Free Convection

Circulation of bulk fluid motion is caused by changes in fluid density resulting from temperature gradient between the solid surface and the main mass of fluid. The stagnation layer of fluid in the immediate vicinity of hot body gets thermal energy by conduction.

The circulation pattern, upward movement of the warm fluid and downward movement of cold fluid is called convection currents.

Forced Convection

Flow of fluid is caused by a pump, a fan or by the atmospheric winds. These mechanical devices provide a definite circuit for the circulating currents and speeds up the heat transfer rate.

(i) Newton law: convection rate equation

The rate of convection heat transfer is proportional to temperature difference Between the fluid and the surface and expressed by **Newton's law of cooling** given by

$$Q_{conv} = h\,A\left(T_s - T_\infty\right) \quad \text{------} \quad (3.1)$$

Where,

Q_{conv} Convective heat flow rate

A Surface in fluid contact

T_s Temperature of surface

T_∞ Temperature of fluid

The constant of proportionality 'h' is known as heat transfer coefficient. Convection heat transfer coefficient can be defined as the rate of heat transfer between a solid surface and a fluid per unit surface area per unit temp difference. Its unit is W/m^2K

It is a parameter which depends on flowing variables:

1) Surface condition roughness & cleanliness
2) Geometry & orientation of the surface
3) Thermo physicalproperties: - Density, viscosity, specific heat, coefficient of expansion & thermal conductivity
4) Nature of fluid flow: Laminar or turbulent

3.2 CONCEPT OF BOUNDARY LAYER

(i) Velocity Boundary Layer

Consider flow over the flat plate .When fluid particles make contact with the surface, they assume zero velocity. These particles then act to retard the motion of particles in the adjoining fluid layer, which

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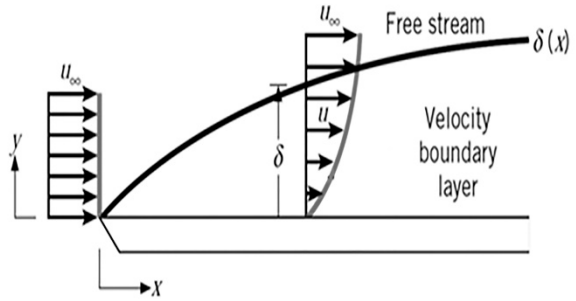
act to retard the motion of particles in the next layer, and so on until, at a distance y= **δ**from the surface, the effect becomes negligible. With increasing distance y normal to the surface, the xvelocity component of the fluid, U must then increase until it approaches the free stream value. A thin fluid layer near the surface in which velocity gradients and shear stresses are large is called **Velocityboundary layer**. The region above boundary layer, velocity gradients and shear stresses are negligible.

The quantity δ_v is termed the **velocityboundary layer thickness**, and it is typically defined as the value of y for which **U= 0.99U∞.**With increasing distance from the leading edge, the effects of viscosity penetrate further into the free stream and the boundary layer grows (**δ**increases with x).

The **boundary layer velocityprofile** refers tothe manner in which u varies with y through the boundary layer.

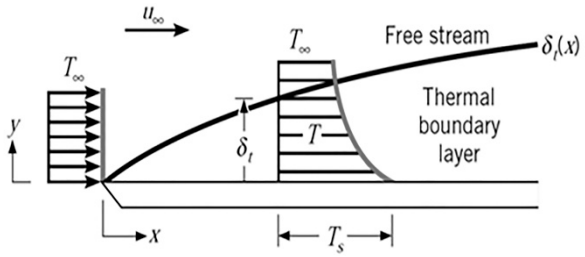


Velocity Boundary layer over flat plate

(ii) Thermal Boundary Layer

As a velocity boundary layer develops when there is fluid flow over a surface, a **thermal boundary layer**develops when the fluid free stream temperature (T_∞) and surface temperatures (T_S) differ. Consider flow over an isothermal flat plate. At the leading edge the temperature profileis uniform, with T(y) = T_∞. However, fluid particles that come into contact with the plate achieve thermal equilibrium at the plate's surface temperature. In turn, these particles exchange energy with those in the adjoining fluid layer, and temperature gradients develop along normal to the surface. The region of the fluid in which these temperature gradients exist is the **Thermal boundary layer.**

Thermal boundary layer thickness δ_t is defined as the value of y for which the ratio $[(T_s-T)/ (T_s- T_\infty)]$ = 0.99.With increasing distance from the leading edge, the effects of heat transfer penetrate further into the free stream and the thermal boundary layer grows.



Thermal boundary layer over flat plate

The fluid layer in contact to surface stops completely. This condition is known as no slip condition. An implication of the 'no slip' condition is that heat transfer from the solid surface to fluid layer adjacent to the surface is by pure conduction.

$$Q''_{conv} = Q''_{cond} = -k\left(\frac{dT}{dy}\right)_{y=0}$$

$$h\left(T_s - T_\infty\right) = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}$$

$$h = \frac{-k\left(dT/dy\right)_{y=0}}{\left(T_s - T_\infty\right)}$$

The temperature gradient at the surface decreases along the flow direction .Thermal conductivity is assumed constant. The temperature difference(T_s- T_∞)does not vary along the flow. Therefore, the heat transfer coefficient decreases along the flow direction.

(iii) Local and average heat transfer coefficient

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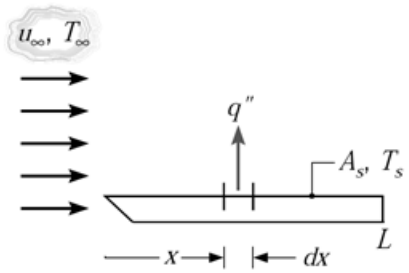

The heat transfer coefficient varies along the flow direction. The total heat transfer rate Q may be obtained by integrating the local flux over the entire surface

$$Q = (T_s - T_\infty)\int h dA_s$$

The heat transfer rate in terms of average heat transfer coefficient can be expressed as

$$Q = \bar{h}A_s(T_s - T_\infty)$$

$\bar{h}$ Isaverage heat transfer coefficient

$$\bar{h} = \frac{1}{L}\int_0^L h_x \, dx.$$

3.3 NON DIMENSIONAL NUMBERS

In convection studies, it is common to Nondimensionalize the governing equations and combine variables, which group together into dimensionless numbers in order to reduce total number of variables.

(i) Nusselt Number(Nu)

Heat transfer coefficient is commonly expressed in terms of non-dimensional number**Nusselt Number(Nu).**It is expressed as

$$Nu = h\frac{\delta}{k}$$

Where 'h' is heat transfer coefficient, k is thermal conductivity of **fluid** and δ is characteristic length. Heat transfer through the fluid in motion is by convection and for fluid at rest is by pure conduction

Physical significance

$$Q_{conv} = h\Delta T \;\&\; Q_{cond} = \frac{k\Delta T}{\delta}$$

$$\frac{Q_{conv}}{Q_{cond}} = \frac{h\Delta T}{\frac{k\Delta T}{\delta}} = \frac{h\delta}{k} = Nu$$

The Nusselt Number, represents the enhancement of heat transfer through fluid layer as a result of convection relative to conduction. The larger the Nusselt Number larger is the convection. Nu =1 for a fluid layer represents heat transfer by pure conduction.

(ii) Reynolds Number (Re)

The transition from laminar to turbulent flow depends on the surface geometry, surface roughness, free stream velocity, surface temperature and type of fluid. The non-dimensional number which accounts for above mentioned parameters is **Reynolds Number (Re)**

Reynolds number is the ratio of inertia force to viscous force.

$$R_e = \frac{\rho V_\infty \delta}{\mu}$$

$\delta$is characteristic length.

Note: For flat plate δ is the distance from leading edge and the diameter D for cylinder or sphere.

(iii) Prandtl Number (Pr)

The development of velocity boundary layer relative to thermal boundary layer will have a strong effect on convection heat transfer.

The relative thickness of the velocity and thermal boundary layer is described by dimensionless parameter **Prandtl Number (Pr).**

$$P_r = \frac{\text{Molecular diffusivity of Momentum}}{\text{Molecular diffusivity of heat}} = \frac{v}{\alpha}$$

where,

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$\nu = \frac{\mu}{\rho}$ is kinematic viscosity

$\alpha = \frac{k}{\rho C_p}$ is thermal diffusivity

For $P_r \ll 1$ the heat diffuses very quickly as in liquid metals and very slowly in oils with $P_r \gg 1$ relative to momentum. Consequently thermal boundary layer is much thicker for liquid metal and much thinner for oil.

(iv) Reynold's Analogy

Reynolds analogy relates the friction coefficient with heat transfer coefficient.
Shear stress is given by

$$\tau = \mu \frac{du}{dy}$$

Heat transfer rate is given by

$$Q = -kA \frac{dT}{dy}$$

Prandtl number is given by

$$\frac{\mu C_p}{k} = P_r$$

For most gases P_r is between 0.6 to 1

$$\frac{\mu C_p}{k} = 1, \quad \frac{k}{\mu} = C_p$$

By rearranging above equation, we get

$$Q = -C_p A \tau \frac{dT}{du}$$

Boundary condition for solving above equation
$T = T_s$ and $u = 0$ at plate surface
$T = T_\infty$ and $u = u_\infty$ at the outer edge of boundary layer.

$$\frac{Q}{C_p A \tau_w} \int_0^{u_\infty} du = -\int_{T_s}^{T_\infty} dT$$

$$\Rightarrow \frac{Q}{A(T_s - T_\infty)} = \frac{C_p}{u_\infty}$$

Skin friction coefficient is given by

$$C_{f_x} = \frac{\tau}{\frac{1}{2}\rho u_\infty^{\,2}}$$

$$\frac{h_x}{\rho C_p u_\infty} = \frac{C_{f_x}}{2}$$

$\frac{h_x}{C_p u_\infty}$ **is known as Stanton Number** $\left(S_{t_x}\right)$

$$S_{t_x} = \frac{N_{u_x}}{R_{e_x} P_r}$$

Physical significance of Stanton No.

$$S_t = \frac{h_x \, \Delta t}{\rho C_p u_\infty \Delta t} = \frac{\text{actual heat flux to the fluid}}{\text{heat flux capacity of the fluid}}$$

The inter relationship can be used directly to infer heat transfer data from measurement of shear stress.

3.4 EXTERNAL FLOWS AND INTERNAL FLOWS

EXTERNAL FLOWS

(i) Flow over Flat Plates

The friction and the heat transfer coefficient for a flat plate can be determined theoretically by solving the conservation of mass, momentum, and energy equations approximately or numerically. They can also be determined experimentally and expressed by empirical correlations. The average Nusselt number can be expressed in terms of the Reynolds and Prandtl numbers in the form.

$$\overline{Nu} = \frac{\bar{h}L}{K} = C \, Re_L^m Pr^n$$

The fluid properties also vary with temperature, and thus with position across the boundary layer. In order to account for the variation of the properties with temperature properly, the fluid properties are usually evaluated at the so called film temperature, defined as. $T_f = \frac{T_s + T_\infty}{2}$

(ii) Laminar flow

In laminar flow, fluid flows in smooth streamlines and highly ordered motion

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The local friction coefficient and Nusselt No. at Location x is given by

$$C_f, x = \frac{0.664}{\sqrt{R_{e_x}}}$$

And Nusselt No. is given by

For constant surface temperature surface

$$N_{u_x} = \frac{h_x . x}{k_f} = 0.332\ R_{e_x}^{1/2} P_r^{1/3},\ Re_x < 5\times10^5$$

1. When a flat plate is subjected to constant uniform heat flux

$$N_{u_x} = \frac{h_x . x}{k_f} = 0.453\ R_{e_x}^{1/2} P_r^{1/3}$$

The average friction coefficient and Nusselt no. over length 'L'

$$C_f = \frac{1}{L}\int_o^L C_f,\ x\ dx = \frac{1.328}{R_{e_L}^{1/2}}$$

$$h = \frac{1}{L}\int_o^L h_f,\ x = 0.664\ R_{e_L}^{\frac{1}{2}} P_r^{\frac{1}{3}}$$

From above results, it can be inferred that $C_f = 2C_{f_x}$, $h = 2h_{f,x}$ for laminar flow

(iii) Turbulent Flow

The local coefficient of friction and average friction coefficient

$$C_f,\ x = \frac{0.0592}{R_{e_x}^{1/5}},\ 5\times10^5 \le R_{e_x} \le 10^7$$

$$C_f = \frac{1}{L}\int_o^L C_f,\ x\ dx = \frac{0.074}{R_{e_L}^{1/5}}$$

Local heat transfer coefficient is given by

1. Constant surface temperature surface

$$N_{u_x} = \frac{h_x x}{k}$$

$$= 0.0296\ R_{e_x}^{\frac{4}{5}} . P_r^{\frac{1}{3}} \left\{ \begin{matrix} 0.6 \le P_r \le 60 \\ 5\times10^5 \le R_e \le 10^7 \end{matrix} \right\}$$

2. When a flat plate is subjected to constant uniform heat flux

$$N_{u_x} = \frac{h_x x}{k}$$

$$= 0.0308\ R_{e_x}^{\frac{4}{5}} . P_r^{\frac{1}{3}} \left\{ \begin{matrix} 0.6 \le P_r \le 60 \\ 5\times10^5 \le R_e \le 10^7 \end{matrix} \right\}$$

Where 'x' is the distance from leading edge of the plate.

Average Nusselt no is given by

1. Constant surface temperature surface

$$N_{u_x} = \frac{h_x x}{k}$$

$$= 0.037\ R_{e_x}^{\frac{4}{5}} . P_r^{\frac{1}{3}} \left\{ \begin{matrix} 0.6 \le P_r \le 60 \\ 5\times10^5 \le R_e \le 10^7 \end{matrix} \right\}$$

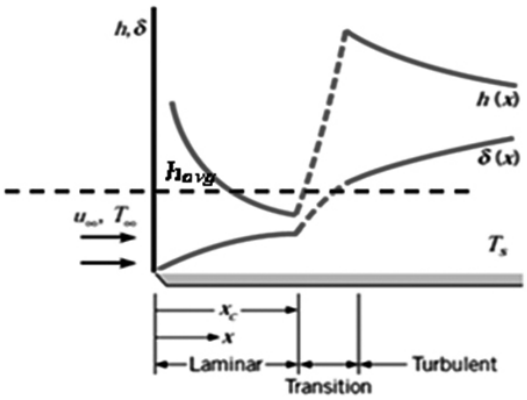
2. When a flat plate is subjected to constant uniform heat flux

$$N_{u_x} = \frac{h_x x}{k}$$

$$= 0.0385\ R_{e_x}^{\frac{4}{5}} . P_r^{\frac{1}{3}} \left\{ \begin{matrix} 0.6 \le P_r \le 60 \\ 5\times10^5 \le R_e \le 10^7 \end{matrix} \right\}$$

(iv) Combined Laminar & Turbulent Flow

In some cases, a flat plate is sufficiently long for the flow to become turbulent, but not long enough to neglect laminar region.



$$h = \frac{1}{L}\left[\int_o^{x_{cr}} h_v, \text{Laminar}\ dx + \int_{x_{cr}}^L h_x \text{turb}\ dx\right]$$

$$\overline{N_{u_L}} = \frac{hL}{k} = \left(0.037 R_{e_L}^{4/5} - A\right) P_r^{1/3} \left\{ \begin{matrix} 0.6 \le P_r \le 60 \\ 5\times10^5 \le R_e \le 10^7 \end{matrix} \right\}$$

Where, $A = 0.037 R_{ex,cr}^{4/5} - 0.664 R_{ex,cr}^{1/2}$

INTERNAL FLOWS

(i)FLOW IN TUBES

There is a fundamental difference between external and internal flows. In external flow, which we have considered so far, the

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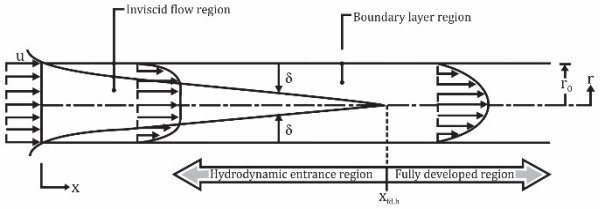
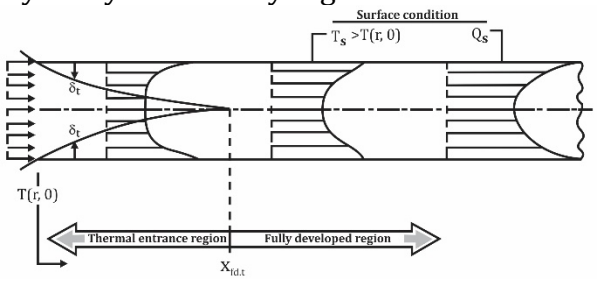
fluid had a free surface, and thus the boundary layer over the surface was free to grow indefinitely. In the internal flow, however, the fluid is completely confined by the inner surfaces of the tube, and thus there is a limit on how much the boundary layer can grow. In external flow, the fluid has free surface, and thus boundary layer over the surface was free to grow indefinitely.

In Internal flow, the fluid is confined by inner surface of the tube, and thus there is a limit on how much the boundary layer can grow.

(ii) Hydrodynamic & Thermal Entry Length



Hydrodynamic entry region



Thermal entry region

The hydrodynamic & thermal entry length in Laminar flow.The region from the tube inlet to the point at which the boundary layer merges at the centerline is called the hydrodynamic entry region, and the length of this region is called hydrodynamic entry length L_h. The region beyond the hydrodynamic entry region in which the velocity profile is fully developed and remains unchanged is called hydrodynamically developed region. The velocity profile in the hydrodynamically developed region is parabolic in laminar flow and somewhat flatter in turbulent flow.

The region of flow over which the thermal boundary layer develops and reaches the tube center is called the thermal entry region, and the length of this region is called the thermal entry length L_t. The region beyond the thermal entry region in which the dimensionless temperature expressed as $(T-T_s)/(T_m-T_s)$ remains unchanged is called the thermally developed region. The region in which the flow both hydrodynamically and thermally developed is called the fully developed flow.

$L_{h,\ laminar} \approx 0.05ReD$

$L_{t,\ laminar} \approx 0.05RePrD$

$L_{h,\ trubulent} \approx L_{t,trubulent} \approx 10D$

(iii) Average velocity & Average temperature

The fluid velocity in a tube changes from zero at the surface to a maximum at the tube center. In fluid flow, it is convenient to work with an average or mean velocity $V_m$which remains constant in incompressible flow when the cross sectional area of the tube is constant.The value of mean velocity V_m is determined from the requirement that the conservation of mass principle.

$m=\rho V_m A$

When a fluid heated or cooled as it flows through a tube,the temperature of a fluid at any cross section changes from T_s at the surface of the wall at that cross section to some maximum(or minimum in the case of heating) at the tube Centre. In the fluid flow it is convenient to work with an average or mean temperature(T_m) that remains constant at a cross section. The mean temperature T_m will change in the flow direction.

The value of the mean temperature T_m is determined from the requirement that the conservation of energy.

$$E_{fluid} = \dot{m}C_pT_m = C_pT\delta m = C_pT(\rho v dA_c)$$

$$T_m = \frac{\int C_pT(\rho v dA_c)}{\dot{m}C_p}$$

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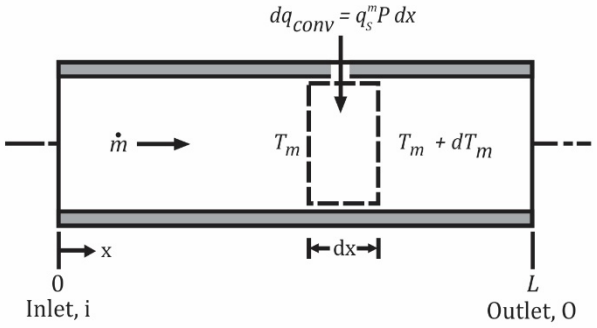
(iv) Constant heat flux

The thermal conditions at the surface of a tube can usually be approximated with reasonable accuracy to be constant surface temperature(T_s=constant) or constant surface heat flux(q_s=constant). For example,
The convection heat flux at any location on the tube can be expressed as
$Q''=h(T_s-T_m)$
Where h is the local heat transfer coefficient and T_s and T_m are the surface and the mean fluid temperatures at that location.



By energy balance, heat gained by fluid is equal to heat incoming as flux

$$dQ_{conv} = h(T_s - T_m) = Q''Pdx = \dot{m}C_p dT_m$$

By solving above differential equation Boundary condition At x= 0, $T_m = T_i$& at x= x,$T_m=T_{mx}$.The mean temperature at any location 'x' from inlet is given by

$$T_{mx} = T_i + \frac{Q_s Px}{\dot{m}C_p}$$

Putting x= L in above equation, we get

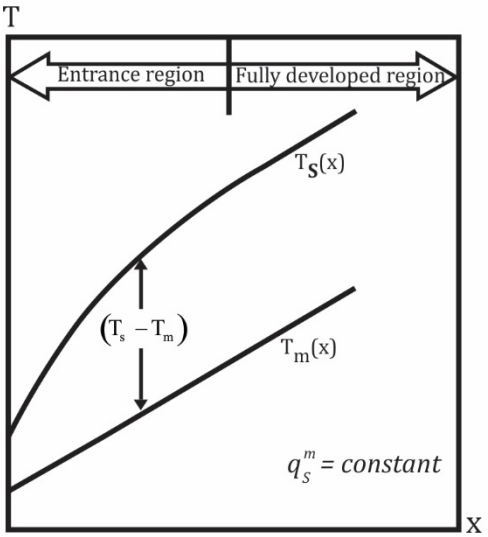
$$T_e = T_i + \frac{Q''A}{\dot{m}C_p}$$

Fluid temperature increases linearly in the direction of flow.
The convection heat transfer (heat flux) at any location on the tube is given by

$$Q'' = h\left(T_s - T_{mx}\right)$$

$$T_s = \frac{Q''}{h} + T_i + \frac{Q''Px}{\dot{m}C_p}$$

Where, h is local heat transfer coefficient,$T_s$surface temp,$T_m$mean temp.



Variation of surface and fluid temperature along tube length of tube

(v) Constant Surface Temp

The constant surface condition is realized when a phase change process such as boiling or condensation occurs at the outer surface of a tube.

$$Q = hA\,\Delta T_{avg} = hA\left(T_s - T_m\right)_{avg}$$

$\Delta T_{avg}$is average temp difference between the fluid & surface.$T_s$is constant
Arithmetic mean temp difference.

$$\Delta T_{am} = \frac{\Delta T_i + \Delta T_e}{2} = \frac{\left(T_s - T_i\right) + \left(T_s - T_e\right)}{2} = \left(T_s - T_b\right)$$

$T_b = 1/2(T_i + T_e)$is bulk mean fluid temp.
The evaluation of ΔT_{am} gives acceptable result, but not always.
Consider the heating of a fluid in a tube of constant cross section. By energy balance
$\dot{m}C_p dT_m = h(T_s - T_m)dA$.
By solving the above differential equation

$$T_e = T_s - \left(T_s - T_i\right)e^{-hA/\dot{m}C_p}$$

$\frac{hA}{\dot{m}C_p}$is a dimensionless parameter known as NTU and is measure of effectiveness of heat transfer system.

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(vi) Laminar flow in tubes

Velocity profile is symmetric about the tube center give the following parabolic velocity profile for thehydrodynamically developed laminar flow:

$$v(r) = 2V_m\left(1-\frac{r^2}{R^2}\right)$$

The Nusselt number in the fully developed laminar flow region in a circular tube is determined in a similar manner from the conservation of energy equation

Nu=3.66 for T_s= constant

Nu=4.36 for q_s= constant

A general relation for the average Nusselt number for the hydrodynamically and / or thermally developing laminar flow in a circular tube is given by Sieder and Tate

$$Nu = 1.86\left(\frac{RePrD}{L}\right)^{1/3}\left(\frac{\mu_b}{\mu_s}\right)^{0.14}$$

Where,μ_b& μ_s is dynamic viscosity evaluated at bulk mean temperature and surface temperature respectively

(vii) Turbulent Flow in tubes

Heat transfer coefficient in turbulent flow are based on experimental studies because of the difficulty in dealing with turbulent flow theoretically.

For smooth tubes, the friction factor is fully developed turbulent flow can be determined from

$f=0.184Re^{0.2}$

The Nusselt number in turbulent flow is related to the friction factor through the famous **Chilton-Colburn analogy** expressed as

$Nu=0.125fRePr^{1/3}$

Nusselt number for fully developed turbulent flow in smooth tubes, which is known as the **Colburn equation**

$Nu=0.023Re^{0.8}Pr^{1/3}$

Another equation often preferred to colburn equation is known as the **Dittus-Boulter** equation given by

$Nu=0.023Re^{0.8}Pr^{n}$

Where n=0.4 for heating and 0.3 for cooling of the fluid flowing through the tube. The fluid properties are evaluated at the bulk mean fluid temperature $T_b=1/2(T_i+T_e)$, which is the arithmetic average of the mean fluid temperatures at the inlet and exit of the tube.

Note: The relations above are not very sensitive to the thermal conditions at the tube surfaces and can be used for both $T_s = \text{const}$ & $Q_s = \text{const}$.

Note that tubes with rough surfaces have much higher heat transfer coefficient than tubes with smooth surfaces. Therefore, Tube surfaces are often intentionally roughened, corrugated, or finned in order to enhance the convection heat transfer coefficient and thus the convection heat transfer rate.

3.5 NATURAL CONVECTION

In natural or free convection fluid motion is due to buoyancy forces within the fluid, while in forced convection it is externally imposed. Buoyancy is due to the combined presence of a fluid density gradient and a body force that is proportional to density. Free convection strongly influences the operating temperatures of power generating and electronic devices. It plays a major role in a vast array of thermal manufacturing applications. Free convection is important in establishing temperature distributions within buildings and in determining heat losses or heat loads for heating, ventilating, and air conditioning systems.

(i)PHYSICAL MECHANISM

Consider a wall at temperature higher than atmospheric temperature. The density of air adjacent to wall is lower than density of air away from wall. The warmer air rises up and cool air come to take its place. The motion that results from continuous replacement of heated air with cooler air is called **natural convection currents.** The

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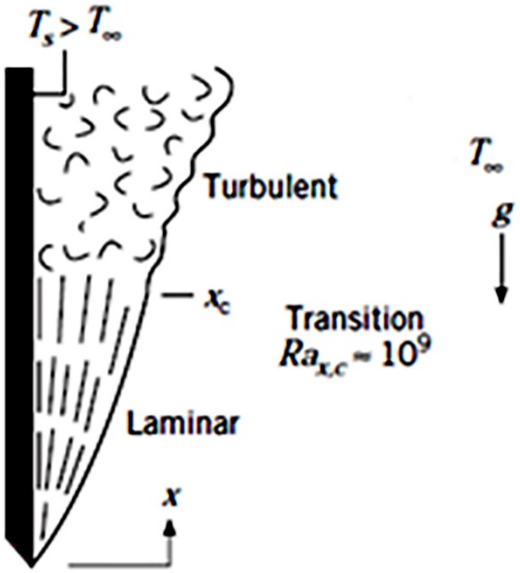
heat transfer enhanced due to natural convection currents is known as **natural convection or free convection**

(a) Boundary layer

Consider a plate is immersed in fluid, and with $T_s > T_\infty$, the fluid close to the plate is less dense than fluid that is further removed. Buoyancy forces therefore induce a free convection boundary layer in which the heated fluid rises vertically, entraining fluid from the quiescent region. The resulting velocity distribution is unlike that associated with forced convection boundary layers. The velocity distribution is shown in figure



(ii) NON DIMENSIONAL NUMBERS

(a) Grashof's Number

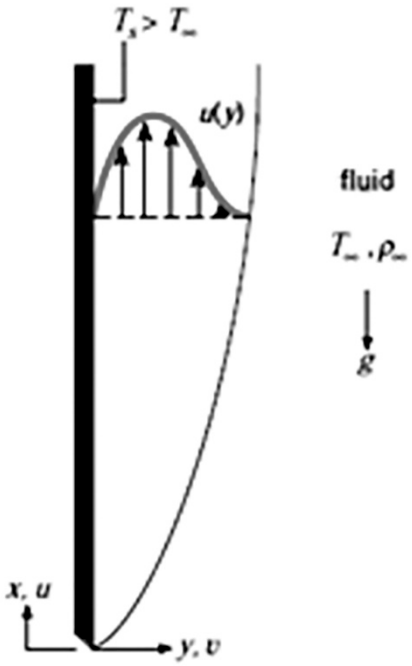
Grashof's number plays the same role in free convection that the Reynolds number plays in forced convection. Reynolds number provides a measure of the ratio of the inertial to viscous forces acting on a fluid element. In contrast, the Grashof number is a measure of the ratio of the buoyancy forces to the viscous forces acting on the fluid.
Grashof's(Gr) Number is given by

$$Gr_L = \frac{g(T_s - T)L^3}{2}$$

Where, β is expansion coefficient
$\left(\beta = \frac{1}{T} \text{ for ideal gas, where T in kelvin}\right)$,
T_s is surface temperature
T_∞ is the ambient temperature
L is the characteristic length
ν is kinematic viscosity.

(b) Rayleigh number

It is important to note that free convection boundary layers are not restricted to laminar flow. As with forced convection, hydrodynamic instabilities may arise. That is, disturbances in the flow may be amplified, leading to transition from laminar to turbulent flow. This process is shown schematically in Figure for a heated vertical plate. Transition in a free convection boundary layer depends on the relative magnitude of the buoyancy and viscous forces in the fluid. It is customary to correlate its occurrence in terms of the **Rayleigh number,** which is simply the product of the Grashof and Prandtl number.



$$Ra_L = Gr_L . P_r$$

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For vertical plates, transition from laminar takes place for $R_{a,x}>10^9$

3.6 NATURAL CONVECTION EMPIRICAL CORRELATIONS (EXTERNAL FLOWS)

(i) Vertical plate

Although we understand the mechanism of natural convection, the complexities make it very difficult to obtain analytical relations. The simple Empirical correlations for the average Nusselt Number

$$\overline{Nu}=\frac{\bar{h}L_c}{k}=C\left(G_rP_r\right)^n=C\left(R_a\right)^n$$

Where, R_a is Rayleigh Number, L_c is characteristic length

The value of constant c and n depend on the geometry of the surface & flow

For vertical plate $L_c=L$

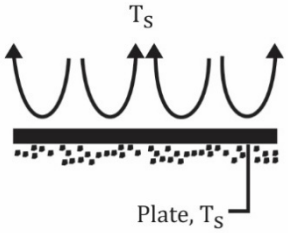
$\overline{Nu}=0.59Ra^{1/4}$ $10^4<Ra<10^9$ for Laminar

$\overline{Nu}=0.1Ra^{1/3}$ $10^{10}<Ra<10^{13}$ for Turbulent

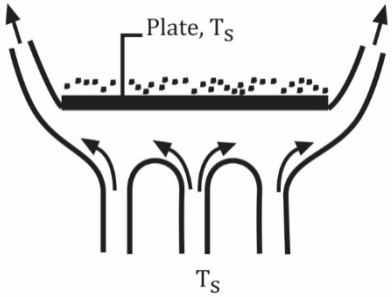
(ii) Horizontal Plate

For horizontal plates characteristic Length($L_c=A/P$). For horizontal, the buoyancy force is exclusively normal to the surface. As for the inclined plate, flow patterns and heat transfer depend strongly on whether the surface is cooled or heated and on whether it is facing upward or downward. The flow of fluid is shown in figure

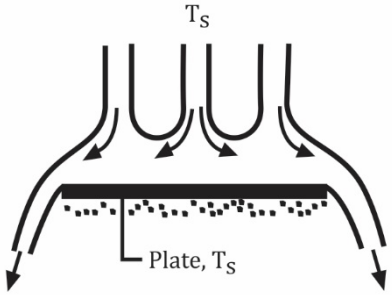
(a)Top surface of hot plate



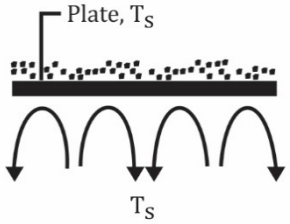
(b)Bottom surface of hot plate



(c)Top surface of cold plate



(d)Bottom surface of cold plate



Lower Surface of Hot Plate or Upper Surface of Cold Plate

$N_u=0.27\ R_a^{1/4}$ $10^5<Ra<10^{11}$

Upper Surface of Hot Plate or Lower Surface of Cold Plate

$N_u=0.59\ R_a^{1/4}$

$10^4<Ra<10^7$ Laminar

$N_u=0.15\ R_a^{1/3}10^7<Ra<10^{11}$ Turbulent

(iii) Flow Cylinder & Sphere Horizontal Cylinder (Isothermal /Morgan suggested)

For an isothermal cylinder, Morgan suggests an expression of the form

$$\bar{N}_{u_D}=\frac{\bar{h}D}{K}=CR_a^{\ n}$$

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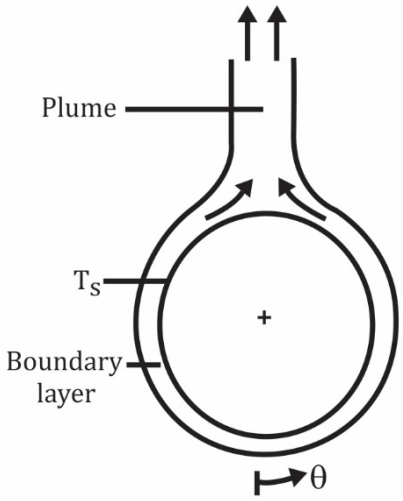
For a heated cylinder, Local Nusselt numbers are influenced by boundary layer development, which begins at θ=0 and concludes at θ=π with formation of a plume ascending from the cylinder. if flow remains laminar over entire range, the distribution of Local Nusselt No. with θ this decay would be disrupted at $R_{a_D} \geq 10^9$



R_{aD}	C	n
$10^{-10}-10^{-2}$	0.675	0.058
$10^{-2}-10^{2}$	1.02	0.148
$10^{2}-10^{4}$	0.850	0.188
$10^{4}-10^{7}$	0.480	0.250
$10^{7}-10^{12}$	0.125	0.333

Churchill & chu relation for entire Range Ra<10^{12}

$$\bar{N}_{u_D} = (0.60 + \frac{0.387R_{a_D}^{1/6}}{\left[1+\left(0.559/P_r\right)^{9/16}\right]^{8/27}})^2$$

Vertical l Cylinder

Vertical cylinder can be treated as vertical plate with (characteristic length Lc = L) for the given condition

$$D \geq \frac{35L}{Gr_L^{\frac{1}{4}}}$$

Sphere

For sphere the relation is given by

$$\bar{N}_{u_D} = \frac{\bar{h}D}{K} = 2 + \frac{0.589R_{a_D}^{1/6}}{\left[1+\left(0.469/P_r\right)^{9/16}\right]^{84/9}}$$

3.7 FORCED CONVECTION

For forced convection,

$$Nu = \phi(Re, Pr)$$

For fully developed flow turbulent flow in smooth pipes, the exprssion proposed by Dittus and Boelter is as follows:

$$Nu_d = 0.023 Re_d^{0.8} Pr^n$$

$$n = \begin{cases} 0.4 \text{ for heating of the fluid} \\ 0.3 \text{ for cooling of the fluid} \end{cases}$$

This equation is valid for fully developed turbulent flow in smooth pipes with Prandtl numbers ranging from 0.6 to 100 and with moderate temperature differences between wall and fluid conditions.

For laminar flow through pipes, the Sieder-Tate equation can be used. (for Re < 2100):

$$Nu_d = 1.86\left(Re_d\, Pr \frac{d}{L}\right)^{1/3}\left(\frac{\mu}{\mu_w}\right)^{0.14}$$

Where d = pipe diameter, m
L = pipe length before mixing occurs in pipe, m
μ = fluid viscosity at the bulk average temperature, Pa-s
μ_w = fluid viscosity at wall temperature

This equation is valid for

$$Re_d\, Pr \frac{d}{L} > 10$$

The viscosity correction factor $\left(\frac{\mu}{\mu_w}\right)$ is needed for highly viscous fluids.

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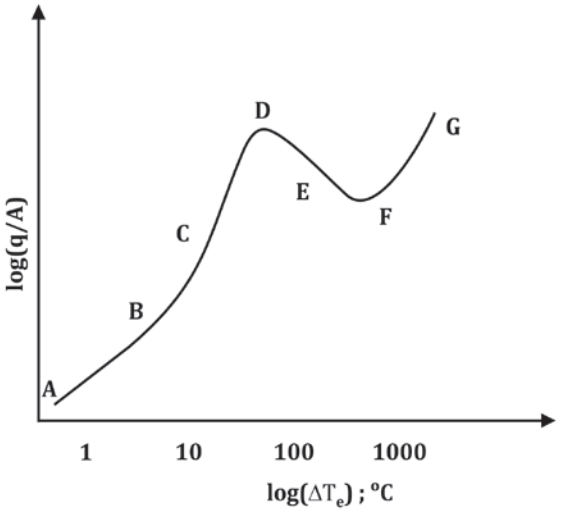
3.8 BOILING AND CONDENSATION

BOILING

Boiling is associated with transformation of liquid to vapour at a solid-liquid interface. Heat transfer to a boiling liquid is important in evaporation and distillation.

When a hot suraface is immersed in a liquid and is maintained at a temperature above the saturation temperature of the liquid, boiling may occur. The heat flux depends on the difference in temperature of the surface and the saturation temperature. Vapour bubbles are formed at the heating surface and rise through the liquid. These bubbles disengage at the liquid surface and are accumulated in a vapour space above the liquid. This type of boiling is referred to as **pool boiling.**

The heat flux data obtained from an electrically heated wire immersed in a liquid are plotted against temperature excess ΔT_e. ΔT_e is the temperature difference between temperature of the heating surface and the saturation temperature. The curve reveals range of conditions for different regimes of boiling.



Region on the curve	Description of the boiling regime
A-B	Pure Convection: The liquid rises to the surface for evaporation
B-C	Nucleate boiling, bubbles form at the heating surface and condensing in the superheated liquid
C-D	Nucleate boiling, bubbles rise to the liquid surface
D	Peak flux
D-E	Partial nucleate boiling and unstable film boiling
E-F	Stable film boiling
F-G	Heat transfer by radiation becomes dominant

At lower temperature drops, the line AB is straight and has a slope of 1.25. Liquid motion in this region is mostly due to single phase natural convection. The vapour formation in this region is very little.

The slope of BC is usually between 3 and 4. Bubbles are generated at the surface of the wire and are condensed in the liquid after breaking away from the surface. This is called **nucleate boiling**. The liquid motion in this regime is strongly influenced by the nucleation of bubbles. Bubbles occupy a very small portion of the heating area at a time. Most of the heating surface is in contact with the liquid.

The curve CD indicates nucleate boiling with the vapour bubbles rising to the liquid surface. Increase in the number of nucleation sites causes bubble interactions due to which they coalesce into jets and slugs. The contact between

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the surface and the liquid becomes impaired. Heat flux continue to increase with ΔT_e.

The temperature drop corresponding to point D is called the critical temperature drop and the heat flux at point D is called the **peak flux**. The maximum heat transfer coefficient is obtained at point D.
Once the number of bubbles reaches to a maximum, The bubbles nearby the heating surface coalesce and a film of vapour is formed on the heating surface. This film is highly unstable. The thermal resistence of this film causes a reduction in heat flux. This region is indicated by the curve DE and is called **parital nucleate boiling**. This curve shows transition from nucleate boiling to film boiling and is also called **transition boiling**.
If the excess temperature is furthur increased, the bubbles form and coalesce so rapidly that they cover the heating surface to form a stable vapour film. The heat transfer throught this vapour film is only by conduction.

The heat flux, decreasing with increase in ΔT_e, reaches a minimum at point F. Point F is called the **Leidenfrost point**.

At very high excess temperature, the heat transfer by radiation through the vapour film becomes dominant. This leads to an increase in the heat transfer coefficient. The excess temperature in this region is so high that the heating wire may get melted. This is called boiling crises. The regime of DE, EF and FG together is called the **film boiling** regime.

Film boiling is not commercially desireable because the rate of heat transfer is low for the large temperature drop.

CONDENSATION:

Consider a vertical flat plate exposed to a condensable vapour. The temeperature of the plate is less than the saturation temperature of the vapour.
Dropwise condensation: If the liquid does not wet the surface, liquid droplets are formed on the surface that fall down the surface.
Film condensation: If the liquid wets the surface, a smooth liquid film is formed on the surface.
Heat transfer rates in dropwise condensation may be about 10 times higher than in film condensation.
Because of higher heat transfer rates, dropwise condensation would be preferable, but it is very difficult to maintain. Most surfaces become wetted after being exposed to a condnsing vapour for long periods of time.
Analysis of film condensation was proposed by Nusselt:
It is assumed that:

A. The condensate layer flows downwards in laminar flow under the effect of gravity and resistance to heat flow is offered only by this layer of condensate.
B. No- slip condition, i.e. the velocity of the liquid at the wall is zero.
C. A linear temperature distribution exists between the wall and the vapour conditions.
D. The viscous shear of the vapour on the liquid film at its outer surface is negligible.

Nusselt equation for vapour condensing on a vertical flat plate is given by:

$$\bar{h} = 0.943\left[\frac{k_f^3 \rho_f^2 \lambda g}{\mu_f L \Delta T_f}\right]$$

where $T_f = \frac{1}{2}(T_v + T_w)$

and $\Delta T_f = T_f - T_w$

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Where,

T_w = Plate temperature

λ = latent heat of condensation of the vapour

For laminar film condensation on a horizontal tube,

$$\bar{h} = 0.725\left[\frac{k_f^3 \rho_f^2 \lambda g}{\mu_f d \Delta T_f}\right]$$

Where d is the diameter of the tube.

If there are n tubes placed over one another in the vertical direction, the heat transfer coefficient can be calculated by replacing *d* in the above equation with *nd*.

Vertical tubes: A condensate film starts forming at the top of the tube. The thickness the film increases rapidly at first and then slowly in the remaining length of the tube. It is assumed that the heat flow through the condensate film is onlly by conduction. The local HTC is given by:

$$h_x = \frac{k_f}{\delta}$$

Example:1 The temperature profile at a particular location on the surface of plate is prescribed by the identities:

(i) $\dfrac{t_s - t}{t_s - t_\infty} = \sin\left(\dfrac{\pi y}{0.015}\right)$

(ii) $\dfrac{t_s - t}{t_s - t_\infty} = \dfrac{1}{2}\left(\dfrac{y}{0.0075}\right)^3 + \dfrac{3}{2}\left(\dfrac{y}{0.0075}\right)$

If thermal conductivity of air is stated to be 0.03 W/m-^{o}c, determine the value of convective heat transfer coefficient in each case

Solution:

The convective heat transfer coefficient is prescribed by the relation

$$h = -\frac{k}{(t_s - t_\infty)}\left(\frac{dt}{dy}\right)_{y=0} \quad \ldots(i)$$

$$\frac{d}{dy}\left[\frac{t_s - t}{t_s - t_\infty}\right] = d\left[\sin\left(\frac{\pi y}{0.015}\right)\right]$$

$$= \frac{\pi}{0.015}\cos\left(\frac{\pi y}{0.015}\right) \quad \text{Or}$$

$$\left(\frac{dt}{dy}\right)_{y=0} = -(t_s - t_\infty)\frac{\pi}{0.015}(\cos 0)$$

$$= -\frac{\pi}{0.015}(t_s - t_\infty)$$

$$\therefore \quad h = -\frac{k}{(t_s - t_\infty)} \times \left\{-\frac{\pi}{0.015}(t_s - t_\infty)\right\}$$

$$\frac{\pi k}{0.015} = \frac{\pi \times 0.03}{0.015} = 6.28 \text{ W/m}^2\text{K} \quad \ldots(ii)$$

$$\frac{d}{dy}\left(\frac{t_s - t}{t_s - t_\infty}\right) = \frac{d}{dy}\left[\frac{1}{2}\left(\frac{y}{0.0075}\right)^3 + \frac{3}{2}\left(\frac{y}{0.0075}\right)\right]$$

$$= \frac{3}{2}\left(\frac{y}{0.0075}\right)^2 \times \frac{1}{0.0075} + \frac{3}{2} \times \frac{1}{0.0075}$$

$$\text{Or} \left(\frac{dt}{dy}\right)_{y-0} = -(t_s - t_\infty) \times \left(\frac{3}{2 \times 0.0075}\right)$$

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$$\therefore\ h = -\frac{k}{(t_s - t_a)} \times \left[-(t_s - t_\infty) \times \frac{3}{2 \times 0.0075} \right]$$

$$= \frac{3k}{2 \times 0.0075} = \frac{3 \times 0.03}{2 \times 0.0075} = 6\ W/m^2K$$

Example:2

Air enters a rectangular duct measuring 30 cm x 40 cm with a velocity of 8.5 m/s and a temperature of 40° C. The flowing air has a thermal conductivity 0.028 W/m-°c, kinematic viscosity 16.95 x 10-6 m2/s and from empirical correlations the Nusselt number has been approximated to be 425. Work out the equivalent diameter of the flow passage, the flow Reynolds number and the convective heat flow coefficient.

Solution:

For a duct of rectangular cross-section with length l and breadth b, the equivalent or hydraulic diameter is given by:

$$d_e = \frac{2lb}{l+b} = \frac{2 \times 0.4 \times 0.3}{0.4 + 0.3} = 0.3428$$

(i) Reynolds number R_e is equal to

$$\frac{Vd_e\rho}{\mu} = \frac{Vd_e}{\upsilon} = \frac{8.5 \times 0.3428}{16.95 \times 10^{-6}} = 0.1719 \times 10^6$$

Clearly the flow if turbulent in character.

(ii) The Nusselt number and the convective film coefficient are related to each other by the expression

$$N_u = \frac{hl}{k} = \frac{hd_e}{k}$$

$\therefore$ Convective coefficient

$$h = \frac{N_u k}{d_e} = \frac{425 \times 0.028}{0.3428} = 34.71\ W/m^2K$$

Example:3

Consider a cubical block 10 cm x 10 cm in size and suspended in still air at 20°C. All surfaces of the block are maintained at 160°C and one of its surface lies in the horizontal position. Determine the total heat loss from the block.

Solution:

At the mean film temperature,

$t_f = (160 + 20)/2 = 90°C$,

the thermo-physical properties of air are:

$\nu = 22.10 \times 10^6\ m^2/s;$

$k = 0.03217\ W/m - ^o c;$

$Pr = 0.690$ And

$$\beta = \frac{1}{T} = \frac{1}{90 + 273} = \frac{1}{363}$$

$= 0.00275$ per degree kelvin

$$Gr = \frac{l_c^3 \rho^2 \beta g \Delta t}{\mu^2}$$ (l_c is the reference length)

$$= l_c^3 \frac{\beta g \Delta t}{\nu^2} = l_c^3 \times \frac{0.00275 \times 9.81 \times (160 - 20)}{(22.10 \times 10^{-6})^2}$$

$$= 7.73 \times 10^9 l_c^3$$

$$Gr \times Pr = 7.73 \times 10^9 l_c^3 \times 0.690 = 5.33 \times 10^9 l_c^3$$

(i) For four vertical faces of the cube

$= l_c = l = 10cm = 0.1m$

$$Gr \times Pr = 5.33 \times 10^9 \times (0.1)^3 = 5.33 \times 10^6$$

Obviously the boundary layer is laminar, and accordingly

$$Nu = \frac{hl}{k} = 0.59\ (GrPr)^{0.25}$$

$$= 0.59\ (5.33 \times 10^6)^{0.25} = 28.348$$

$$\therefore h = 28.348 \times \frac{0.03217}{0.1} = 9.12\ W/m^2 - ^o c$$

Heat loss,

$$Q_1 = hA\Delta t = 9.12 \times (4 \times 0.1 \times 0.1) \times (160 - 20)$$

$= 51.07\ W$

(ii) For top face of the cube

$$l_c = \frac{A}{P} = \frac{ab}{2(a+b)} = \frac{0.1 \times 0.1}{2(0.1 + 0.1)} = 0.025\ m$$

$$Gr \times Pr = 5.53 \times 10^9 (0.025)^3 = 83281$$

Obviously the boundary layer is laminar for the horizontal top face of the cube, and accordingly

$Nu = 0.54\ (GrPr)^{0.25}$ ....(heated surface up)

$Nu = 0.54\ (83281)^{0.25} = 9.17$

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$$\therefore h = Nu\frac{k}{l_c} = 9.17 \times \frac{0.03217}{0.025}$$

$$= 11.8 \text{ W/m}^2 - {}^o\text{c}$$

Heat loss,

$$Q_2 = hA\Delta t = 11.8 \times (0.1 \times 0.1) \times (160 - 20)$$

$$= 16.52 \text{ W}$$

(iii)For bottom face of the cube

$$l_c = \frac{A}{P} = \frac{0.1 \times 0.1}{2(0.1 + 0.1)} = 0.025$$

$$Gr \times Pr = 5.33 \times 10^9 \left(0.025\right)^3 = 83281$$

Obviously the boundary layer is laminar for the horizontal bottom face of the cube, and accordingly

$Nu = 0.27\ (GrPr)^{0.25}$ ..(heated surface down)

$$Nu = 0.27\ (83281)^{0.25} = 4.587$$

$$\therefore h = Nu\frac{k}{l_c} = 4.587 \times \frac{0.03217}{0.025}$$

$$= 5.9 \text{ W/m}^2 - {}^o\text{c}$$

Heat loss,

$$Q_3 = hA\Delta t = 5.9 \times (0.1 \times 0.1) \times (160 - 20)$$

$$= 8.26 \text{ W}$$

Total heat loss from the cubical block

$$= 51.07 + 16.52 + 8.26 = 75.85 \text{ W}$$

Example:4 Air moving at 0.3 m/s blows over the top of a chest-type freezer. The top of the freezer measures 0.9 m by 1.5 m and is poorly insulated so that the surface remains at 10°C. If the temperature of air is 30°C, make calculations for the maximum heat transfer by forced convection from the top of the freezer.

Solution:

At the mean film temperature, $t_f = (10 + 30)/2 = 20^o\text{C}$, the thermo-physical properties of air are:

$v = 15.06 \times 10^{-6}\ \text{m}^2/\text{s}$; $k = 0.0259 \text{ W/m} - {}^o\text{c}$;

$Pr = 0.703$

The maximum average heat transfer occurs when the air flows is in the direction of the shorter dimension.

$$Re = \frac{\rho Vl}{\mu} = \frac{Vl}{\upsilon} = \frac{0.3 \times 0.9}{15.06 \times 10^{-6}} = 17928$$

(laminar)

$$Nu = 0.664(Re)^{0.5}(Pr)^{0.33}$$

$$= 0.664(17928)^{0.5} \times (0.703)^{0.33} = 79.14$$

$$h = Nu \times \frac{k}{l} = 79.14 \times \frac{0.0259}{0.9}$$

$$= 2.28 \text{ W/m}^2 - {}^o\text{c}$$

$\therefore$ Convection heat flow

$$Q = hA\Delta t = 2.28 \times (0.9 \times 1.5) \times (10 - 30)$$

$$= -61.56 \text{ W}$$

The negative sign indicates that heat transfer is towards the freezer.

Example:5

Air flows through a 10 cm internal diameter tube at the rate of 75 kg/hr. Measurements indicate that at a particular point in the tube, the pressure and temperature of air are 1.5 bar and 325 K respectively whilst the tube wall temperature is 375 K. Make calculations for the heat transfer rate from one meter length in the region of this point. The general non-dimensional correlations for turbulent flow in the tube is:

$Nu = 0.023\ Re^{0.8}\ Pr^{0.4}$

where the fluid properties are evaluated at the bulk temperature.

Solution:

For air at $t_f = 325$ K and $\rho = 1$ atm, the thermo-physical properties of air are:

$\mu = 1.967 \times 10^{-5}$ Kg/ms;

$k = 0.02792$ W/mK and $Pr = 0.713$

These properties may be assumed to be independent of pressure to an excellent approximation.

$$Re = \frac{\rho Vd}{\mu} = \frac{md}{A\mu} = \frac{4m}{\pi d\mu} = \frac{4 \times (75/3600)}{\pi \times 0.1 \times 1.967 \times 10^{-5}}$$

$$= 13492$$

This is well in excess of the critical Reynolds number for flow in tubes (2500); the flow is turbulent and the given correlation applies

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$Nu = 0.023(13492)^{0.8}(0.713)^{0.4} = 40.46$

Convective film coefficient,

$$h = \frac{Nu \times k}{d} = \frac{40.46 \times 0.02792}{0.1} = 11.296 \text{ W/m}^2\text{K}$$

Heat flow rate,

$Q = hA\Delta t = 11.296 \times (\pi \times 0.1 \times 1) \times (375 - 325)$

$= 177.35$ W

Example:6

A square channel with a side 10 mm and length 1.5 m carries water with a velocity of 5 m/s. Measurements indicate that lengthwise mean temperature of water is 30°C whilst the inner surface of channel is at 80°C. Calculate the convective coefficient of heat transfer from the channel wall to the water. Use the correlation:

$$Nu = 0.021 Re^{0.8} Pr^{0.43} \left(\frac{Pr}{Pr_w}\right)^{0.25}$$

where the thermo-physical properties pertain to those at the mean bulk temperature of water. Pr_w corresponds to the value of Prandtl number at the channel surface temperature and equivalent diameter is the reference dimension.

$\rho = 995.07$ Kg/m^3; $c_p = 4174$ J/kgK;

$k = 0.6172$ W/mK;

$\mu = 2.88$ Kg/m-hr; $\nu = 0.805 \times 10^{-6}$ m^2/s;

Pr=5.42

At wall: temperature

$t_w = 80^oC$ and $Pr_w = 2.21$

Solution: Equivalent diameter of the channel, $d_{eq} = 4\frac{A}{P} = 4\frac{0.01 \times 0.01}{4 \times 0.01} = 0.01$ m

where A is cross sectional area of the channel and P is perimeter of the channel.

$$Re = \frac{Vd_{eq}}{\nu} = \frac{5 \times 0.01}{0.805 \times 10^{-6}} = 62100$$

$$Nu = 0.021(62100)^{0.8}(5.42)^{0.43}\left(\frac{5.42}{2.21}\right)^{0.35} = 371.1$$

$$\therefore h = \frac{Nu \times k}{d_{eq}} = \frac{371.4 \times 0.6172}{0.01} = 22923 \text{ W/m}^2\text{K}$$

Example:7

Air at atmospheric pressure and 20°C flows with 6 m/s velocity through main trunk duct of air conditioning system. The duct is rectangular in cross-section and measures 40 cm x 80 cm. Determine heat loss per meter length of duct corresponding to unit temperature difference. The relevant thermo-physical properties of air are

$\nu = 15 \times 10^{-6}$ m^2/s; $\alpha = 7.7 \times 10^{-2}$ m^2/hr

and $k = 0.026$ W/m-^{o}c

Solution:

For the rectangular duct, characteristic length l

$$l = \frac{2ab}{a+b} = \frac{2 \times 80 \times 40}{80 + 40} = 53.3 \text{ cm}$$

$$Pr = \frac{\mu c_p}{k} = \frac{\nu}{\alpha} = \frac{15 \times 10^{-6}}{(7.7 \times 10^{-2})/3600} = 0.701$$

$$Re = \frac{\rho Vl}{\mu} = \frac{Vl}{\nu} = \frac{6 \times 0.533}{15 \times 10^{-6}} = 0.213 \times 10^6$$

Obviously the flow is turbulent and accordingly

$$Nu = \frac{hl}{k} = 0.023(Re)^{0.8} \times (Pr)^{0.4}$$

$Nu = 0.023(0.213 \times 10^6)^{0.8} \times (0.701)^{0.4} = 365.34$

$\therefore$ Convection coefficient

$$h = 365.34 \times \frac{k}{l} = 365.34 \times \frac{0.026}{0.533}$$

$= 17.82$ W/m^2 – ^{o} c

Area of duct per meter length

$= 2(a+b) \times 1 = 2(0.8 + 0.4) \times 1 = 2.4 \text{ m}^2$

$\therefore$ Convective heat loss,

$Q = hA\Delta t = 17.82 \times 2.4 \times 1 = 42.768$ W

Example: 8

Air at a temperature of 25°C is blown across a flat plate at a mean velocity of 7.5m/s. If the plate surface temperature is 575°C, make calculations for the heat

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transferred per meter width from both sides of the plate over distance of 20 cm from the leading edge.

For heat transfer from a plate with large temperature between the plate and the fluid, the local Nusselt number is given by :

$$Nu_x = 0.332(Pr)^{1/3}(Re)^{1/2}\left(\frac{T_s}{T_a}\right)^{0.117}$$

where all the properties are at the mean film temperature, T_s and T_a are the absolute temperature of the plate surface and the free stream of air respectively. The characteristics linear dimension is the distance from the leading edge.

Solution:

At the mean film temperature $t_f = (575+25)/2 = 300^oC$,

the thermo-physical properties of air are:

$\rho = 0.615 kg/m^3$; $c_p = 1.0465 kJ/kgK$;

$k = 0.1659 kJ/mhrK$; and

$\mu = 29.724\times10^{-6} kg/ms$

$$Re = \frac{\rho Vl}{\mu} = \frac{0.615\times7.5\times0.2}{29.724\times10^{-6}} = 31035$$

$$Pr = \frac{\mu c_p}{k} = \frac{(29.724\times10^{-6}\times3600)\times1.0465}{0.1659}$$

$$= 0.675$$

$$Nu_x = 0.332(0.675)^{1/3}(31035)^{1/2}\times\left(\frac{273+575}{273+25}\right)^{0.117}$$

$= 58 \therefore$ Local heat transfer coefficient,

$$h_x = \frac{58\times0.1659}{0.2} = 48.11 kJ/m^2hrK$$

The local and average convective coefficients are related by an expression of the form

$$\bar{h} = \frac{1}{l}\int_0^1 h_x dx$$

$$= \frac{1}{l}\int_0^1 \frac{k}{x}\times0.332(Pr)^{1/3}\left(\frac{T_s}{T_a}\right)^{0.117}\left(\frac{\rho V}{\mu}\right)^{1/2} x^{1/2}dx$$

$$= 0.322\frac{k}{l}(Pr)^{1/3}\left(\frac{T_s}{T_a}\right)^{0.117}\left(\frac{\rho v}{\mu}\right)^{1/2}\int_0^1\frac{dx}{\sqrt{x}}$$

$$= 0.322\frac{k}{l}(Pr)^{1/3}\left(\frac{T_s}{T_a}\right)^{0.117}\left(\frac{\rho v}{\mu}\right)^{1/2} 2l^{1/2}$$

$$= 2\times0.322\frac{k}{l}(Pr)^{1/3}\left(\frac{\rho vl}{\mu}\right)^{1/2}\left(\frac{T_s}{T_a}\right)^{0.117}$$

$= 2\times$ local heat transfer coefficient

$= 2\times48.11 = 96.22 kJ/m^2hrK$

Heat loss from both sides of plat $= 2(hA\Delta t)$

$$= 2\left[96.22(0.2\times1)(575-25)\right] = 21168.4 kJ/hr$$

Example:9

A flat plate $1\,m\times1\,m$ is placed in a wind tunnel. The velocity and temperature of free stream air are 80 m/s and 10^oC respectively. The flow over the whole length of the plate is made turbulent by turbulizing grid placed upstream of the plate. Make calculations for the following parameters:

a) thickness of hydrodynamic boundary layer at trailing edge of the plate

b) heat flow from the surface of the plate

The plate is maintained at 50^oC and use the following thermo-physical properties of a

$\rho = 1.25 Kg/m^3$; k

$= 0.022 W/m-^oc$;

$v = 14.15\times10^{-6} m^2/s$;

$c_p = 1000 J/KgK$

Solution:

Flow Reynolds number

$$Re_1 = \frac{\rho u_\infty l}{\mu} = \frac{u_\infty l}{v} = \frac{80\times1}{14.15\times10^{-6}} = 5.65\times10^6$$

For turbulent boundary layer, the boundary layer thickness is prescribed by the relation, $\frac{\delta}{l} = \frac{0.37}{(Re_1)^{0.2}}$

$$\therefore \delta = \frac{0.37}{\left(5.65\times10^6\right)^{0.2}}\times1 = 0.01655m = 16.55mm$$

(b) Prandtl number

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$$Pr = \frac{\mu c_p}{k} = \frac{\rho v c_p}{k} = \frac{1.25 \times 14.15 \times 10^{-6} \times 1000}{0.022}$$
$$= 0.804$$
$$\overline{Nu} = \frac{\overline{h} l}{k} = 0.036 \left(Re_1\right)^{0.8} \left(Pr\right)^{0.33}$$
$$= 0.036 \times (5.65 \times 10^6)^{0.8} (0.804)^{0.33} = 8437$$
$$\overline{h} = \overline{Nu} \frac{k}{l} = 8437 \times \frac{0.022}{1} = 185.61 W/m^2 - deg$$
$\therefore$ Heat flow from the plate (one side)
$$= kA\Delta t = 185.61 \times (1 \times 1) \times (50 - 10) = 7424.5 W$$

Example:10

Air at atmospheric pressure and 20^oC flows past a flat plate with a velocity of 4 m/s. The plate is 30 cm wide, is heated uniformly throughout its entire length and is maintained at a surface temperature of 60^oC. Make calculations for the following parameters at 40 cm distance from the leading edge:

a) local and average friction coefficient
b) local & average heat transfer coefficient

Solution:

a) local friction coefficient at $x = 0.4m$ is given by

$$C_{fx} = \frac{0.664}{\sqrt{Re_x}} = \frac{0.664}{\sqrt{9.41 \times 10^4}} = 0.00216$$

Average friction coefficient
$$C_f = 2C_{fx} = 2 \times 0.00216 = 0.00432$$

b) The local Nusselt number at $x = l = 0.4m$ is given by

$$Nu_x = h_x \frac{x}{k} = 0.332 \left(Re_x\right)^{0.5} \left(Pr\right)^{0.33}$$
$$= 0.332 (9.41 \times 10^4)^{0.5} (0.742)^{0.33} = 92.27$$

Local heat transfer coefficient
$$h_x = Nu_x \times \frac{k}{x} = 92.27 \times \frac{0.0272}{0.4}$$
$$= 6.274 W/m^2K$$
$\therefore$ Average heat transfer coefficient
$$\overline{h} = 2h_x = 2 \times 6.274 = 12.548 W/m^2K$$

Example:11

Atmospheric air at 25^oC flows with a velocity of $40m/s$ past a flat plate 0.75m long and 0.4 m wide. The surface of the plate is maintained at 275^oC. Estimate the total heat transferred and the total drag force on one side of the plate.

Solution:

At the mean film temperature,
$$t_f = (275 + 25)/2 = 150^o$$
the thermo-physical properties of air are:
$$\rho = 0.834 kg/m^3; \mu = 23.84 \times 10^{-6} kg/ms$$
$$c_p = 1017 J/kg - ^o c \text{ and}$$
$$k = 34.59 \times 10^{-3} W/m - ^o c$$

From these properties, the relevant parameters are:

$$Pr = \frac{\mu c_p}{k} = \frac{23.84 \times 10^{-6} \times 1017}{34.59 \times 10^{-3}} = 0.70$$
$$Re_1 = \frac{Vl\rho}{\mu} = \frac{40 \times 0.75 \times 0.834}{23.84 \times 10^{-6}} = 1.05 \times 10^6$$

Presuming that transition occurs at critical Reynolds number of 5×10^5, the average Nusselt number is

$$Nu = \left[0.036 Re_l^{0.8} - 836\right](Pr)^{0.33} +$$
$$= \left[0.036 \times (1.05 \times 10^6)^{0.8} - 836\right] \times (0.70)^{0.33}$$
$$= 1356$$
$$h = Nu \times \frac{k}{l} = \frac{1356 \times 34.59 \times 10^{-3}}{0.75}$$
$$= 62.54 W/m^2 - ^o c$$

and heat transfer from one side of the plate is,
$$hA\Delta T = 62.54 \times (0.75 \times 0.4) \times (275 - 25) = 6690 W$$

Further skin friction or drag co-efficient is

$$C_D = \frac{0.072}{(Re_l)^{0.2}} - \frac{1670}{Re_l}$$
$$= \frac{0.072}{(1.05 \times 10^6)^{0.2}} - \frac{1670}{1.05 \times 10^6} = 0.0029$$

and drag force on one side of the plate is
$$= C_D \times \frac{1}{2} \rho V^2 \times \text{area of plate surface}$$

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$$= 0.0029 \times \left(\frac{1}{2} \times 0.834 \times 40^2 \right) \times (0.75 \times 0.4)$$

$$= 0.58\text{N}$$

Example: 12

In a gas turbine system, hot gases at 1000°C flow at 75m/s past the surface of combustion chamber which is at a uniform temperature of 300°C. What would be the heat loss from the gases to the combustion chamber which can be idealized as a flat plate measuring $100\text{cm} \times 50\text{cm}$? The flow is parallel to the 100 cm side and the transition from laminar to turbulent conditions is anticipated to be abruptly at a critical Reynolds number of $\text{Re}_c = 5 \times 10^5$

Take the following properties of gas:

$\rho = 0.496\text{kg/m}^3$; $v = 93.5 \times 10^{-6}\,\text{m}^2/\text{s}$

$k = 0.0744\text{W/mK}$ and $\text{Pr} = 0.625$

Solution:

The average value of Nusselt number over a flat plate when both laminar and turbulent boundary layers are present is given by:

$$\text{Nu} = \text{Pr}^{0.33}\left[0.664(\text{Re}_l)^{0.5} + 0.036\left\{(\text{Re}_l)^{0.8} - (\text{Re}_c)^{0.8}\right\}\right]$$

When transition occurs at a critical Reynolds number of $\text{Re}_c = 5 \times 10^5$, the above expression simplifies to

$$\overline{\text{Nu}} = \text{Pr}^{0.33}\left[0.036(\text{Re}_1)^{0.8} - 836\right]$$

The Reynolds number at the plate end is

$$\text{Re}_1 = \frac{l\rho u_\infty}{\mu} = \frac{l u_\infty}{\nu} = \frac{1 \times 75}{93.5 \times 10^{-6}} = 8.02 \times 10^5$$

Apparently the boundary layer becomes turbulent and the above correlation applies.

$$\overline{\text{Nu}} = (0.625)^{0.33}\left[0.036(8.02 \times 10^5)^{0.8} - 836\right]$$

$$= 914.33$$

$$\overline{h} = \overline{\text{Nu}} \times \frac{k}{l} = 914.33 \times \frac{0.744}{1} = 68.02\text{W/m}^2\text{K}$$

Heat loss $Q = h\,A\,\Delta t$

$$= 68.02 \times (1 \times 0.5) \times (1000 - 300) = 23809\text{W}$$

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GATE QUESTIONS

Q.1 Heat transfer by natural convection is enhanced in systems with
(A) High viscosity
(B) High coefficient of thermal expansion
(c) Low temperature gradients
(d) Low density change with temperature

[GATE-2001]

Q.2 The Sieder-Tate correlation for heat transfer in turbulent from in a pipe gives $Nu \propto Re^{0.8}$, where Nu is the Nusselt number and Re is the Reynolds number for the flow. Assuming that this relation is valid, the heat transfer coefficient varies with pipe diameter D as

[GATE-2001]

(A) $D^{-1.8}$ (B) $D^{-0.2}$
(C) $D^{0.2}$ (D) $D^{1.8}$

Q.3 A 200 W heater has a spherical casing of diameter 0.2 m. The heat transfer co-efficient for conduction and convection from the casing to the ambient air is obtained from $Nu = 2 + 0.6\,Re^{1/2}\ Pr^{1/3}$, with Re=$10^4$ and Pr =0.69. The temperature of the ambient air is 30 °C and the thermal conductivity of air is k = 0.02 W/m K.

(a) Find the heat flux from the surface at steady state.
(b) Find the steady state surface temperature of the casing.
(c) Find the temperature of the casing at steady state for stagnant air. Why is this situation physically infeasible?

[GATE-2001]

Q.4 1000 kg of liquid at 30 °C in a well-stirred vessel has to be heated to 120 °C using immersed coils carrying condensing steam at 150 °C. The area of the steam coils is 1.2 m^2 and overall heat transfer coefficient to the liquid is 1500 W/m^2 °C . Assuming negligible heat loss to surrounding and specific heat capacity of the liquid to be 4 kJ/kg °C, the time taken for the liquid to reach desired temperature will be
(A) 15 min (B) 22 min
(C) 44 min (D) 51 min

[GATE-2002]

Q.5 Three solid objects of the same material and of equal masses a sphere, a cylinder (length = diameter) and a cube are at 500 °C initially. These are dropped in a quenching bath containing a large volume of cooling oil each attaining the bath temperature eventually. The time required for 90% change of temperature is smallest for
(A) Cube (B) Cylinder
(C) Sphere (D) Equal for all the three

[GATE-2003]

Q.6 A dilute aqueous solution is to be concentrated in an evaporator system. High pressure steam is available. Multiple effect evaporator system is employed because
(A) Total heat transfer area of all the effects is less than that in a single effect evaporator system
(B) Total amount of vapour produced per kg of feed steam in a multi-effect system is much higher than in single effect
(C) Boiling point elevation in a single effect system is much higher than that in any effect in a multi-effect system
(D) Heat transfer coefficient in a single effect is much lower than that in any effect in a multi-effect system

[GATE-2003]

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Q.7 In forced convection, the Nusselt number (Nu) is a function of
(A) Re and Pr (B) Re and Gr
(C) Pr and Gr (D) Re and Sc
[GATE-2004]

Q.8 A metal ball of radius 0.1 m at a uniform temperature of 90 °C is left in air at 30 °C. The density and the specific heat of the metal are 3000 kg/m^3 and 0.4 kJ/kg-K, respectively. The heat transfer coefficient is 50 W/m^2-K. Neglecting the temperature gradient inside the ball, The time taken (in hour) for the ball to cool to 60 °C is
(A) 555 (B) 55.5
(C) 0.55 (D) 0.15
[GATE-2004]

Q.9 It is desired to concentrate a 20 % salt solution (20 kg of salt in 100 kg of solution) to a 30 % salt solution in an evaporator. Consider a feed of 300 kg/min at 30 °C. The boiling point of the solution is 110 °C, the latent heat of vaporization is 2100 kJ/kg, and the specific heat of the solution is 4 kJ/kg-K. The rate at which heat has to be supplied (in kJ/min) to the evaporator is
(A) 3.06×10^5 (B) 6.12×10^5
(C) 7.24×10^5 (D) 9.08×10^5
[GATE-2004]

Q.10 An electrically heated element is submerged in a pool of water at its saturation temperature. As the temperature of the element increases, the maximum heat transfer coefficient is observed
(A) In the free convection regime
(B) Between the nucleate boiling and partial nucleate boiling mixed with unstable film boiling regimes
(C) In the incipient nucleate boiling regime
(D) In the stable film boiling regime without significant radiation effects.
[GATE-2005]

Q.11 In film type condensation of liquid along a vertical tube, the thickness of the condensate layer increases towards the bottom This implies that the local heat transfer co-efficient.
(A) Increases from top to bottom
(B) Decreases from top to bottom
(C) Remains constant from top to bottom
(D) First increases and then decreases from top to bottom
[GATE-2005]

Q.12 Consider the flow of a gas with density 1 kg/m^3, viscosity 1.5×10^{-5} kg/m-s, specific heat C_p = 846 J/kg–K and thermal conductivity K = 0.01665 W/mK, in a pipe of diameter D = 0.01 m and length L = 1 m and assume the viscosity does not change with temperature. The Nusselt number for a pipe with (L/D) ratio greater than 10 and Reynolds number greater than 20000 is given by $Nu = 0.026\ Re^{0.8}\ Pr^{1/3}$,while the Nusselt number for a laminar flow for Reynolds number less than 2100 and $\left(Re\ Pr\ \frac{D}{L}\right) < 10$ is $Nu = 1.86\left[Re^{0.8}Pr(D/L)\right]^{1/3}$. If the gas flows through the pipe with an average velocity of 0.1 m/s, the heat transfer coefficient is
(A) 0.68 W/m^2-K (B) 1.14 W/m^2-K
(C) 2.47 W/m^2-K (D) 24.7 W/m^2-K
[GATE-2005]

Common Data Question
A liquid of mass 7 kg and specific heat 4 kJ/kg °C is contained in a cylindrical heater of diameter 0.15 m and height

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0.40 m. The cylindrical surface of the heater is exposed to air at 25 °C while the end caps are insulated, so that the heat transfer takes place only through the cylindrical surface.
Thickness of the wall of the heater=2 mm
The wall thermal conductivity=10W/mK
The heat transfer coefficient in the liquid = 100 W/m^{2}K
The heat transfer coefficient in air = 10 W/m^2–K
The liquid is initially maintained at a temperature of 75 °C. At time t=0, the heater is switched off and the temperature of the liquid in the heater decreases due to heat loss across the cylindrical surface.

Q.13 What is the overall heat transfer coefficient in W/m^2-K?

[GATE-2005]

(A) 1 (B) 4.04
(C) 9.07 (D) 10

Q.14 What is the time required for the temperature of the liquid to reduce to 50 °C after the heater is switched off, assuming lumped system analysis is valid?
(A) 7.874 ×10^3 s (B) 11.346×10^3 s
(C) 16.828×10^3 s (D) 23.213×10^3 s

[GATE-2005]

Q.15 Water enters a thin walled tube (L=1 m, D=3 mm) at an inlet temperature of 97 °C and mass flow rate 0.015 kg/s. The tube wall is maintained at a constant temperature of 27 °C.
Given the following data for water,
Density = 1000 kg/m^3
Viscosity (μ) = 489×10^{-6} N s/m^2
Specific heat Cp = 4184 J/kg–K
Inside heat transfer coefficient(h)= 12978 W/m^2–K
The outlet temperature of water in °C is
(A) 28 (B) 37
(C) 62 (D) 96

[GATE-2007]

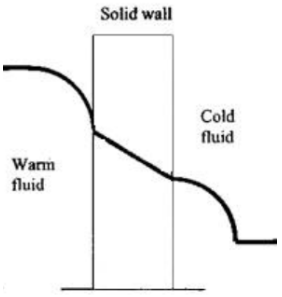
Q.16 A metallic ball [ρ = 2700 kg/m^3 and C_p =0.9 kJ/kg–°C] of diameter 7.5 cm is allowed to cool in air at 25 °C. When the temperature of the ball is 125 °C, it is found to cool at the rate of 4 °C/min. If thermal gradients inside the ball are neglected, the heat transfer coefficient (in W/m^2-°C) is
(A) 2.034 (B) 20.34
(C) 81.36 (D) 203.4

[GATE-2008]

Q.17 Hot liquid is flowing at a velocity of 2 m/s through a metallic pipe having an inner diameter of 3.5 cm and length of 20m. The temperature at the inlet of the pipe is 90 °C.
Following data is given for liquid at 90 °C
Density = 950 kg/m^3;
Specific heat = 4.23 kJ/kg–°C
Viscosity=2.55×10^{-4} kg/m-s
Thermal conductivity = 0.685 W/m–°C
The heat transfer coefficient[in kW/m^2-°C] inside the tube is
(A) 222.22 (B) 111.11
(C) 22.22 (D) 11.11

[GATE-2008]

Q.18 The temperature profile for heat transfer from one fluid to another separated by a solid wall is
(A)



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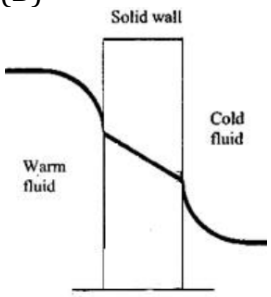
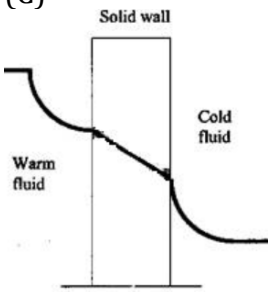
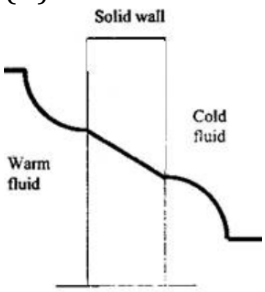
(B)



(C)



(D)



[GATE-2008]

Q.19 During the transient convective cooling of a solid object, Biot number $\rightarrow 0$ indicates

(A) Uniform temperature throughout the object

(B) Negligible convection at the surface of the object

(C) Significant thermal resistance within the object

(D) Significant temperature gradient within the object

[GATE-2009]

Q.20 In film type condensation over a vertical tube, local heat transfer coefficient is

(A) inversely proportional to local film thickness

(B) directly proportional to local film thickness

(C) equal to local film thickness

(D) independent of local film thickness

[GATE-2011]

Q.21 An aqueous sodium chloride solution(10 wt%) is fed into a single effect evaporator at a rate of 10000kg/hr. It is concentrated to a 20 wt% sodium chloride solution. The rate of consumption of steam in the evaporation is 8000 kg/hr. The evaporator capacity (kg/hr) and economy are

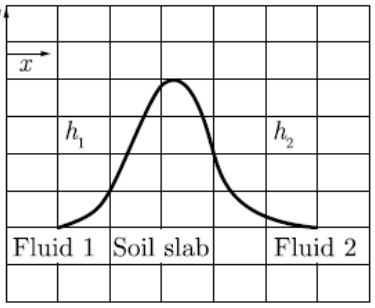
(A) 5000, 0.625 (B) 10000, 0.625

(C) 5000, 1.6 (D) 10000, 1.6

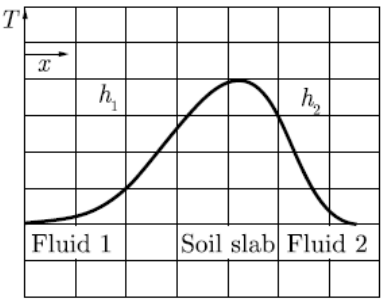
[GATE-2011]

Q.22 74 Heat is generated uniformly within a solid slab. The slab separates fluid-1 from fluid-2. The heat transfer coefficients between the solid slab and the fluid are h_1 and $h_2(h_2>h_1)$ respectively. The steady state temperature profile (T vs x) for one-dimensional heat transfer is correctly shown by,

(A)



(B)



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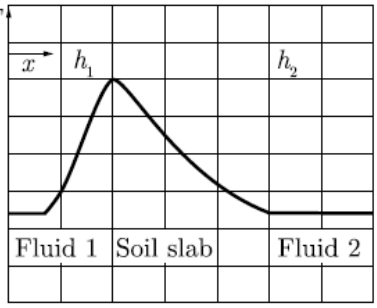
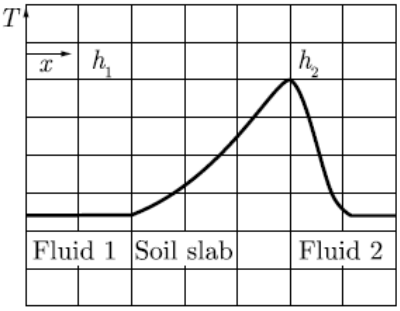
(C)



(D)



[GATE-2011]

Q.23 For heat transfer across a solid-fluid interface, which one of the following statements is NOT true when the Biot number is very small compared to 1 ?

(A) Conduction resistance in the solid is very small compared to convection resistance in the fluid

(B) Temperature profile within the solid is nearly uniform

(C) Temperature drop in the fluid is significant

(D) Temperature drop in the solid is significant

[GATE-2012]

Q.24 A solid sphere with an initial temperature T_i is immersed in a large thermal reservoir of temperature T_0. The sphere reaches a steady temperature after a certain time t_1. If the radius of the sphere is doubled, the time required to reach steady-state will be

(A) $\frac{t_1}{4}$ (B) $\frac{t_1}{4}$

(C) $2t_1$ (D) $4t_1$

[GATE-2012]

Q.25 If the Nusselt number (Nu) for heat transfer in a pipe varies with Reynolds number (Re) as $Nu \propto Re^{0.8}$ then for constant average velocity in the pipe, the heat transfer co-efficient varies with the pipe diameter D as

(A) $D^{-1.8}$ (B) $D^{-0.2}$

(C) $D^{1.2}$ (D) $D^{1.8}$

[GATE-2012]

Q.26 In a pool boiling experiment, the following phenomena were observed.

P. Natural Convention

Q. Film boiling

R. Transition Boiling

S. Nucleate boiling

What was CORRECT sequence of their occurrence?

A)P, Q, R, S

B)S,R,Q,P

C)Q,R,P,S

D)P,S,R,Q

[GATE-2013]

Q.27 Steam economy of a multiple effect evaporator system is defined as

(A) kilogram of steam used per hour

(B) kilogram of steam consumed in all effects for each kilogram of steam fed

(C) kilogram of steam used in all the effects for each kilogram of water vaporized per hour

(D) kilogram of water vaporized from all the effects for each kilogram of steam fed to the first effect

[GATE-2014]

Q.28 In the figure below, the temperature profiles of cold and hot fluids in counter current double pipe heat exchangers (in different modes of operation) are shown on the left. For each case, match the heat exchange process for the fluid represented by the

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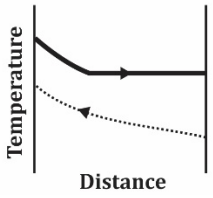
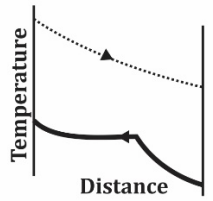
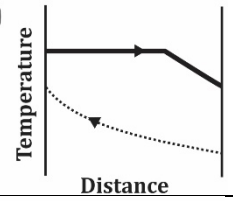
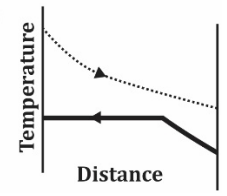
bold curve with the options given on the right.

Table-I	Table-II
(I) 	(P) Heating of sub-cooled feed to superheated vapour
(II) 	(Q) Condensation of superheated vapour
(III) 	(R) Boiling of sub-cooled liquid
(IV) 	(S) Condensation of saturated vapour followed by sub-cooling

(A) I-P, II-Q, III-R, IV-S
(B) I-P, II-Q, III-S, IV-R
(C) I-Q, II-P, III-S, IV-R
(D) I-Q, II-S, III-P, IV-R

[GATE-2015]

Q.29 A heated solid copper sphere (of surface area A and volume V) is immersed in a large body of cold fluid. Assume the resistance to heat transfer inside the sphere to be negligible and the heat transfer coefficient (h), density (ρ), heat capacity (C), and thermal conductivity (k) to be constant. Then, at time *t*, the temperature difference between the sphere and the fluid is proportional to:

(A) $\exp\left[-\frac{hA}{\rho VC}t\right]$

(B) $\exp\left[-\frac{\rho VC}{hA}t\right]$

(C) $\exp\left[-\frac{4\pi k}{\rho CA}t\right]$

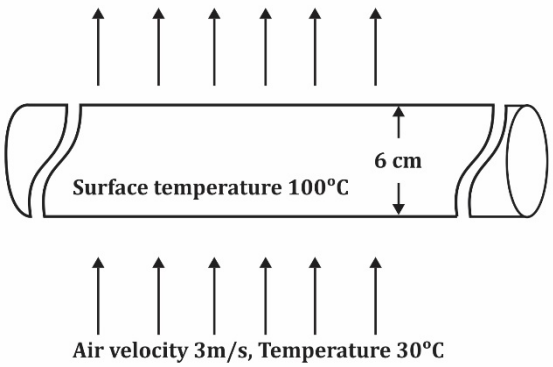
(D) $\exp\left[-\frac{\rho CA}{4\pi k}t\right]$

[GATE-2015]

Q.30 Air is flowing at a velocity of 3 m/s perpendicular to a long pipe as shown in the figure below. The outer diameter of the pipe is d = 6 cm and temperature at the outside surface of the pipe is maintained at 100°C. The temperature of the air far from the tube is 30°C.

Data for air: Kinematic viscosity, ν = 18 × 10^{-6} m^2/s, Thermal conductivity, k= 0.03 W/(m-K)

Using the Nusselt number correlation: $Nu = \frac{hd}{k} = 0.024 \times \mathrm{Re}^{0.8}$, The rate of heat loss per unit length (W/m) from the pipe to air (up to one decimal place) is _____



[GATE-2015]

Q.31 Steam at 100 °C is condensing on a vertical plate. The condensate flow is laminar. The average Nusselt numbers are Nu_1 and Nu_2, when the plate

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temperatures are 10 °C and 55 °C, respectively. Assume the physical properties of the fluid and steel to remain constant within the temperature range of interest. Using the Nusselt equations for film type condensation, what is the value of the ratio $\frac{Nu_2}{Nu_1}$?

(A) 0.5 (B) 0.84

(C) 1.19 (D) 1.41

[GATE-2016]

Q.32 A jacketed stirred tank with a provision for heat removal is used to mix sulphuric acid and water in a steady state flow process. H_2SO_4 (I) enters at a rate of 4 kg/h at 25°C and H_2O (II) enters at a rate of 6 kg/h at 10°C. The following data are available.

Specific heat capacity of water = 4.2 $kJ\,kg^{-1}K^{-1}$.

Specific heat capacity of aqueous solution of 40 mass% $H_2SO_4 = 2.8\,kJ\,(kg\ solution)^{-1}K^{-1}$

Assume the specific heat capacities to be independent of temperature.

Based on reference states of H_2SO_4 (I) and H_2O (II) at 25°C, the heat of mixing for aqueous solution of 40 mass % H_2SO_4 = -650 kJ $(kg\ H_2SO_4)^{-1}$.

If the mixed stream leaves at 40°C, what is the rate of heat removal (in kJ/h)?

(A) 1802 (B) 2558

(C) 5702 (D) 6458

[GATE-2016]

Q.33 A fluid flows over a heated horizontal plate maintained at temperature T_w. The bulk temperature of the fluid is T_∞. The temperature profile in the thermal boundary layer is given by:

$$T = T_w + (T_w - T_\infty)\left[\frac{1}{2}\left(\frac{y}{\delta_t}\right)^3 - \frac{3}{2}\left(\frac{y}{\delta_t}\right)\right],$$

$$0 \le y \le \delta_t$$

Here, y is the vertical distance from the plate, δ_t is the thickness of the thermal boundary layer and k is the thermal conductivity of the fluid.

The local heat transfer coefficient is given by

(A) $\frac{k}{2\delta_t}$

(B) $\frac{k}{\delta_t}$

(C) $\frac{3}{2}\frac{k}{\delta_t}$

(D) $2\frac{k}{\delta_t}$

[GATE-2017]

Q.34 In nucleate boiling, the pressure inside a bubble is higher than the pressure of the surrounding liquid. Assuming that both the liquid and vapour are saturated, the temperature of the liquid will ALWAYS be

(A) At 100°C

(B) Lower than the temperature of the vapour

(C) Equal to the temperature of the vapour

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(D) Higher than the temperature of the vapour

[GATE-2017]

Q.35 Segmental baffles in a 2-4 shell and tube heat exchanger

(A) Changes the flow pattern of the tube side fluid and increase the overall heat transfer co-efficient

(B) Increase the heat transfer coefficient in the shell side and support the tubes

(C) Help to reduce the thermal expansion of the tubes and increase the heat transfer coefficient in the tube side

(D) Increase the number of passes in the shell side and increase the heat transfer coefficient in the tube side

[GATE-2018]

Q.36 An insulated storage tank contains 1000 kg liquid of specific heat 10 $\mathrm{kJ\,kg^{-1}K^{-1}}$. The liquid is heated by saturated steam, condensing in a helical coil at a temperature of 180°C. The heat transfer area of the coil is 0.1 m^2. If the overall heat transfer coefficient is constant at 1000 $\mathrm{W\,m^{-2}\,K^{-1}}$, then the time (in hours) required to raise the temperature of the liquid in the tank from 20°C to 80°C is _____ (rounded off to second decimal place).

[GATE-2018]

Q.37 Vapour bubbles are formed in the nucleate boiling regime at a frequency of 10 bubbles per second per nucleation site. There are 100 nucleation sites per m^2 of heating area. The latent heat of vaporization and the density of vapour under the operating condition are 1000 kJ/kg and 1 kg/m^3 respectively. The diameter of each bubble is 10^{-3} m. Assume that the entire heat supplied is used for vapour generation. The heat flux (in Watt per m^2 of heating area) is _____ (rounded off to third decimal place).

[GATE-2018]

Q.38 Economy of evaporators used for concentrating sugarcane juice is

(A) $\dfrac{\text{kg of concentrated juice produced}}{\text{kg of steam supplied}}$

(B) $\dfrac{\text{kg of steam supplied}}{\text{kg of sugarcane juice fed}}$

(C) $\dfrac{\text{kg of water vaporized}}{\text{kg of steam supplied}}$

(D) $\dfrac{\text{kg of sugarcane juice fed}}{\text{kg of water vaporized}}$

[GATE-2018]

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ANSWER KEY:

1	2	3(a)	3(b)	3(c)	4	5	6	7	8	9	10	11	12
(B)	(B)	1592.35	319.2	7987	(D)	(A)	(B)	(A)	0.15	(A)	(B)	(B)	(C)
13	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**
(C)	(B)	(B)	(B)	(D)	(B)	(A)	(A)	(A)	(A)	(D)	(C)	(B)	(D)
27	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**		
(D)	(C)	(A)	250.9	(C)	(A)	(C)	(B)	(B)	13.05	0.533	(C)		

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EXPLANATIONS

Q.1 (B)

High coefficient of thermal expansion.

Q.2 (B)

$$\begin{pmatrix}\text{Sieder}-\text{tata}\\ \text{correlation}\end{pmatrix} \Rightarrow N_u \propto Re^{0.8}\,(\text{Given})$$

$$\frac{hD}{k} \propto \left(\frac{\rho VD}{\mu}\right)^{0.8}$$

$$h \propto \frac{D^{0.8}}{D}$$

$$\Rightarrow h \propto D^{-0.2}$$

Q.3

Given

$Q = 200W$

Spherical casing $d = 0.2m$

$N_u = 2 + 0.6\,Re^{1/2}\,Pr^{1/3}$

$Re = 10^4$

$Pr = 0.69$

$T_\infty = 30^\circ C$

$k_{air} = 0.02\dfrac{W}{m\ K}$

$N_u = 2 + (0.6)\left(10^4\right)^{1/2}(0.69)^{1/3}$

$\dfrac{h(0.2)}{0.02} = 55.01$

$h = 5.50\dfrac{W}{m^2K}$

(a) (1592.35)

$$\frac{Q}{A} = \frac{200}{\pi d^2} = \frac{200}{\pi(0.2)^2} = 1592.35\frac{W}{m^2}$$

(b) (319.2)

$Q = hA\Delta T$

$\dfrac{Q}{A} = h\ \Delta T$

$1529.35 = (5.50)(T_c - 30)$

$T_c = 319.2\ ^\circ C$

(c) (7987)

At stagnant air, $Nu = 2$

$\dfrac{h(0.2)}{0.02} = 2$

$h = 0.2\dfrac{W}{m^2K}$

$\dfrac{Q}{A} = h\ \Delta T$

$(1529.35) = (0.2)(T_c - 30)$

$T_c = 7987\ ^\circ C$

Situation is physically infeasible, because temperature gradient at the surface starts the convection current. So air do not stay stagnant at all.

Q.4 (D)

$m = 1000\ kg$

$T_1 = 30$

$T_2 = 120$

$Ts = 150$

$A = 1.2\ m^2$

$U = 1500\dfrac{W}{m^2\,^\circ C}$

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$C_p = 4000 \ \frac{J}{kg\ °C}$

$$\ln\left[\frac{T_S - T_2}{T_S - T_1}\right] = -\frac{h\,A\,t}{\rho V C_p}$$

$$\ln\left[\frac{150-120}{150-30}\right] = -\frac{1500\times1.2\times t}{1000\times4000}$$

t = 3080.11 sec

t = 51.11 min

Q.5 (A)

$A_{sphere} = \pi D^2$

$A_{cylinder} = \pi D^2$

$A_{cube} = 6D^2$

The time required for change of temperature is smallest for largest surface Area.

Here, Area of cube is maximum So, time required minimum is for Cube.

Q.6 (B)

Total amount of vapour produced per feed steam in a multi effect system is much higher than a single effect.

Q.7 (A)

Re and Pr

Q.8 (0.15)

Lump Parameter model

$T = 60\ ^oC$

$T_i = 90\ ^oC$

$T_\infty = 30\ ^oC$

$h = 50 \frac{W}{m^2\ K}$

$C_p = 400 \left(\frac{J}{kg\ K}\right)$

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp\left[\frac{-h\,A\,t}{\rho\,V\,C_p}\right]$$

$$\left[\frac{60-30}{90-30}\right] = \exp\left[\frac{50(3)t}{3000(0.1)400}\right]$$

$$\ln(0.5) = \frac{-150\ t}{(300)(400)}$$

$$\frac{(0.693)(120000)}{150} = t$$

t = 554.51 sec

t = 0.15 hour

Q.9 (A)

V = 100 Kg solution

$\dot{m} = 300 \ \frac{kg}{min}$

$T_1 = 30^oC$

$T_2 = 110^oC$

$\lambda = 2100 \ \frac{kJ}{kg}$

$C_p = 4 \frac{kJ}{kg\ K}$

$$Q = \dot{m} C_p \Delta T + V\lambda$$

$$= (300)(4)(110-30) + (100\times2100)$$

$$Q = 3.06\times10^5 \ \frac{kJ}{min}$$

Q.10 (B)

Between the nucleate boiling and partial nucleate boiling mixed with unstable film boiling regime.

Q.11 (B)

Decreases from Top to bottom

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Q.12 (C)

Given:

$\rho = 1\left(\frac{kg}{m^3}\right)$ $\quad D = 0.01 m$

$\mu = 1.5\times10^{-5}\left(\frac{kg}{ms}\right)$ $\quad L = 1\ m$

$C_p = 846\ \frac{J}{kg\ K}$ $\quad v = 0.1\ \frac{m}{s}$

$k = 0.01665\ \frac{W}{m\ K}$

$Nu = 0.026\ Re^{0.8}\ Pr^{\frac{1}{3}}$ for $\frac{L}{D} > 10$ and $Re > 20,000$

$Nu = 1.86\left[Re\ Pr\left(\frac{D}{L}\right)\right]^{\frac{1}{3}}$ for $Re < 2100$ $\left[Re\,Pr\frac{D}{L}\right] < 10$

$Re = \frac{\rho VD}{\mu} = \frac{(1)(0.1)(0.01)}{(1.5\times10^{-5})} = 66.66 < 2100$

$\frac{\mu C_P}{k} = \frac{(1.5\times10^{-5})(846)}{(0.01665)} = 0.762$

$\frac{L}{D} = \frac{1}{0.01} = 100$

$\left[Re\,Pr\,\frac{D}{L}\right] = \left[(66.66)\,(0.762)\left(\frac{1}{100}\right)\right]\ 0.508 < 10$

So, $Nu = 1.86\left[Re\ Pr\ \frac{D}{L}\right]^{\frac{1}{3}}$

$\frac{h(0.01)}{(0.01665)} = (1.86)\,(0.508)^{\frac{1}{3}}$

$h = 2.47\ \frac{W}{m^2K}$

Q.13 (C)

Given:

$m = 7\,kg$ $\quad x = 2\,mm = 0.002\ m$

$C_p = 4000\ \frac{J}{kg\ K}$ $\quad k = 10\ \frac{W}{m\ K}$

$d = 0.15\ m$ $\quad h_{liquid} = 10\ \frac{W}{m^2\ K}$

$H = 0.40\,m$ $\quad h_{air} = 10\ \frac{W}{m^2\ K}$

$T_{air} = 25\ ^oC$ $\quad T_1 = 75\ ^oC$

$\frac{1}{U} = \frac{1}{h_{liquid}} + \frac{x}{k} + \frac{1}{h_{air}}$

$= \frac{1}{10} + \frac{0.002}{10} + \frac{1}{100}$

$U = 9.07\ \frac{W}{m^2K}$

Q.14 (B)

$T_2 = 50^oC$

For Lumped Heat System

$\ln\left(\frac{T_2 - T_\infty}{T_1 - T_\infty}\right) = -\frac{h\,A\,t}{\rho\,V\,C_p}$

$\ln\left(\frac{50-75}{25-75}\right) = \frac{(10)(\pi DH)t}{(7)(4000)}$

$\frac{0.693(7)(4000)}{10\,\pi(0.15)(0.40)} = t$

$t = 11.36\times10^3\ sec$

Q.15 (B)

$L = 1\ m$

$D = 0.003\,m$

$T_i = 97\ ^oC$

$\dot{m} = 0.015\ \frac{kg}{s}$ $\quad \mu = 487\times10^{-9}\ \frac{N\,s}{m^2}$

$T_{wall} = 27\ ^oC$ $\quad C_p = 4184\ \frac{J}{kg\ K}$

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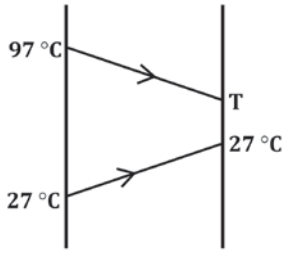
$\rho = 1000 \frac{kg}{m^3}$ $\quad h = 12978 \frac{W}{m^2\ K}$

$A = \pi DL = (3.14)(0.003)(1) = (9.42 \times 10^{-3})$



$$\Delta T_{LMTD} = \frac{(97-27)-(T-27)}{\ln \frac{(97-27)}{(T-27)}}$$

$m\ C_p\ \Delta T = U\ A\ \Delta T_{LMTD}$

$$(0.015)(4184)(97-T) = (12978)(9.42\times10^{-3})\left[\frac{97-T}{\ln\left(\frac{70}{T-27}\right)}\right]$$

$$\ln\left[\frac{70}{T-27}\right] = 1.94$$

$$\frac{70}{T-27} = 7.014$$

$T_o = 36.97\ ^oC$

Q.16 (B)

$\rho_P = 2700 \frac{kg}{m^3}$

$C_P = 0.9 \frac{kJ}{kg\ ^oC}$

$d_p = 7.5\ cm = 0.075\ m$

$T_{air} = 25\ ^oC$

$T_1 = 125\ ^oC$

$\frac{dT}{dt} = 4\frac{^oC}{min} = \frac{4}{60}\frac{^oC}{sec}$

Lumped parameter energy Balance :

$$h_A(T - T_{air}) = m C_p \frac{dT}{dt}$$

$$h = \frac{\rho V C_P \left(\frac{dT}{dt}\right)}{A(T - T_{air})}$$

$$= \frac{(2700)(0.9\times10^3)}{\pi(0.075)^2(125-25)} \frac{4}{(60)} \frac{(\pi)}{6}(0.075)^3$$

$h = 20.25 \frac{W}{m^2\ ^oC}$

Q.17 (D)

$v = 2\ m/s$

$d = 3.5\ cm = 0.035\ m$

$k = 0.685 \frac{W}{m\ ^oC}$

$L = 20\ m$

$T = 90\ ^oC$

$\rho = 950 \frac{kg}{m^3}$

$C_P = 4.23 \times 10^3 \frac{J}{kg\ ^oC}$

$\mu = 2.55 \times 10^{-4} \frac{kg}{m\text{-}s}$

$$Re = \frac{\rho v d}{\mu} = \frac{(950)(2)(0.035)}{(2.55\times10^{-4})} = 260784$$

So, Turbulent Flow,

$$Pr = \frac{\mu C_P}{k} = \frac{(2.55\times10^{-4})(4.23\times10^3)}{(0.685)} = 1.574$$

By using Dittus-Bolter equation

$Nu = 0.023\ Re^{0.8}\ Pr^{0.3}$

$$\frac{hD}{k} = 0.023\ (260784)^{0.8}\ (1.145)^{0.3}$$

$$\frac{h(0.035)}{(0.685)} = 0.023\ (21529.04)(1.145)$$

$$= \frac{566.96\ (0.685)}{(0.035)}$$

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$$h = 11.11 \frac{W}{m^2\ ^oC}$$

Q.18 (B)

Q.19 (A)

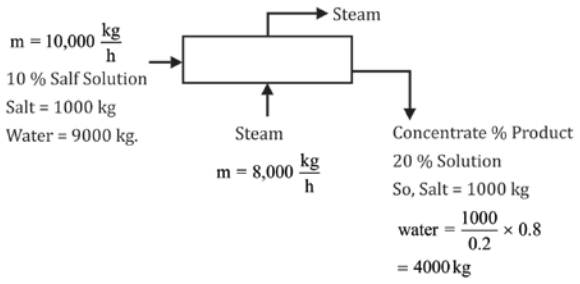
Q.20 (A)

Film Condensation on a vertical tube

$$h_x \propto \frac{k_f}{\delta}$$

$$\text{So, } h_x \propto \frac{1}{\delta}$$

Q.21 (A)



$$\begin{pmatrix}\text{Capacity of}\\ \text{Evaporator}\end{pmatrix} = \begin{pmatrix}\text{kg of water}\\ \text{Evaporator}\end{pmatrix}$$

$$= 9000 - 4000$$

$$= 5000 \text{ kg}$$

$$\text{Economy of Evaporator} = \frac{\text{kg water Evaporated}}{\text{kg steam fed}}$$

$$= \frac{5000}{8000} = 0.625$$

Q.22 (A)

Q.23 (D)

Temp. drop in the Solid is significant

Q.24 (C)

Lumped heat System

$$\ln\left(\frac{T - T_\infty}{T_1 - T_\infty}\right) = \frac{h_i A t}{\rho V C}$$

$$t \propto \frac{V}{A}$$

$$t = \frac{\frac{4}{3}\pi r^3}{4\pi r^2} = \frac{r}{3}$$

Now, $r_1 = r$

$r_2 = 2r$ (given)

$$\text{So, } \frac{t_2}{t_1} \propto \frac{r_2}{r_1}$$

$$t_2 = 2t_1$$

Q.25 (B)

Nusselt number Relation :

$$Nu \propto Re^{0.8}$$

$$\frac{hD}{k} \propto \left(\frac{\rho VD}{\mu}\right)^{0.8}$$

$$h \propto \frac{D^{0.8}}{D}$$

$$h \propto D^{-0.2}$$

Q.26 (D)

Natural convection>Nucleate boiling

>Transition boiling> Film boiling

Q.27 (D)

Q.28 (C)

Q.29 (A)

Assume resistance to heat transfer inside the sphere is negligible. Temperature inside the sphere is uniform (say T)

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This is valid when $\text{Bi}=\frac{hD}{k}<0.1$

$$hA\left(T_\infty - T\right)dt = \rho V C_p dT$$

$$\int_{T_0}^{T}\frac{dT}{\left(T_\infty - T\right)} = \frac{hA}{\rho V C_p}\int_0^t dt$$

$$\left(T - T_\infty\right) = \left(T_0 - T_\infty\right)e^{\left(-\frac{hAt}{\rho V C_p}\right)}$$

$$\therefore \Delta T \propto \exp\left[-\frac{hA}{\rho V C_p}t\right]$$

Q.30 (250.9)

Given: $T = 30\,°C$

$$v = 3\frac{m}{s}$$

$$d = 6\,cm = 0.06\,m$$

$$\nu = 18\times10^{-6}\frac{m^2}{s}$$

$$k = 0.03\frac{W}{mK}$$

$$T_{air} = 30\,°C$$

$$\text{Nu} = \frac{hD}{k} = 0.024\text{Re}^{0.8}$$

$$\frac{h(0.06)}{0.03} = (0.024)\left(\frac{3\times0.06}{18\times10^{-6}}\right)^{0.8}$$

$$h = 19.091\frac{W}{m^2K}$$

$$Q = hA\Delta T$$

$$= (19.091)\times\pi(0.06)(1)\times(100-30)$$

$$Q = 250.9\frac{W}{m}$$

Q.31 (C)

For condensation on a vertical plate and laminar flow of condensate

$$\text{Nu} = 1.13\left[\frac{\rho_L(\rho_L-\rho_G)ghL^3}{\mu\, k\, \Delta T}\right]^{1/4}$$

$$\text{So, Nu} \propto \frac{1}{(\Delta T)^{1/4}}$$

$$\text{So, } \frac{Nu_2}{Nu_1} \propto \left[\frac{\Delta T_1}{\Delta T_2}\right]^{\frac{1}{4}}$$

$$= \left[\frac{100-10}{100-55}\right]^{\frac{1}{4}}$$

$$\frac{Nu_2}{Nu_1} = 2^{\frac{1}{4}} = 1.189$$

Q.32 (A)

Consider aqueous solution of 40 mass% H_2S0_4 in the tank as system for applying steady state energy balance.

Enthalpy Of entering H_2S0_4 stream

$= 4\ C_p\ (25 - 25) = 0$

Enthalpy of entering water stream

$= 6 \times 4.2 \times (10 - 25) = -378$ kJ/h

Heat of mixing for aqueous solution of 40 mass% $H_2S0_4 = -650 \times 4 = -2600$ kJ/h

Heat generated in mixing of H_2S0_4, and water = 2600 kJ/h

For steady state flow process,

Flow rate of aqueous solution leaving from tank = 4 + 6 = 10 kg/h

fraction of H_2S0_4 in leaving stream

= 4/10 = 0.4

Enthalpy of leaving aqueous solution of 40 mass% $H_2S0_4 = 10\times2.8\times(40-25)$

$= 420$ KJ/h

Let heat removal rate= Q kJ/h

Steady state energy balance provides

$0 - 378 + 2600 = 420 + Q$

or $Q = 1802$ KJ/h

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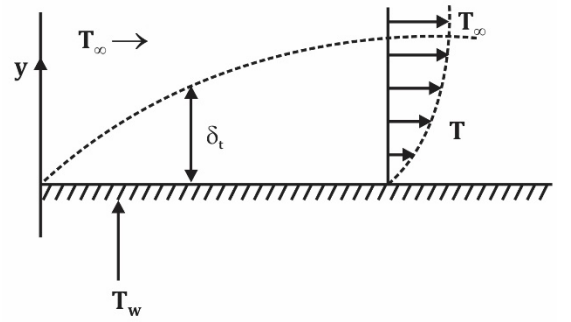
Q.33 (C)



Local heat transfer coefficient is related to Fourier equation by:

$$Q = h_x\left(T_w - T_\infty\right) = -k\left(\frac{\partial T}{\partial y}\right)\Bigg|_{y=0} \quad \ldots(1)$$

Given:

$$T = T_w + \left(T_w - T_\infty\right)\left[\frac{1}{2}\left(\frac{y}{\delta_t}\right)^3 - \frac{3}{2}\left(\frac{y}{\delta_t}\right)\right], \quad 0 \le y \le \delta_t$$

$$\frac{\partial T}{\partial y} = \left(T_w - T_\infty\right)\left[\frac{1}{2\delta_t^3}\left(3y^2\right) - \frac{3}{2\delta_t}\right]$$

$$\frac{\partial T}{\partial y}\Bigg|_{y=0} = \left(T_w - T_\infty\right)\left(-\frac{3}{2\delta_t}\right) \quad \ldots(2)$$

From (1) and (2),

$$h_x\left(T_w - T_\infty\right) = -k\left(T_w - T_\infty\right)\left(-\frac{3}{2\delta_t}\right)$$

$$h_x = \frac{3k}{2\delta_t}$$

Q.34 (B)

$P_{vap,\,sat.} > P_{liq,\,sat.}$ (given)

By Antoine equation,

When temperature increases, pressure increases

$T_{v,\,sat} > T_{L,sat}$

Temperature of liquid > Temperature of vapour

Q.35 (B)

Q.36 (13.05)

Given: $m = 1000\,kg,\ C_p = 10\frac{kJ}{kg\,K}$

$$A = 0.1\,m^2,\ U = 1000\frac{W}{m^2K}$$

$$T_1 = 20\,°C,\ T_2 = 80\,°C,\ T_{steam} = 180\,°C$$

Energy balance:

$$mC_p\frac{dT}{dt} = UA\left(T_s - T\right)$$

$$\int_{T_i}^{T_f}\frac{dT}{\left(T_s - T\right)} = \frac{UA}{mC_p}\int_0^t dt$$

$$-\ln\left(T_s - T\right)\Big|_{T_i}^{T_f} = \frac{UAt}{mC_p}$$

$$\ln\left(\frac{T_s - T_i}{T_s - T_f}\right) = \frac{UAt}{mC_p}$$

$$t = \frac{mC_p}{UA}\ln\left(\frac{T_s - T_i}{T_s - T_f}\right)$$

$$t = \frac{1000\times10000}{1000\times0.1}\ln\left(\frac{180-20}{180-80}\right) = 47000\,sec$$

$$\therefore t = \frac{47000}{3600}\,hr = 13.05\,hr$$

Q.37 (0.533)

Diameter of each bubble $d = 10^{-3}\,m$

Bubbles generated per second

per nucleation site $\dot{n} = 10$

Nucleate sites per meter square area s = 100

$$\lambda_v = 1000\frac{kJ}{kg}$$

Density of vapour $\rho_v = 1\,\frac{kg}{m^3}$

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Volume of each bubble $= \frac{\pi d^3}{6} = \frac{\pi\left(10^{-3}\right)^3}{6}$

Amount of vapour in each bubble $m = \rho_v V$

$$m = 1 \times \frac{\pi\left(10^{-3}\right)^3}{6} = \frac{\pi}{6}\left(10^{-9}\right) \text{ kg}$$

Heat supplied to form the vapour contained $= m\lambda_v$

$\therefore$ Total heat required $= m\,\lambda_v \times \dot{n} \times s$

$$= \frac{\pi}{6}\left(10^{-9}\right) \times 1000 \times 10^3 \times 10 \times 100$$

$$= \frac{\pi}{6} = 0.5325 \frac{W}{m^2}$$

Q.38 (C)

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4 RADIATION

4.1 Basics of Radiation

4.1.1 Radiation: Process & Properties

Thermal Radiation is the transmission of thermal Energy without any physical contact between the bodies involved. Unlike conduction and convection no medium is required for energy transfer to take place. The mechanism of emission is related to energy released as a result of oscillations or transitions of the many electrons that constitute matter. These oscillations are, in turn, sustained by the internal energy, and therefore the temperature, of the matter. Hence we associate the emission of thermal radiation with thermally excited conditions within the matter. Energy released by radiating surface is in the form of successive and separate packets of / Quanta energy called Photons. Photons travel with unchanged frequency in straight paths with speed equal to that of light. When photons reach the receiving surface, there occurs reconversion of wave motion into thermal energy.

4.1.2 Salient features and characteristics of Radiation Electromagnetic waves

The electromagnetic waves are emitted as a result of vibration and rotational movement of the molecular, atomic, particles comprising the matter. Electromagnetic waves travel at the speed of light. Temperature is the strength of these activities at microscopic level. Thermal radiation is a type of electromagnetic radiations.

C is Speed of light

$C = \lambda\nu$

λ is Wavelength

ν is Frequency

Each photon can be assumed to be a particle of mass m, energy and momentum

$$e = mc^2 = h\nu \Rightarrow m = \frac{h\nu}{c^2}$$

$$\text{Momentum} = mc = \frac{h\,\nu}{c}$$

Speed of light depends on the medium

$C = C_o / n$

'n' is the index of refraction

The short wavelength gamma rays, X rays, and ultraviolet (UV) radiation are primarily of interest to the high-energy physicist and the nuclear engineer, while the long wavelength microwaves and radio waves are of concern to the electrical engineer. It is the intermediate portion of the spectrum, which extends from approximately 0.1 to 100 μm and includes a portion of the UV and all of the visible and infrared (IR), that is termed **thermal radiation** because it is both caused by and affects the thermal state or temperature of matter. For this reason, thermal radiation is pertinent to heat transfer. Thermal radiation emitted by surface encompasses a range of wavelengths

Different types of electromagnetic waves are produced by different mechanisms.

Gamma rays – Nuclear reactions

X-rays - bombardment of metals with high-energy electrons.

Radio waves - Flow of alternating current through conductor.

4.2 RADIATION LAWS

Black body

Black body has following characteristics

1. A blackbody absorbs all incident radiation, regardless of wavelength and direction.

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2. For a prescribed temperature and wavelength, no surface can emit more energy than a blackbody.
3. Although the radiation emitted by a blackbody is a function of wavelength and temperature, it is independent of direction. That is, the blackbody is a diffuse emitter

4.2.1 Planck's Law

The energy emitted by a black surface varies in accordance with wavelength, temperature, surface characteristics of the body.

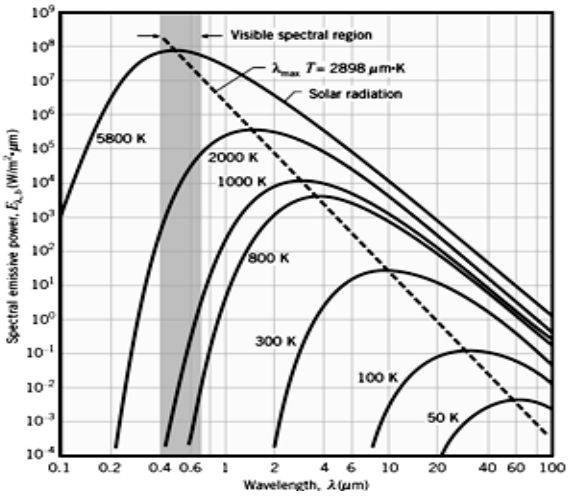
$$(E_\lambda)_b = 2\pi C^2 h \frac{\lambda^{-5}}{\exp[ch/k\lambda T]-1}$$

h is Plank's Constant

k is Boltzman Const, 13.802×10^{-24}. J/k

C is 3×10^8 m/s (Speed light)

$(E_\lambda)_b$ Monochromatic (single wavelength) emissive power and is defined as the energy emitted by the black body in all directions at given wavelength (λ) per unit wavelength interval around.



Characteristics of emissions

1. The emitted radiation varies continuously with wavelength.
2. At any wavelength the magnitude of the emitted radiation increases with increasing temperature.
3. The spectral region in which the radiation is concentrated depends on temperature, with comparatively more radiation appearing at shorter wavelengths as the temperature increases.
4. A significant fraction of the radiation emitted by the sun, which may be approximated as a blackbody at 5800 K, is in the visible region of the spectrum. In contrast, for T = 800 K, emission is predominantly in the infrared region of the spectrum and is not visible to the eye.

   At any temp, the area under the mono chromatic emissive power versus wavelength gives the rate of radiant energy emitted with the wavelength interval $d\lambda$. Thus$dE_b = (E_\lambda)d\lambda$.

$$E_b = \int_{\lambda=0}^{\lambda=\infty} (E_\lambda)_b \, d\lambda$$

E_b is total Emissive power/unit area (radiant energy flux density).

4.2.2 Wein's Displacement

The blackbody spectral distribution has a maximum value and that the corresponding wavelength λ_{max} depends on temperature. The nature of this dependence may be obtained by differentiating plank's law with respect to temperature and setting the result equal to zero

$$\lambda_{max} T = 2900\ \mu m^{\circ} k$$

4.2.3 Stefan Boltzmann Law

The total emissive power per unit area of a blackbody may be expressed as

$$E_b = \int_0^\infty (E_\lambda)_b .d\lambda = \int_0^\infty \frac{c_1 \lambda^{-5}}{\exp(c_2/\lambda T)-1} d\lambda$$

After performing integration

$$E_b = \sigma_b T^4$$

Where,

$$\text{Stefan} - \text{Boltzmann constant} = 5.67 \times 10^{-8}\ w/m^2 k^4$$

& T is temperature in K

A body radiating is simultaneously receiving heat from other bodies. Then net radiant heat flux is given by

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$$Q_{net} = \sigma_b \left(T_1^4 - T_2^4 \right)$$

4.2.4 RADIATION PROPERTIES

Most of the materials are opaque to thermal radiation and radiation is considered surface phenomena.

Emissivity

The blackbody is an ideal emitter and no surface can emit more radiation than a blackbody at the same temperature. Therefore blackbody is chosen as a reference in describing emission from a real surface. A surface radiative property known as the **emissivity** 'ϵ'. Emissivity is defined as the ratio of radiation emitted by the surface to the radiation emitted by black body at same temperature.

$O \le \epsilon \le 1$

The emissivity of real surface is not constant. It varies with temperature, wavelength and direction of the emitted radiations.

For real surface, $\epsilon = f(\theta, \lambda, T)$

ϵ_λ Is Spectral emissivity

ϵ_θ is Directional emissivity, where θ is the angle between the direction of radiation and Normal to the surface.

In practice it is convenient to use properties averaged over all directions called as hemispherical properties. The emissivity averaged in all the direction is called **hemispherical emissivity.**

Spectral emissive power is defined as integral of rate of radiation energy emitted at a specified wavelength per unit surface area over the entire hemisphere.

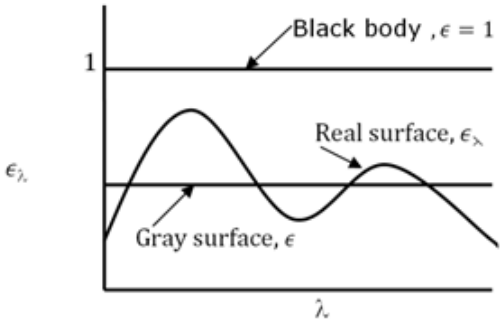
Spectral hemispherical emissivity is expressed as

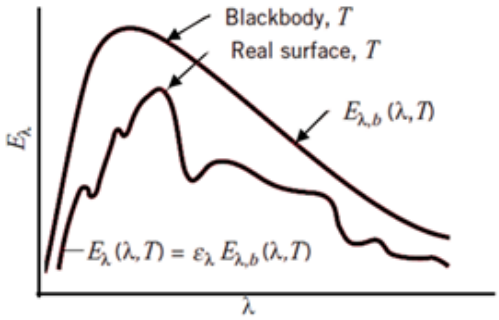
$$\epsilon(\lambda, T) = \frac{E_\lambda(\lambda, T)}{E_{\lambda b}(\lambda, T)}$$

The emissivity averaged over all wavelengths is called **Total emissivity** expressed as

$$\epsilon(T) = \frac{E(T)}{E_b(T)} = \frac{\int_0^\infty \epsilon(\lambda, T)(E_\lambda)_b \, d\lambda}{\int_0^\infty (E_\lambda)_b \, d\lambda}$$

$$= \frac{\int_0^\infty \epsilon(\lambda, T)(E_\lambda)_b \, d\lambda}{\sigma_b T^4}$$





Spectral hemispherical emissivity & spectral emissive power

A surface is said to be **diffuse** if its properties are independent of direction. Blackbody is considered a diffuse body. A gray body has its properties independent of wavelength.

Surface	emissivity
Black body	$\epsilon = 1$
Real Surface	$\epsilon_\theta \neq$ const $\epsilon_\lambda \neq$ const
Gray Surface	$\epsilon_\theta \neq$ const $\epsilon_\lambda =$ const
Diffuse Surface	$\epsilon_\theta =$ const $\epsilon_\lambda \neq$ const
Gray & Diffuse	$\epsilon_\theta =$ const $\epsilon_\lambda =$ const

Absorption, Reflection, and Transmission by Real Surfaces

The radiation energy incident or a surface per unit surface area per unit time is called

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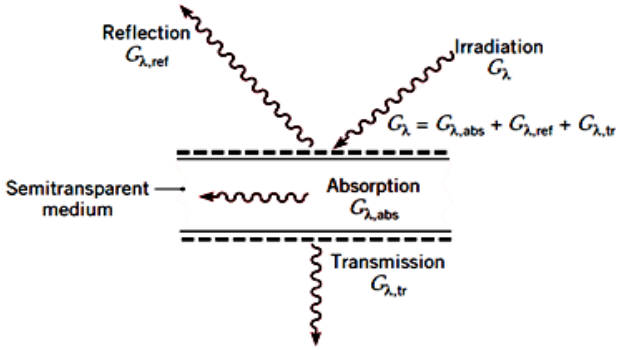
Irradiation (G).The spectral irradiation G_λ($W/m^2\mu m$) is defined as the rate at which radiation of wavelength $\lambda$is incident on a surface, per unit area of the surface and per unit wavelength interval $d\lambda$about λ



$$G_{\lambda,abs} + G_{\lambda,ref} + G_{\lambda,tr} = G_\lambda$$

$$\frac{G_{\lambda,abs}}{G_\lambda} + \frac{G_{\lambda,ref}}{G_\lambda} + \frac{G_{\lambda,tr}}{G_\lambda} = 1$$

Spectral, hemispherical absorptive$\alpha_\lambda$as

$$\alpha_\lambda = \frac{G_\lambda,\ abs}{G_\lambda}$$

The **total, hemispherical absorptivity, 'α'** represents an integrated average over both direction and wavelength. It is defined as the fraction of the total irradiation absorbed by a surface

$$\alpha = \frac{\int_0^\infty \alpha_\lambda G_\lambda d\lambda}{\int_0^\infty G_\lambda d\lambda}$$

Similarly,

Spectral, hemispherical reflectivity ρ_λ and Spectral, hemispherical transmissivity $\tau_\lambda$as

$$\rho_\lambda = \frac{G_\lambda,\ ref}{G_\lambda},\ \tau_\lambda = \frac{G_\lambda,\ tr}{G_\lambda}$$

The **total, hemispherical reflectivity 'ρ'** is then defined as

$$\rho = \frac{\int_0^\infty \rho_\lambda G_\lambda d\lambda}{\int_0^\infty G_\lambda d\lambda}$$

The **total, hemispherical reflectivity 'τ'** is then defined as

$$\tau = \frac{\int_0^\infty \tau_\lambda G_\lambda d\lambda}{\int_0^\infty G_\lambda d\lambda}$$

In terms of total hemispherical properties

$$\alpha + \rho + \tau = 1$$

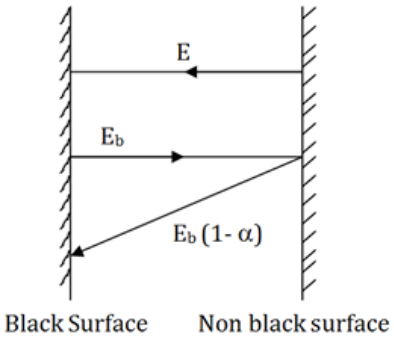
For blackbodies $\alpha = 1, \rho = 0, \tau = 0$

If the medium is opaque, there is no transmission, and absorption and reflection are surface processes for which

$$\alpha + \rho = 1$$

KIRCHOFF'S LAW

Consider a two body enclosure with Black surface Temperature(T_b) and Non-black surface Temperature(Ts).



E is the radiant energy by Non-black.

$E_b$is the radiant energy by black body.

α is the absorptivity of Non-black body.

Total energy change in Non-black body = Energy radiated – Energy absorbed

$$= E - \alpha E_b$$

If $Ts = T_b$ then, No exchange takes place.

$$E - \alpha E_b = 0$$

$$\frac{E}{\alpha} = E_b$$

For different surfaces

$$\frac{E_1}{\alpha_1} = \frac{E_2}{\alpha_2} = \frac{E_3}{\alpha_3} \ldots E_b = f(T)$$

$$\epsilon = \frac{E}{E_b}\ ,\ \text{at same temp } (T = T_b)$$

$$\epsilon = \alpha$$

The emissivity ϵ and absorptivity α of a real surface are equal. This is known as Kirchhoff's Law. A good absorber is also good emitter of radiation.

4.2.5 INTENSITY OF RADIATION

The intensity of radiation 'I' is the energy emitted in a particular direction per unit surface area and through a solid angle.

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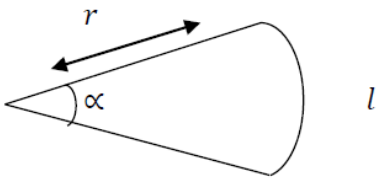
Plane angle (α)

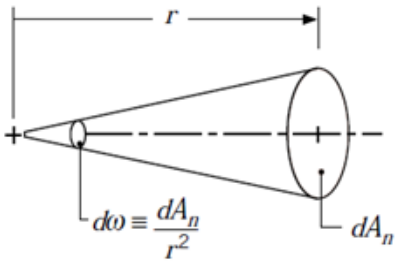
$\propto = l/\alpha$.



Solid Angle (ω)

The differential solid angle 'dω'is defined by a region between the rays of a sphere and is measured as the ratio of the area dA_non the sphere to the sphere's radius squared. It unit is steradian (Sr)

$$d\omega = \frac{dA_n}{r^2}$$



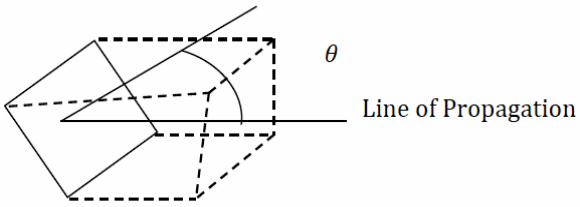
dA_n =Projection of incident surface normal to the line of propagation

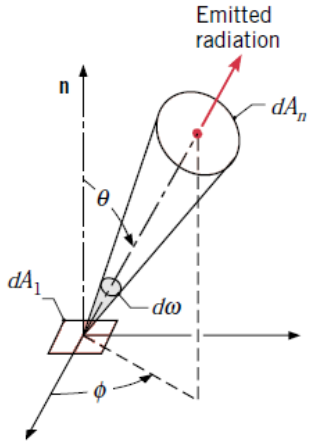
dA = Area of incident surface.

θ = Angle b/w the normal to the incident surface and the line propagation

r =Length of the line of propagation b/w the radiating & incident surface

Line of Propagation





$dA_n = dA\cos\theta$

Spectral Intensity ($I_{(e,\lambda)}$)

It is the rate at which radiant energy is emitted at the wavelength λin the (θ,ϕ) direction, per unit area of the emitting surface normal to this direction, per unit solid angle about this direction, and per unit wavelength interval dλaboutλ.its unit is W/m²Srμm.

$$I_{e,\lambda}(\lambda,\theta,\phi) = \frac{dQ}{dA_1 \cos\theta \, d\omega \, d\lambda}$$

Radiation energy per unit wavelength is given by

$$dQ_\lambda = \frac{dQ}{d\lambda} = I_{e,\lambda}(\lambda,\theta,\phi)\, dA_1 \cos\theta \, d\omega$$

The spectral radiation flux associated with dA_1 is

$$dQ''_\lambda = \frac{dQ_\lambda}{dA_1} = I_{e,\lambda}(\lambda,\theta,\phi)\, \cos\theta \, d\omega$$

Spectral, hemispherical emissive power

It is the rate at which radiation of wavelength λis emitted in all directions from a surface per unit wavelength interval dλ about λ and per unit surface area. Its unit is W/m2 μm

$$d\omega = \frac{dA_n}{r^2} = \frac{r^2 \sin\theta \, d\theta \, d\phi}{r^2}$$

$$E_\lambda = \int dQ''_\lambda = \int_0^{\pi/2} \int_0^{2\pi} I_{e,\lambda}(\lambda,\theta,\phi)\, \cos\theta \sin\theta \, d\theta \, d\phi$$

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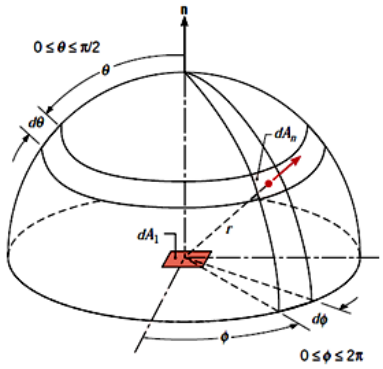
Total, hemispherical emissive power

It is the rate at which radiation is emitted per unit area at all possible wavelengths and in all possible directions.



$$\mathbf{E} = \int_0^\infty E_\lambda d\lambda$$

Or

$$\mathbf{E} = \int_0^\infty \int_0^{\pi/2} \int_0^{2\pi} I_{e,\lambda}(\lambda, \theta, \phi) \cos\theta \sin\theta \, d\theta \, d\phi \, d\lambda$$

By solving the integration

$E = \pi I$

Where I is the total intensity of the emitted radiations

For black body

$$I_n = \frac{E_b}{\pi} = \frac{\sigma_b T^4}{\pi}$$

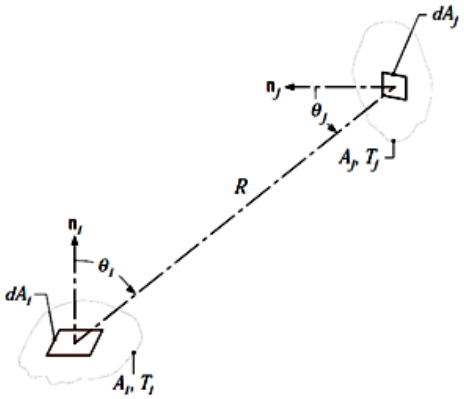
Where I_n is the intensity along the normal direction

4.2.6 RADIATION EXCHANGE B/W SURFACE

The heat exchange between surfaces depends on the surface orientation and properties.We assume that the surfaces are separated by a non-participating medium. Since, such a medium neither emits, absorbs, nor scatters, it has no effect on the transfer of radiation between surfaces. A vacuum meets these requirements exactly, and most gases meet them to an excellent approximation. Our first objective is to establish geometrical features of the radiation exchange problem by developing the notion of a view factor. Our second objective is to develop procedures for predicting radioactive exchange between surfaces that form an enclosure.

(a) Shape Factor

To account for the effects of orientation on radiation heat transfer between two surfaces we define a new parameter called shape factor, which mainly depends on the geometry of surfaces and has on effect of surface properties. It is also called as view factor, configuration factor, and angle factor. The shaper factor based on the assumption that surfaces are diffuse emitters & reflectors is called diffuse shape factor.



F_{ij} is radiation shape factor from surface 'i' to surface 'j'

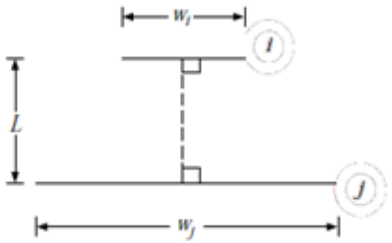
$$F_{ij} = \frac{\text{direct radiation from surface 'i' incident upon 'j'}}{\text{total radiation from emitting surface 'i'}}$$

Shape factor is given by

$$F_{ij} = \frac{1}{A_i} \iint \frac{1}{\pi R^2} \cos\theta_i \cos\theta_j dA_i \, dA_j$$

Shape factor for infinitely long two dimensional geometries

Parallel Plates with Midlines Connected by Perpendicular



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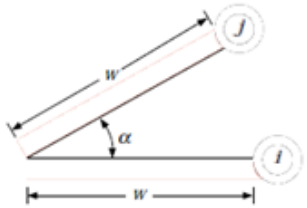
$$Fij = \frac{\left[\left(W_i + W_j\right)^2 + 4\right]^{1/2} - \left[\left(W_j + W_i\right)^2 + 4\right]^{1/2}}{2W_i}$$

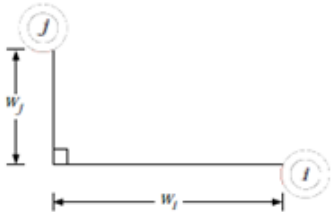
$$W_i = W_i / L, W_j = W_j / L$$

Inclined Parallel Plates of Equal Width and a Common Edge



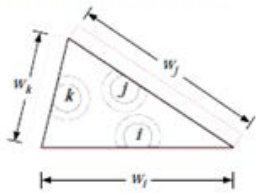
$$F_{ij} = 1 - \sin\left(\frac{\alpha}{2}\right)$$

Perpendicular Plates with a Common Edge



$$F_{ij} = \frac{1 + \left(w_j / w_i\right)\left[1 + \left(w_j / w_i\right)^2\right]^{1/2}}{2}$$

Three-Sided Enclosure



$$F_{ij} = \frac{w_i + w_j - w_k 1}{2w_i}$$

(b) Radiation heat Exchange: Black Surfaces

Consider two black surfaces at temperature T1 & T2 the two bodies form an enclosure.Net radiation heat transfer between two surfaces is given by

$$Q_{net} = \begin{matrix}\text{Radiation Energy} \\ \text{emitted by surface 1} \\ \text{striking surface2}\end{matrix} - \begin{matrix}\text{Radiation Energy} \\ \text{emitted by surface 2} \\ \text{striking surface1}\end{matrix}$$

$$Q_{net} = A_1F_{12}T_1^4 - A_2F_{21}T_2^4$$

Consider enclosure of 'N' black bodies maintained at specified temperatures. The net radiation heat transfer from any surface 'i' of the enclosure is evaluated by adding net heat radiation from surface 'i' to each surface of the enclosure.

$$Q_i = \sum_{j=1}^{N} A_i F_{ij} (T_i^4 - T_j^4)$$

4.2.7 RADIATION HEAT EXCHANGE: GRAY, DIFFUSE SURFACES

As compared to black surface the heat transfer of non-black surfaces may experience multiple reflections off all surfaces, with partial absorption occurring at each. Analyzing radiation exchange in an enclosure may be simplified by making certain assumptions. Each surface of the enclosure is assumed to be isothermal, Opaque, diffuse and gray. the medium within the enclosure is taken to be non-participating

(a) Radiation heat transfer between Infinite Parallel Plates

The assumptions made to carry out analysis are

1. The gap between the plates is small and radiations cannot get out. The two pates form enclosure.
2. The area of plates is same, therefore $F_{12} = F_{21} = 1$.
3. Surfaces are separated by non-absorbing medium such as air.

The amount of energy leaving surface '1' per unit area Q_1

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$Q_1 = \text{Energy emitted by 1} - \text{Energy absorbed by 1 after multiple reflections}$

$$Q_1 = E_1 - [\alpha_1(1-\alpha_2)E_1 + \alpha_1(1-\alpha_1)(1-\alpha_2)^2 E_1 + \alpha_1(1-\alpha_1)^2(1-\alpha_2)^3 E_2 \ldots.]$$

$$Q_1 = E_1 - E_1\alpha_1(1-\alpha_2)[1+(1-\alpha_1)(1-\alpha_2)+(1-\alpha_1)^2(1-\alpha_2)^2 \ldots]$$

The equation forms an infinite series with common ratio, $P = (1-\alpha_1)(1-\alpha_2)$

$$Q_1 = E_1 - E_1\alpha_1(1-\alpha_2)[1+P+P^2+P^3 \ldots.]$$

By solving the above equation we get

$$Q_1 = E_1 - E_1\alpha_1(1-\alpha_2)\left[\frac{1}{1-P}\right]$$

Or

$$Q_1 = E_1 - \frac{\alpha_1(1-\alpha_2)E_1}{1-(1-\alpha_1)(1-\alpha_2)}$$

According to Kirchhoff''s Law

$$\alpha_1 = \epsilon_1 \ \& \ \alpha_2 = \epsilon_2$$

$$Q_1 = E_1\left[\frac{\epsilon_2}{\epsilon_1+\epsilon_2-\epsilon_1\epsilon_2}\right]$$

Similarly, The amount of energy leaving surface '2' per unit area

$$Q_2 = E_2\left[\frac{\epsilon_2}{\epsilon_1+\epsilon_2-\epsilon_1\epsilon_2}\right]$$

Net heat exchange between surface '1' & '2' is $Q_{12} = Q_1 - Q_2$

Or

$$Q_{net} = \frac{E_1\epsilon_2 - E_2\epsilon_1}{(\epsilon_1+\epsilon_2-\epsilon_1\epsilon_2)}$$

According to stefan Boltzman Law

$$E_1 = \epsilon_1\sigma_b T_1^4 \ ; \ E_2 = \epsilon_2\sigma_b T_2^4$$

$$\therefore Q_{12} = \frac{\epsilon_1\epsilon_2}{(\epsilon_1+\epsilon_2-\epsilon_1\epsilon_2)}.\sigma_b(T_1^4 - T_2^4)$$

The above equation can be written as

$$Q_{12} = f_{12}\sigma_b(T_1^4 - T_2^4)$$

Where, f_{12} is interchanging factor given by

$$f_{12} = \frac{\epsilon_1\epsilon_2}{\epsilon_1+\epsilon_2-\epsilon_1\epsilon_2}$$

(b) Infinite Long Concentric Cylinders

From reciprocity theorem

$$A_1F_{12} = A_2F_{21}$$

Inner cylinder is completely enclosed by outer cylinder, i.e. $F_{12} = 1$

$$F_{21} = \frac{A_1}{A_2}$$

Analysis similar to infinite plate is carried out for infinite long cylinders

Heat energy leaving Inner cylinder is given by

$$Q_1 = \frac{E_1\epsilon_2}{\left[(A_1/A_2)\epsilon_1\right]+\epsilon_2-\left[(A_1/A_2)\epsilon_1\epsilon_2\right]}$$

Heat energy leaving outer cylinder is given by

$$Q_2 = \frac{\epsilon_1 E_2(A_1/A_2)}{\left[(A_1/A_2)\epsilon_1\right]+\epsilon_2-\left[(A_1/A_2)\epsilon_1\epsilon_2\right]}$$

$$E_2 = \epsilon_2\sigma T_2^4, \ E_1 = \epsilon\ \sigma T_1^4$$

Net heat exchange between surface inner cylinder & outer cylinder is

$$Q_{12} = f_{12}A_1\sigma_b(T_1^4 - T_2^4)$$

Where, f_{12} is interchanging factor given by

$$f_{12} = \frac{1}{1/\epsilon_1 + A_1/A_2(1/\epsilon_2 - 1)}$$

The above relation is also applicable for concentric spheres

(c) Small Gray bodies

The radiant energy emitted by surface I would be partly absorbed by surface 2, and the unabsorbed reflected portion would be lost in space.

Energy emitted by body $1 = A_1\epsilon_1\sigma_b T_1^4$

Energy incident on body $2 = F_{12}A_1\epsilon_1\sigma_b T_1^4$

Energy absorbed by body

$2 = \propto_2 F_{12}A_1\epsilon_1\sigma_b T_1^4$

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The radiant energy transfer from body 1 to body 2 is $Q_1 = \epsilon_1\epsilon_2 A_1 f_{12}\sigma_b T_1^4$

Likewise energy transfer from body 2 to body 1 is

$Q_{21} = \epsilon_1\epsilon_2 A_\partial F_{21}\sigma_b T_2^4$

$Q_{net} = \epsilon_1\epsilon_2 A_1F_{12}\ \sigma_b\left(T_1^4 - T_2^4\right)..\left[A_1F_{12} = A_2F_{21}\right]$

$Q_{12} = f_{12}A_1F_{12}\ \sigma_b\left(T_1^4 - T_2^4\right)$

$f_{12} = \epsilon_1\epsilon_2$ for small bodies

(d) Small bodies in a large Enclosure

The large gray enclosure acts like a black body; it absorbs practically all the radiation incident upon it and reflects negligibly small energy back to the small gray body.

$F_{12} = 1$

Energy Emitted by small body

$1 = A_1\epsilon_1\sigma_b T_1^4$

=Energy absorbed by large enclosure

$\left(A_1\epsilon_1F_{12}\sigma_b T_1^4\right)$

Energy Emitted by body $2 = A_2\epsilon_2\sigma_b T_2^4$

Energy absorbed by body

$1 = F_{21}A_2\epsilon_2\sigma_b T_2^4.\propto_1$

$Q_{12} = A_1\epsilon_1\ \sigma_b T_1^4 - \epsilon_1\epsilon_2 A_2F_{21}\sigma_b T_2^4$

Assume $T_1 = T_2, Q_{12} = 0$

$A_1\epsilon_1 = A_2F_{21}\epsilon_1\epsilon_2$

$A_1 = A_2F_{21}\epsilon_2$

$Q_{12} = A_1\epsilon_1\ \sigma_b T_1^4 - A_1\epsilon_1\ \sigma_b T_2^4$

$= A_1\epsilon_1\ \sigma_b\left(T_1^4 - T_2^4\right)$

$f_{12} = \epsilon_1$

4.2.8 ELECTRICAL NETWORK APPROACH FOR RADIATION HEAT EXCHANGE

The approach to evaluate heat exchange between non-black surface needs more efforts as heat exchange involves multiple reflections .To reduce the efforts electrical network approach is followed.

Radiosity (J): A surface emits the as well as reflects the thermal radiations. To evaluate heat exchange it is required to know total radiations leaving the surface with no regards of its origin. The total radiation energy leaving the surface per unit time per unit area is known as **Radiosity**

Irradiation (G) :

The total radiation energy falling on the surface per unit time per unit area is known as **Irradiations**

For a gray & opaque surface Radiosity is given by

J=Radiation emitted by surface + Radiation reflected by surface

$J = E + \rho G$

Energy Emitted by body $E = \epsilon E_b$

where 'E_b 'is the emissive power of black body.

$J = \epsilon E_b + \rho G$

Now $\alpha + \rho = 1, \alpha = \epsilon$

$J = \epsilon E_b + (1-\epsilon)G$

$G = \dfrac{J - \epsilon E_b}{(1-\epsilon)}$

Net heat transfer rate is given by

$\dfrac{Q_{net}}{A}$ = Radiation leaving the surface per unit area – Radiation falling on the surface per unit area

$\dfrac{Q_{net}}{A} = J - G$

$\dfrac{Q_{net}}{A} = J - \dfrac{J - \epsilon E_b}{(1-\epsilon)}$

$\dfrac{Q_{net}}{A} = \dfrac{\epsilon(E_b - J)}{(1-\epsilon)}$

$Q_{net} = \dfrac{E_b - J}{(1-\epsilon)/A\epsilon}$

In an electrical analogy to ohm's law, the above equation can be expressed as

$Q_{net} = \dfrac{E_b - J}{R}$

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Where, $(1-\epsilon)/A\epsilon$ is related to the surface properties of the radiating body and is called the **surface resistance**.$E_b - J$Represents the potential difference and net rate of radiation heat transfer corresponds to current in electrical analogy. The direction net heat transfer depends on the magnitude of 'J' and 'E_b' for Blackbody

$\epsilon = 1 \& E_b = J$

Some surfaces encountered in numerous practical heat transfer applications are modeled as being adiabatic. Since their backs are well insulated and net heat transfer through them is zero.

$Q = 0, E_b = J$

Such surfaces are called reradiating surfaces.

Note that temp of reradiating surface is independent of its emissivity.

Now consider radiant heat exchange b/w two non-black surfaces of the total radiation J, leaving the surface 1, only fraction $J_1 A_1 F_{12}$ is received by the other surface. Similarly heat received by other surface $= J_2 A_2 F_{21}$.

Net heat transfer is given by

$$Q_{net} = J_1 A_1 F_{12} - J_2 A_2 F_{21}$$

From reciprocity theorem $[A_1 F_{12} = A_2 F_{21}]$

$$Q_{net} = \frac{J_1 - J_2}{1/A_1 F_{12}}$$

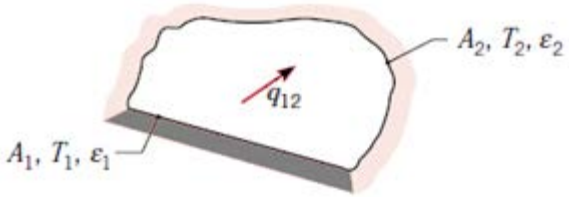
In an electrical analogy to ohm's law, the above equation can be expressed as

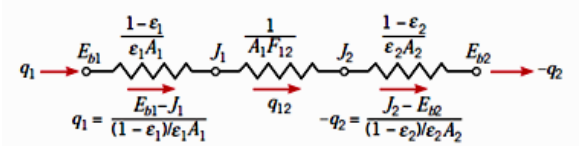
$$Q_{net} = \frac{J_1 - J_2}{R}$$

Where, $R=1/A_1 F_{12}$ is related to the orientation of surfaces and is called **special resistance**

7.7.1 Two- Surface enclosure

The enclosure involving two surfaces that exchange radiation only with each other. Such a two-surface enclosure is shown schematically in Figure. Since there are only two surfaces, the net rate of radiation transfer from surface 1, Q_1, must equal the net rate of radiation transfer to surface 2,Q_2, and both quantities must equal the net rate at which radiation is exchanged between 1 and 2.





Net heat transfer rate is given by

$$(Q_{12})_{net} = \frac{E_{b1} - E_{b2}}{\frac{1-\epsilon_1}{A_1\epsilon_1} + \frac{1}{A_1\epsilon_{12}} + \frac{1-\epsilon_2}{A_2\epsilon_2}}$$

$$(Q_{12})_{net} = \frac{A_1\sigma_b\left(T_1^4 - T_2^2\right)}{1-\epsilon_1/\epsilon_1 + 1/F_{12} + \left(1-\epsilon_2\right)/\epsilon_2 . A_1/A_2}$$

$$(f_g)_{12} = \frac{1}{\left(\frac{1-\epsilon_1}{\epsilon_1}\right) + 1/F_{12} + \left(\frac{1-\epsilon_2}{\epsilon_2}\right) . A_1/A_2}$$

Where, $(f_g)_{12}$ is gray body factor.

Case-I for two **Black surface** enclosures

$\epsilon_1 = \epsilon_2 = 1$

$$(f_g)_{12} = \frac{1}{1/F_{12}} = F_{12}$$

$$(Q_{12}) = A_1 F_{12}\left(T_1^4 - T_2^4\right)$$

Case-II

The radiating surfaces are **infinite parallel plates**. All radiation emitted by one plate is absorbed by other plane.

$F_{12} = F_{21} = 1 \& A_1 = A_2$

$$(f_g)_{12} = \frac{1}{1/\epsilon_1 + 1/\epsilon_2 - 1}$$

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Case-III

The radiating bodies are **concentric cylinders or spheres**. The inner cylinder or sphere of Area A_1 sees only the outer surface not itself

$$F_{12}=1, F_{21}=\frac{A_1}{A_2}$$

$$\left(f_g\right)_{12}=\frac{1}{\left(\frac{1-\epsilon_1}{\epsilon_1}\right)+\frac{1}{F_{12}}+\left(\frac{1-\epsilon_2}{\epsilon_2}\right)\frac{A_1}{A_2}}$$

Or

$$\left(f_g\right)_{12}=\frac{1}{1/\epsilon_1+A_1/A_2\left(1/\epsilon_2-1\right)}$$

4.3 SHAPE FACTOR ALGEBRA

Reciprocity theorem

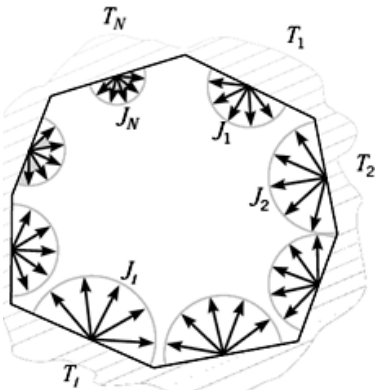
The value of shape factor depends only on the geometry and orientation of surface with respect to each other.

$A_1F_{12} = A_2F_{21}$ (Reciprocity theorem)

Summation Rule

This rule is applicable to the surfaces in enclosures. The rule is based on conservation of energy principle. The radiation leaving any surface 'i' will be intercepted by the surfaces of the enclosure.

$$\sum_{j=1}^{N} F_{ij}=1$$



The term F_{ii} appearing in this summation represents the fraction of the radiation that leaves surface 'i' and is directly intercepted by 'i'. If the surface is concave, it sees itself and F_{ii} is nonzero. However, for a plane or convex surface, $F_{ii}= 0$.

Superposition Rule

Sometimes it is required to express the given geometry as sum or difference of geometries with known shape factors. According to superposition rule, the shape factor from a surface 'i' to a surface 'j' is equal to sum of shape factors from surface 'i' to all parts of 'j'. Let surface 'j' be divided in 'N' parts then shape factor F_{ij} is given by

$$F_{ij}=F_{i1}+F_{i2}\ldots.F_{iN}$$

This is the case when surface intercepting the radiations is divided.

If the surface is emitting the radiation is divided into sub areas A_{i1}, A_{i2}, A_{in}..... then.

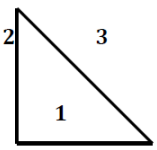
$$A_iF_{ij}=A_{i1}F_{i1j}+A_{i2}F_{i2j}\ldots.A_{in}F_{inj}=\sum_{n=1} A_{in}F_{inj}$$

Symmetry Rule

The determination of shape factor becomes easy if the geometry involves some sort of symmetry. Symmetry rule is expressed as two or more surfaces possesses symmetry about a surface will have same shape factor from that surface.

As shown in figure body with cross section as isosceles triangle.

The surface '1' & '2' are symmetric for surface '3'.therfore,$F_{31}=F_{32}$



4.4 RADIATION SHIELDS

Radiation shields constructed from low emissivity (high reflectivity) materials can be used to reduce the net radiation transfer between two surfaces. Consider placing a radiations shield, surface 3, between the two large, parallel planes.

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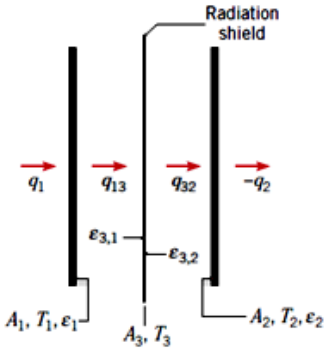
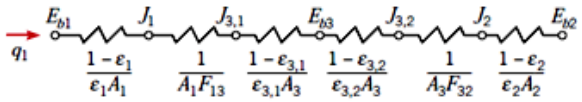
Under steady state T_3 is developed &$(Q_{13})_{net} = (Q_{32})_{net} = (Q_{12})_{net}$

By applying the network analysis concept





$$(Q_{13})_{net} = \frac{(T_2^4 - T_3^4)A\sigma_b}{1/\epsilon_1 + 1/\epsilon_3 - 1} = \frac{A\sigma(T_3^4 - T_2^4)}{\frac{1}{\epsilon_3} + 1/\epsilon_2 - 1}$$

By rearranging the above equation, the steady state temperature T_3 can be evaluated by

$$T_3^4 = \frac{T_1^4(1/\epsilon_3 + 1/\epsilon_2 - 1) + T_2^4(1/\epsilon_1 + 1/\epsilon_2 - 1)}{(1/\epsilon_3 + 1/\epsilon_2 - 1) + (1/\epsilon_1 + 1/\epsilon_3 - 1)}$$

Total heat transfer between surface 1 & 2, with shield is given by

$$(Q_{12})_{net} = \frac{A\sigma(T_1^4 - T_2^4)}{(1/\epsilon_3 + 1/\epsilon_2 - 1) + (1/\epsilon_1 + 1/\epsilon_3 - 1)}$$

The ratio of heat transfer without shield and with shield is given by

$$\frac{(Q_{12})_{net}\text{ with shield}}{(Q_{12})_{net}\text{ without shield}} = \frac{1/\epsilon_1 + 1/\epsilon_2 - 1}{(1/\epsilon_1 + 1/\epsilon_3 - 1) + (1/\epsilon_3 + 1/\epsilon_2 - 1)}$$

If $\epsilon_1 = \epsilon_2 = \epsilon_3$

$$T_3^4 = \frac{1}{2}(T_1^4 - T_2^4)$$

$$\frac{Q_{12}\text{with sheild}}{Q_{12}\text{ without sheild}} = \frac{1/\epsilon_1 + 1/\epsilon_1 - 1}{2/\epsilon - 1 + 2/\epsilon - 1}$$

$$= \frac{2/\epsilon - 1}{2(2/\epsilon - 1)} = 1/2$$

For N radiation shields with same emissivity

$$(Q_{12})_{net} = \frac{A\sigma(T_1^4 - T_2^4)}{(N+1)(2/\epsilon - 1)}$$

$$\frac{Q_{12}\text{with sheild}}{Q_{12}\text{ without sheild}} = \frac{(2/\epsilon - 1)}{(N+1)(2/\epsilon - 1)} = \frac{1}{N+1}$$

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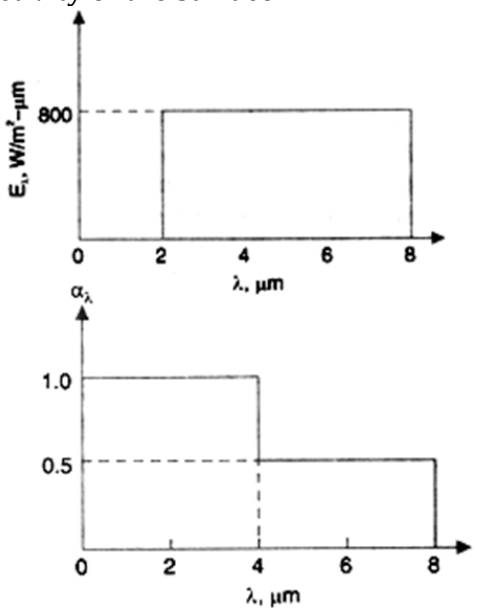
Examples

Example:1

Measurements were made of the monochromatic absorptivity and monochromatic hemispherical irradiation incident on an opaque surface, and the variation of these parameters with wavelength may be approximated by the results shown below. Determine the absorbed radiant flux, the total hemispherical absorptivity and the total reflectivity of the surface.



Solution:

Incident flux $= 800\ (8-2) = 4800 \text{W/m}^2$

Absorbed radiant flux $= \int_0^\infty \alpha_\lambda E_\lambda d\lambda$

$= \int_2^4 (1\times 800) d\lambda + \int_4^8 (0.5\times 800) d\lambda$

$= 800\ (4-2) + 400\ (8-4) = 3200 \text{W/m}^2$

$\therefore$ Absorptivity α=3200/4800=0.667

The requirement that all the radiant energy striking any surface may be accounted for is $\sigma + \tau + \rho = 1$

Here, $\tau = 0$ as the surface is opaque and therefore reflectivity of the surface is

$\rho = 1 - \alpha$

$= 1 - 0.667 = 0.333$

Example:2

What are the ranges of wavelength of electromagnetic waves covering ultra-violets visible, infrared and thermal radiation?

A small black body has a total emissive power of 4.5kW/m^2. Determine its surface temperature and the wavelength of maximum emission. In which range of the spectrum does this wavelength fall?

Solution:

From Stefan Boltzman law, the rate of energy transmission from a black body is

$E = \sigma_b T^4 \quad ; \quad 4.5\times 1000 = 5.67\times 10^{-8} T^4$

$\therefore \quad T = \left[\frac{4.5\times 1000}{5.67\times 10^{-8}}\right]^{\frac{1}{4}} = 530.77 \text{K}$

The wavelength of emission maximum is given by Wien's law. That is

$\lambda_{max} T = 2.8908\times 10^{-3}$

$\lambda_{max} = \frac{2.898\times 10^{-3}}{530.77} = 5.46\times 10^{-6} \text{m} = 5.46 \mu\text{m}$

From Fig, it may be seen that this wavelength falls in the infrared region of the spectrum.

Example:3

A small surface of area $A_1 = 0.0015 \text{m}^2$ emits diffusely, & measurements `indicate that the total intensity associated with

emission in the normal direction $I_n = 6500 \text{W/m}^2 - \text{sr}$. The radiation thus emitted is intercepted by three surfaces of areas $A_2 = A_4 = 0.001\ \text{m}^2$ and $A_3 = 0.00125\ \text{m}^2$ which are at a distance of 0.6m from A_1 and their orientation is as indicated in the figure given below. Make calculations for: a) intensity associated with emission in each of the three directions, (b) solid angles subtended by the intercepting surfaces when viewed from the emitting surface and (c) rates at which radiation emitted by A_1 is

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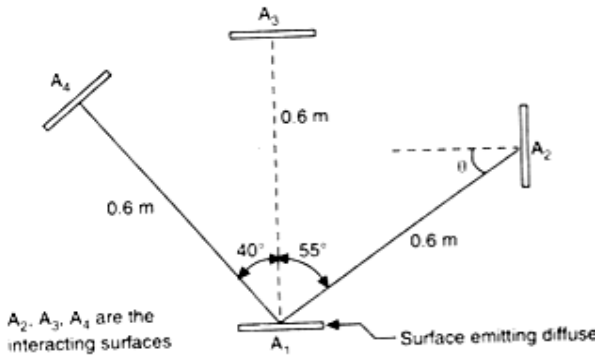
intercepted by the three surfaces A_2, A_3 and A_4 .



Solution:

a) For a diffused emitter, the intensity of the emitted radiation is independent of direction. Therefore

$$I = 6500 W/m^2 - sr$$

For each of the three directions

b) In terms of differential surface and the distance radiation travels, the solid angle is given by:

$$d\omega = \frac{dA_n}{r^2} = \frac{dA\cos\theta}{r^2}$$

Where θ is the angle between the surface normal and the direction of radiation. Therefore solid angle subtended by surface A_2 with respect to surface A_1 is

$$\omega_{2-1} = \frac{0.001 \times \cos(90-55)}{0.6^2}$$

$$= 2.275 \times 10^{-3} sr$$

Similarly

$$\omega_{3-1} = \frac{0.00125\cos 0°}{0.6^2} = 3.47 \times 10^{-3} sr$$

$$\omega_{4-1} = \frac{0.001\cos 0°}{0.6^2} = 2.78 \times 10^{-3} sr$$

c) The rate at which radiation is intercepted by each of three surfaces $A_2, A_3, and A_4$ can be obtained from the expression

$$Q_{1-j} = I A_1 \cos\theta_1 \omega_{j-1}$$

Where θ_1 is the angle between normal to the emitting surface A_1 and the direction of propagation of radiation. Therefore

$$Q_{1-2} = 6500 \times 0.0015 \cos 55 \times 2.275 \times 10^{-3}$$

$$= 12.72 \times 10^{-3} W$$

$$Q_{1-3} = 6500 \times 0.0015 \cos 0 \times 3.47 \times 10^{-3}$$

$$= 33.83 \times 10^{-3} W$$

And

$$Q_{1-4} = 6500 \times 0.0015 \cos 40 \times 2.78 \times 10^{-3}$$

$$= 20.76 \times 10^{-3} W$$

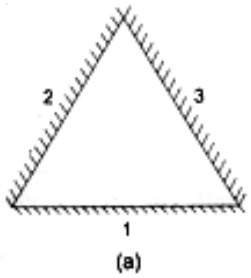
Example:4

Calculate the shape factors for the configurations shown in the figure given below:

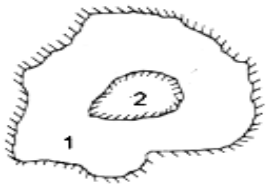
a) long tube with cross-section of an equilateral triangle

b) black body inside a black enclosure

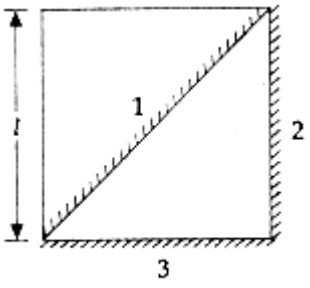
c) diagonal partition within a long square duet.



(a)



(b)



(c)

Solution:

The desired shape factors can be worked out by invoking the summation rule, the

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reciprocity theorem and from inspection of the geometry.

a) By summation rule for radiation from surface 1.

$F_{11}+F_{12}+F_{13}=1$

The flat surface 1 cannot see itself and so $F_{11}=0$. That gives

$F_{12}+F_{13}=1$

Due to symmetry, the radiation from surface 1 is equally divided between surfaces 2 and 3, and therefore

$F_{12}=F_{13}=0.5$

Likewise, considering radiation from surface 2:

$F_{21}+F_{23}=1$ (because $F_{22}=0$)

$F_{23}=1-F_{21}$

By reciprocity:

$F_{21}=\frac{A_1}{A_2}F_{12}=F_{12}$ (because $A_1=A_2$)

$\therefore\ F_{23}=1-F_{12}=1-0.5=0.5$

b) The concave surface 1 can see itself and the rest of radiation falls on the enclosed surface 2. Invoking the conservation principle (summation rule) for surface 1

$F_{11}+F_{12}=1$

By reciprocity:

$A_1F_{12}=A_2F_{21}$ or $F_{12}=\frac{A_2}{A_1}F_{21}$

$\therefore\ F_{11}=1-F_{12}=1-\frac{A_2}{A_1}F_{21}$

Further, all the radiations our from the black surface 2 are intercepted by the enclosing surface, therefore $F_{21}=1$, and so

$F_{11}=1-\frac{A_2}{A_1}$

c) From summation rule: $F_{11}+F_{12}+F_{13}=1$

By inspection $F_{11}=0$ as the flat surface cannot see itself

$\therefore F_{12}+F_{13}=1$

Due to symmetry, the radiation form surface 1 is equally divided between the surfaces 2 and 3 and therefore

$F_{12}=F_{13}=0.5$

By reciprocity:

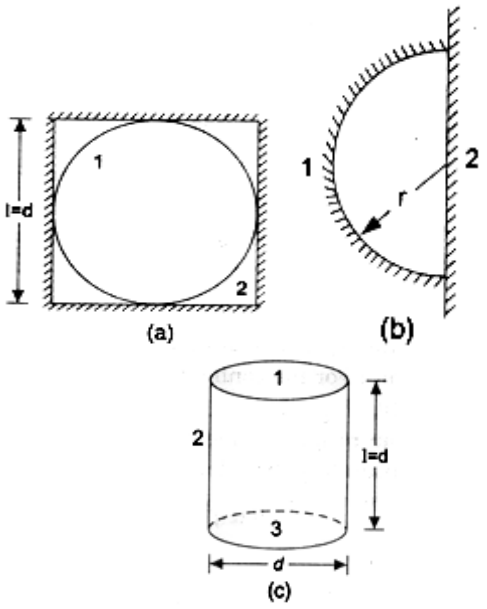
$F_{21}=\frac{A_1}{A_2}F_{12}=\frac{\sqrt{2}l}{l}\times 0.5=0.71$

Example:5 Calculate the shape factors for the configurations shown in the figure given below:

a) sphere of diameter d inside a cubical box of length l=d

b) hemispherical surface closed by a plane surface

c) end and side of circular tube of equal length and diameter.



Solution:

The desired shape factors can be worked out by invoking the summation rule, the reciprocity theorem and from inspection of the geometry.

a) The convex spherical surface cannot see itself and the entire radiation emitted by this surface falls on the enclosing box. Invoking the conservation principle (summation rule) for surface 1,

$F_{11}+F_{12}=1$ or $F_{12}=1$ (because $F_{11}=0$)

By reciprocity:

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$$F_{21}=\frac{A_1}{A_2}F_{12}=\frac{\pi d^2}{6l^2}\times 1=\frac{\pi}{6}$$ (because l=d)

b) The hemispherical surface 1 can see itself and the rest of radiation falls on the flat plane surface.

$\therefore F_{11}+F_{12}=1$

Further all the radiations emitted from the plane surface 2 (which cannot see itself) are intercepted by the hemispherical surface and so $F_{21}=1$

By reciprocity:

$$F_{12}=\frac{A_2}{A_1}F_{21}=\frac{\pi r^2}{\frac{1}{2}\times 4\pi r^2}\times 1=0.5$$

$\therefore F_{11}=1-F_{12}=1-0.5=0.5$

c) From the given geometry,

$$\frac{l}{r_1}=2 \text{ and } \frac{r_3}{l}=0.5$$

With reference to Fig, the shape factor F_{13} is then read as =0.17

Applying summation rule:

$F_{11}+F_{12}+F_{13}=1$

The flat surface 1 cannot see itself and so $F_{11}=0$. That gives

$F_{12}=1-F_{13}=1-0.17=0.83$

By reciprocity:

$$F_{21}=\frac{A_1}{A_2}F_{12}=\frac{\frac{\pi}{4}d^2}{\pi dl}\times 0.83=\frac{0.83}{4}\approx 0.21$$

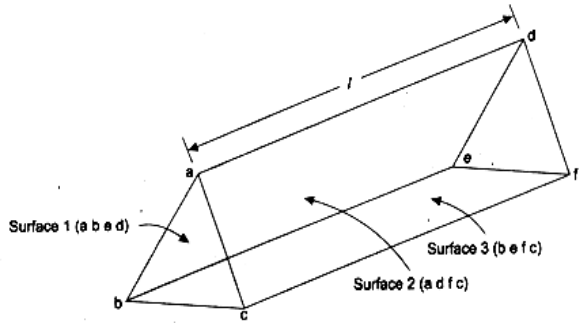
(because l=d)

Example:6

Consider a very long isocelles triangular duct shown in the accompanying figure. For the given dimensions:

$ab=ac=x$ and $bc=\frac{x}{2}$

Show that $F_{12}=0.75$



Solution:

The duct is very long and as such the radiation lost out of the end of the duct can be ignored. This implies that the shape factors of the large surfaces with respect to the triangular end surfaces are vanishingly small

From the summation rule for flat surfaces (1) and (3)

$F_{12}+F_{13}=1$

$F_{31}+F_{32}=1$

By symmetry $F_{31}=F_{32}$

Then from identity (ii), we obtain

$F_{31}=0.5$

By reciprocity theorem; $A_1F_{13}=A_3F_{31}$

Or $$F_{13}=\frac{A_3}{A_1}F_{31}=\frac{l\times\frac{x}{2}}{l\times x}\times 0.5=0.25$$

Therefore from identity (i)

$F_{12}=1-F_{13}=1-0.25=0.75$

Example:7

Show that the radiation shape factor between a disc of radius r located at a distance f from the centre of a small sphere is given by

Solution:

With reference to Fig. , consider an elemental area dA_2 of included angle $d\theta$ at an angle θ .

$dA_2=(2\pi R\sin\theta)\,R\,d\theta$

Then area of the surface of the sector ab of the sphere

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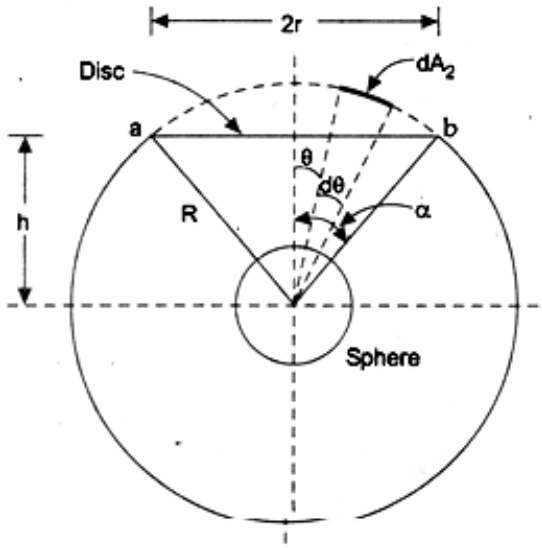
$$A_2 = \int_0^{\alpha} (2\pi R \sin\theta) R \, d\theta$$

$$= 2\pi R^2 \int_0^{\alpha} \sin\theta d\theta = 2\pi R^2 (1 - \cos\alpha)$$



Area of the sphere of radius $R = 4\pi R^2$

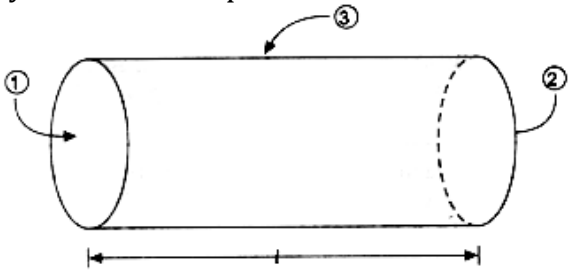
$$\therefore \quad F_{sphere-disc} = \frac{2\pi R^2 (1 - \cos\alpha)}{4\pi R^2}$$

$$= \frac{1}{2}(1 - \cos\alpha) = \frac{1}{2}\left[1 - \frac{h}{R}\right]$$

$$= \frac{1}{2}\left[1 - \frac{h}{\sqrt{r^2 + h^2}}\right]$$

Example:8

Consider a thin hollow cylinder of 8 cm diameter and 10 cm length. If the radiant shape factor of the circular surface of this cylinder is 0.172, make calculations for the shape factor of the curved surface of the cylinder with respect to itself.



Solution:

Refer figure for the nomenclature of the surfaces of the thin hollow cylinder

For the surface 1 and 2,

$A_1 F_{12} = A_2 F_{21}$ (Reciprocity theorem)

Since $A_1 = A_2$, We get

$F_{12} = F_{21} = 0.172$

Also $F_{11} = F_{22} = 0$ and $F_{31} = F_{32}$ because of symmetry.

By summation rule, the shape factor relations among the three surfaces are given by

$F_{11} + F_{12} + F_{13} = 1$ or $F_{12} + F_{13} = 1$ ($Q\, F_{11} = 0$)

$\therefore \quad F_{13} = 1 - F_{12} = 1 - 0.172 = 0.828$

Also, $F_{31} + F_{32} + F_{33} = 1$ or $F_{31} + F_{31} + F_{33} = 1$

($Q\, F_{31} = F_{32}$)

$\therefore \quad F_{33} = (1 - 2F_{31})$

Invoking reciprocity theorem: $A_1 F_{13} = A_3 F_{31}$

$$F_{31} = \frac{A_1}{A_2} F_{13} = \frac{\pi r^2}{2\pi r l} F_{13} = \frac{r}{2l} F_{13}$$

$$= \frac{4}{2 \times 10} \times 0.828 = 0.1656$$

Thus $F_{33} = 1 - 2 \times 0.1656 = 0.6688$

Example:9

Distinguish between the configuration factor and the interchange factor.

Determine the radiation heat flux between two closely spaced, black parallel plates radiating only to each other if their temperatures are 850 K and 425K respectively. Recalculate the heat flux presuming that each of the parallel plates has an emissivity of 0.5. In each case, the plates have an area of $4m^2$.

Solution:

The configuration factor F_{12} considers the orientation and geometry of the black radiating surfaces; how the two surfaces view each other and to what extent the two surfaces radiate solely to each other. The interchange factor f_{12} allows for the departure of the two surfaces from

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complete blackness; a function of the emissivities of the two surfaces.

For the black parallel plates radiating only to each other, $F_{12}=1$ and then the radiant heat exchange is:

$$Q_{12}=F_{12}A_1\sigma_b(T_1^4-T_2^4)$$
$$=1\times4\times(5.67\times10^{-8})\times(850^4-425^4)$$
$$=110\times10^{-3}\,W$$

For the gray surface, the heat exchange is:

$$Q_{12}=\left(F_g\right)_{12}A_1\sigma_b(T_1^4-T_2^4)$$

For the given configuration of parallel plates which see each other and nothing else,

$F_{12}=1$ and $A_1=A_2$

$$\left(F_g\right)_{12}=\frac{1}{\frac{1-\epsilon_1}{\epsilon_1}+\frac{1}{F_{12}}+\frac{1-\epsilon_2}{\epsilon_1}\times\frac{A_1}{A_2}}$$

$$=\frac{1}{\frac{1-\epsilon_1}{\epsilon_1}+1+\frac{1-\epsilon_2}{\epsilon_2}}=\frac{1}{\frac{1}{\epsilon_1}+\frac{1}{\epsilon_2}-1}$$

$$=\frac{1}{\frac{1}{0.5}+\frac{1}{0.5}-1}=0.333$$

$$\therefore\ Q_{12}=0.333\times4\times(5.67\times10^{-8})(850^4-425^4)$$
$$=37\times10^{-3}\,W$$

It may be noted that if the emissivity of each plate is one –half of a black body, heat flux is reduced by a factor of 3.

Example:10

A thermos flask has a double walled bottle and the space between the walls is evacuated so as to reduce the heat flow. The bottle surfaces are silver plated and the emissivity of each surface is 0.025. If the contents of the bottle are at 375K, find the rate of heat loss from the thermos bottle to the ambient air at 300K.

Solution:

The rate of heat interchange between the two bottle surfaces is given by

$$Q_{12}=(F_g)_{12}A_1\sigma_b(T_1^4-T_2^4)$$

The gray body factor $(F_g)_{12}$ is equal to

$$\frac{1}{\left(1-\epsilon_1\right)/\epsilon_1+1/F_{12}+\left(1-\epsilon_2\right)/\epsilon_2\times A_1/A_2}$$

For infinite long parallel planes which see each other and nothing else

$F_{12}=1$ and $A_1=A_2$

$$\therefore(F_g)_{12}=\frac{1}{\left(1-\epsilon\right)/\epsilon_1+1+\left(1-\epsilon_2\right)/\epsilon_2}$$

$$=\frac{1}{1/\epsilon_1+1/\epsilon_2-1}=\frac{1}{1/0.025+1/0.025-1}$$
$$=0.01266$$
$$\therefore Q_{12}=0.01266\times1\times(5.67\times10^{-8})(375^4-300^4)$$
$$=8.38W$$

Example:11

Two larger parallel planes with emissivity 0.4 are maintained at different temperatures and exchange heat only by radiation. What percentage change in net radiative heat transfer would occur if two equally large radiation shields with surface emissivity 0.04 are introduced in parallel to the plates?

Solution:

Case (i) when shields are not used.

$$Q_{12}=(F_g)_{12}A_1\sigma_b(T_1^4-T_2^4)$$
$$=\frac{A_1\sigma_b(T_1^4-T_2^4)}{\left(\frac{1}{\epsilon_1}-1\right)+\frac{1}{F_{12}}+\left(\frac{1}{\epsilon_2}-1\right)\frac{A_1}{A_2}}$$
$$=\frac{A_1\sigma_b(T_1^4-T_2^4)}{\frac{1}{\epsilon_1}+\frac{1}{\epsilon_2}-1}\ \text{as}\ A_1=A_2\ \text{and}\ F_{12}=1$$
$$=\frac{A_1\sigma_b(T_1^4-T_2^4)}{\frac{1}{0.4}+\frac{1}{0.4}-1}=0.2C$$

where $C=A_1\sigma_b(T_1^4-T_2^4)$

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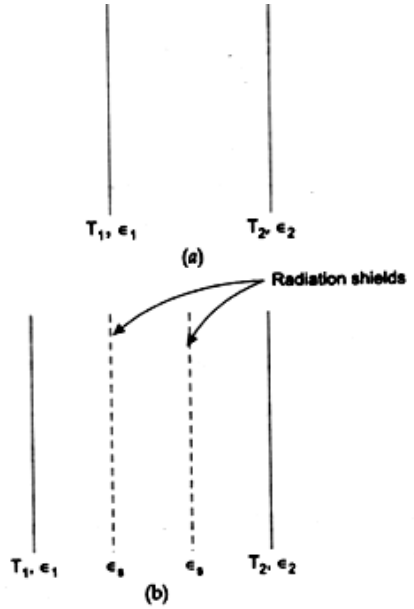

Case (ii) When two radiation shields are used

$$Q_{12}=\frac{A_1\sigma_b(T_1^4-T_2^4)}{\left\{\left(\frac{1}{\epsilon_1}-1\right)+\frac{1}{F_{1s}}+\left(\frac{1}{\epsilon_s}-1\right)\frac{A_1}{A_s}\right\}+\left\{\left(\frac{1}{\epsilon_s}-1\right)+\frac{1}{F_{ss}}+\left(\frac{1}{\epsilon_s}-1\right)\frac{A_s}{A_s}\right\}}$$

$$+\left(\frac{1}{\epsilon_s}-1\right)+\frac{1}{F_{s2}}+\left(\frac{1}{\epsilon_2}-1\right)\frac{A_s}{A_2}$$

As $A_1=A_2=A_s$ and $F_{1s}=F_{ss}=F_{s2}=1$, we get

$$Q_{12}=\frac{A_1\sigma_b(T_1^4-T_2^4)}{\left(\frac{1}{\epsilon_1}+\frac{1}{\epsilon_s}-1\right)+\left(\frac{1}{\epsilon_s}+\frac{1}{\epsilon_s}-1\right)+\left(\frac{1}{\epsilon_s}+\frac{1}{\epsilon_2}-1\right)}$$

$$=\frac{A_1\sigma_b(T_1^4-T_2^4)}{\frac{1}{\epsilon_1}+\frac{1}{\epsilon_2}+\frac{4}{\epsilon_s}-3}=\frac{C}{\frac{1}{0.4}+\frac{1}{0.4}+\frac{4}{0.4}-3}$$

$$=\frac{C}{102}=0.0098C$$

$\therefore$ Percentage change in heat radiant heat exchange

$$=\frac{0.2C-0.0098C}{0.2C}\times 100=95.1\%$$

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GATE QUESTIONS

Q.1 The heat transfer by radiation from a mild steel surface is to be reduced by reducing the emissivity of the surface. This can be best achieved by
(A) Painting the surface black
(B) Painting the surface white
(C) Giving the surface a mirror finish
(D) Roughening the surface

[GATE-2001]

Q.2 For an ideal black body,
(A) Absorptivity = 1
(B) Reflectivity = 1
(C) Emissivity = zero
(D) Transmissivity = 1

[GATE-2004]

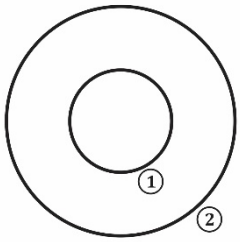
Q.3 A black body at a higher temperature T_H transfers energy by radiation to a black body at a lower temperature T_L. Initially $T_H = 1850\,°C$, $T_L = 500\,°C$ and the net rate of energy transfer is 25 W. After some time, when $T_H = 1500\,°C$ and $T_L = 750\,°C$, what is the net rate of energy transfer?
(A) 8.73 W (B) 9.60 W
(C) 13.89 W (D) 11.01 W

[GATE-2005]

Q.4 An insulated cylindrical pipe of 0.2m diameter has a surface temperature of 45 °C. It is exposed to black body surroundings at 25 °C. The emissivity and absorptivity of the insulation surface are 0.96 and 0.93, respectively. The convective heat transfer coefficient outside the insulation surface is $3.25 W/m^2\text{-}K$. The Stefan-Boltzmann constant is $5.67\ 10^{-8} W/m^2\text{-}K^4$. The surrounding fluid may be assumed to be transparent. Find the percentage contribution from radiation to the total heat transfer rate to the surroundings.
(A) 30.9 (B) 50.0
(C) 57.6 (D) 68.4

[GATE-2006]

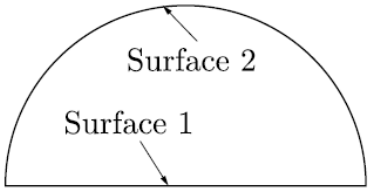
Q.5 For the two long concentric cylinders with surface areas A_1 and A_2, the view factor F_{22} is given by



(A) 0 (B) 1
(C) $1-\frac{A_1}{A_2}$ (D) $\frac{A_1}{A_2}$

[GATE-2007]

Q.6 A well-insulated hemispherical furnace (radius = 1m) is shown below



The self-view factor of radiation for the curved surface 2 is
(A) 1/4 (B) 1/2
(C) 2/3 (D) 3/4

[GATE-2009]

Q.7 The view factor matrix for two infinitely long coaxial cylinders, shown in the figure below, is

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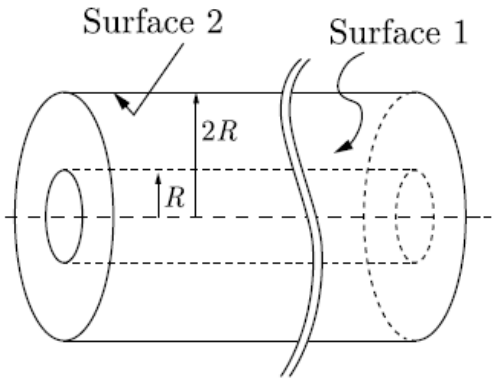


(A) $\begin{pmatrix} 0 & 1 \\ 0.5 & 0.5 \end{pmatrix}$ (B) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

(C) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (D) $\begin{pmatrix} 0.5 & 0.5 \\ 0 & 1 \end{pmatrix}$

[GATE-2010]

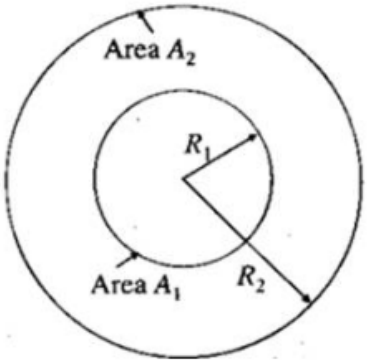
Q.8 Consider two black bodies with surfaces S_1 (area = 1 m^2) and S_2 (area=4m^2). They exchange heat only by radiation, 40% of the energy emitted by S_1 is received by S_2. The fraction of energy emitted by S_2 that is received by S_1 is

(A) 0.05 (B) 0.1

(C) 0.4 (D) 0.6

[GATE-2011]

Q.9 For the enclosure formed between two concentric spheres as shown below ($R_2=2R_1$), the fraction of radiation leaving the surface area A_2 that strikes itself is



(A) $\frac{1}{4}$ (B) $\frac{1}{2}$

(C) $\frac{1}{\sqrt{2}}$ (D) $\frac{3}{4}$

[GATE-2012]

Q.10 A hole of area $1\ cm^2$ is opened on the surface of a large spherical cavity whose inside temperature is maintained at $727^\circ C$. The value of Stefan-Boltzmann constant is $5.67\times10^{-8} W/m^2-K^{-4}$. Assuming black body radiation, the rate at which the energy is emitted (in W) by the cavity through hole, up to 3 digits after the decimal point is, ____

[GATE-2013]

Q.11 In a completely opaque medium, if 50% of the incident monochromatic radiation is absorbed, then which of the following statements are CORRECT?

P. 50% of the incident radiation is reflected

Q. 25% of the incident radiation is reflected

R. 25% of the incident radiation is transmitted

S. No incident radiation is transmitted.

(A) P and S only (B) Q and R only

(C) P and Q only (D) R and S only

[GATE-2014]

Q.12 Two infinitely large parallel plates (I and II) are held at temperatures T_I and T_{II} ($T_I > T_{II}$) respectively, and placed at a distance 2d apart in vacuum. An infinitely large flat radiation shield (III) is placed in parallel in between I and II. The emissivities of all the plates are

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equal. The ratio of the steady state radiative heat fluxes with and without the shield is

(A) 0.5 (B) 0.75

(C) 0.25 (D) 0

[GATE-2015]

Q.13 The space between two below concrete spheres of radii 0.1 m and 0.2 m is under vacuum. Exchange of radiation (uniform in all directions) occurs only between the outer surface (S_1) of the smaller sphere and the inner surface (S_2) of the larger sphere. The fraction (rounded off to the second decimal place) of the radiation energy leaving S_2, which reaches S_1 is _____.

[GATE-2016]

Q.14 Let $I_{b\lambda}$ be the spectral blackbody radiation intensity per unit wavelength about the wavelength λ. The blackbody radiation intensity emitted by a blackbody over all wavelengths is

(A) $\frac{dI_{b\lambda}}{d\lambda}$

(B) $\frac{d^2 I_{b\lambda}}{d\lambda^2}$

(C) $\int_0^\infty I_{b\lambda} d\lambda$

(D) $\int_0^\infty \lambda I_{b\lambda} d\lambda$

[GATE-2017]

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ANSWER KEYS

1	2	3	4	5	6	7	8	9	10
(B)	(A)	(D)	(D)	(C)	(B)	(A)	(B)	(D)	(5.67)
11	12	13	14						
(A)	(A)	(0.25)	(C)						

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EXPLANATIONS

Q.1 (B)

Painting Surface White

Q.2 (A)

Absorptivity = 1

Q.3 (D)

$T_{H\text{-}1} = 1850\ ^{o}C = 2123\ K$

$T_{L\text{-}1} = 500\ ^{o}C = 773\ K$

$Q_1 = 25\ W$

$T_{H\text{-}2} = 1500\ ^{o}C = 1773\ K$

$T_{L\text{-}2} = 750\ ^{o}C = 1023\ K$

$$Q = \sigma A\left(T_1^4 - T_2^4\right)$$

$$\frac{Q_2}{Q_1} = \frac{\sigma A\left(T_{H2}^{\ 4} - T_{L2}^{\ 4}\right)}{\sigma A\left(T_{H1}^{\ 4} - T_{L1}^{\ 4}\right)}$$

$$\frac{Q_2}{25} = \frac{\left(1773^4 - 1023^4\right)}{\left(2123^4 - 773^4\right)}$$

$$Q_2 = 11.01\,W$$

Q.4 (D)

$d = 0.2\ m$

$T_s = 45\,^{o}C = 318\,K$

$T_\infty = 25\,^{o}C, = 298\,K$

$e_1 = 0.96$

$e_2 = 0.93$

$$h = 325\ \frac{W}{m^2K}$$

$$\sigma = 5.67\times10^{-8}\ \frac{W}{m^2K^4}$$

Heat transfer by Radiation $\left(Q_R\right) = \sigma A\left[e_1T_S^4 - e_2T_\infty^4\right]$

$$= \left(5.67\times10^{-8}\right)\left[0.96(318)^4 - 0.93(298)^4\right]$$

$$Q_R = 140.75\ W$$

Heat transfer by convection $\left(Q_C\right) = h\ A\ \Delta T$

$$= 3.25(318 - 298)$$

$$Q_C = 65\,W$$

$$\begin{pmatrix}\%\ \text{Contribution}\\ \text{of radiation}\end{pmatrix} = \frac{Q_R}{Q_C + Q_R} = \frac{140.75}{65 + 140.75}\times 100$$

$$= 68.4\ \%$$

Q.5 (C)

$F_{11} + F_{12} = 1$

$F_{11} = 0$

so, $F_{12} = 1$

Now, $A_1F_{12} = A_2F_{21}$

$$\frac{A_1}{A_2} = F_{21}$$

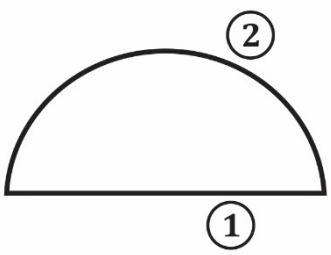
Now, $F_{22} + F_{21} = 1$

$$F_{22} = 1 - \frac{A_1}{A_2}$$

Q.6 (B)

r = 1 m

Energy Conservation Rule on Surface – 1



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$F_{11} + F_{12} = 1$

$0 + F_{12} = 1$

$F_{12} = 1$

Reciprocity Theorom :

$A_1 F_{12} = A_2 F_{21}$

$\pi r^2 (1) = 2\pi r^2 \; F_{21}$

$\frac{1}{2} = F_{21}$

Energy Conservation Rule on Surface – 2

$F_{21} + F_{22} = 1$

$F_{22} = 1 - \frac{1}{2}$

$F_{22} = \frac{1}{2}$

Q.7 (A)

View Factor Matrix $= \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix}$

From fig. $F_{11} = 0$

By Energy Conservation Rule :

$F_{11} + F_{12} = 1$

So, $F_{12} = 1$

Now, By Reciprocity rule

$A_1 F_{12} = A_2 F_{21}$

$\pi DH(F_{21}) = \pi 2DH \; F_{21}$

$F_{21} = \frac{1}{2}$

Now, By Convervation Rule,

$F_{21} + F_{22} = 1$

$F_{22} = \frac{1}{2}$

So, view Factor Matrix becomes $\begin{bmatrix} 0 & 1 \\ 0.5 & 0.5 \end{bmatrix}$

Q.8 (B)

F_{12} = Fraction of energy emitted by S_1 that is received by S_2 = 0.40

F_{21} = Fraction of energy emitted by. S_2 that is received by S_1

By Reciprocity Rule Rule

$A_1 F_{12} = A_2 F_{21}$

Given, $A_1 = 1 m^2$

$A_2 = 4 m^2$

So, $(1)(0.4) = (4) F_{21}$

$F_{21} = 0.1$

Q.9 (D)

Energy Conservation rule at Surface-1

$F_{11} + F_{12} = 1$

$0 + F_{12} = 1$

$F_{12} = 1$

By, Reciprocity Thearom

$A_1 F_{12} = A_2 F_{21}$

$\pi R_1^2 (1) = \pi R_2^2 (F_{21})$

$R_1^2 = 4 R_1^2 \; F_{21}$

$F_{21} = \frac{1}{4}$

By Energy Conservation rule on Surface-2

$F_{21} + F_{22} = 1$

$F_{22} = 1 - \frac{1}{4}$

$F_{22} = \frac{3}{4}$

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Q.10 (5.67)

Given: $T = 727\,^{\circ}C = 1000\,K$

$\sigma = 5.67\times10^{-8}\,\frac{W}{m^2K^4}$

$\varepsilon = 1\,(\text{black body})$

$A = 1cm^2 = 10^{-4}\,m^2$

$Q = \varepsilon\sigma AT^4$

$= (1)\left(5.67\times10^{-8}\right)\left(10^{-4}\right)(1000)^4$

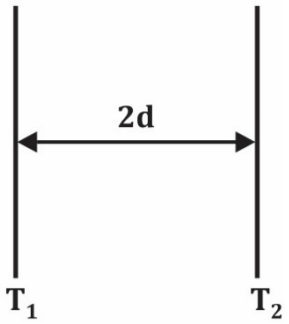
$\therefore Q = 5.67\,W$

Q.11 (A)

P and S only

Q.12 (A)

Without Shield:

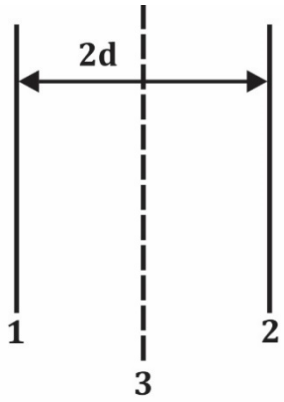


$$Q_{12} = \frac{eA\sigma\left(T_1^4 - T_2^4\right)}{\left[\frac{1}{e_1} - \frac{1}{e_2} - 1\right]}$$

$e_1 = e_2 = e\ (\text{given})$

$$\frac{Q_{12}}{A} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\left[\frac{2}{e} - 1\right]} \quad \ldots(1)$$

With Shield:



For n-Radiation shields

$$\frac{Q_{12}}{A} = \frac{1}{(n+1)}\,\frac{\sigma\left(T_1^4 - T_2^4\right)}{\left(\frac{2}{e} - 1\right)}$$

here, $n = 1$

$$\frac{Q_{12}}{A} = \frac{1}{2}\,\frac{\sigma\left(T_1^4 - T_2^4\right)}{\left(\frac{2}{e} - 1\right)} \quad \ldots(2)$$

From (1) and (2),

$$\frac{Q_{12}\,(\text{with radiation shield})}{Q_{12}\,(\text{without radiation shield})} = \frac{\frac{1}{2}\,\frac{\sigma\left(T_1^4 - T_2^4\right)}{\left(\frac{2}{e} - 1\right)}}{\frac{\sigma\left(T_1^4 - T_2^4\right)}{\left[\frac{2}{e} - 1\right]}}$$

$$\therefore \frac{Q_{12}^*}{Q_{12}} = \frac{1}{2} = 0.5$$

Q.13 (0.25)

Given: $r_1 = 0.1\,m \quad d_1 = 0.2\,m$

$r_2 = 0.2\,m \quad d_2 = 0.4\,m$

F_{21} = fraction of entire radiation leaving the surface-1 falling on surface-2

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By energy balance:

$F_{11} + F_{12} = 1$

$F_{12} = 1 \qquad (Q\ F_{11} = 0)$

By reciprocity theorem,

$A_1F_{12} = A_2F_{21}$

$\left(\pi r_1^2\right)F_{12} = \left(\pi r_2^2\right)F_{21}$

$\frac{(0.1)^2}{(0.2)^2} = F_{21}$

$F_{21} = 0.25$

Q.14 (C)

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5 HEAT EXCHANGER

The process of heat exchange between two fluids that are at different temperatures and separated by a solid wall occurs in many engineering applications. The device used to implement this exchange is termed a heat exchanger, and specific applications may be found in space heating and air-conditioning, power production, waste heat recovery, and chemical processing.

5.1 HEAT EXCHANGER TYPES

Classification of Heat Exchangers

1. Operating Principle
2. Arrangements of flow path
3. Design and certain constructional features

A. Double pipe

The simplest heat exchanger is one for which the hot and cold fluids move in the same or opposite directions in a **concentric tube (or double-pipe)** construction. In the **parallel-flow** arrangement, the hot and cold fluids enter at the same end, flow in the same direction, and leave at the same end. In the **counter flow** arrangement, the fluids enter at opposite ends, flow in opposite directions, and leave at opposite ends.

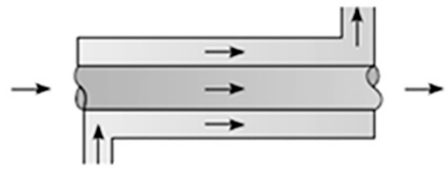


Figure: parallel flow

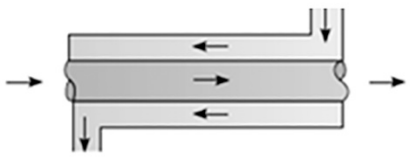
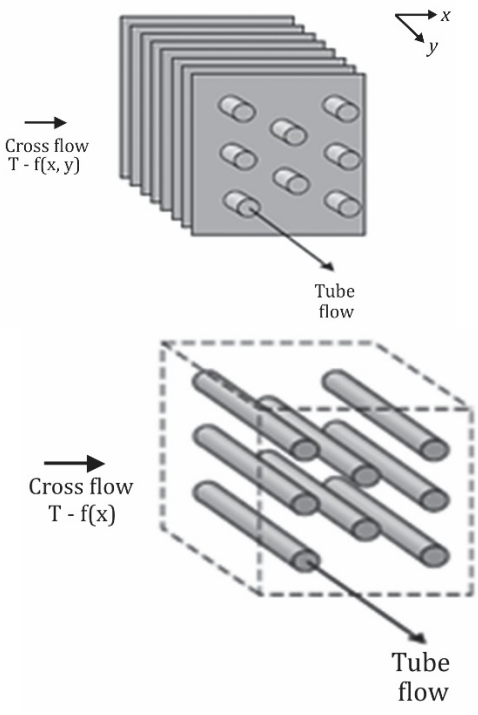


Figure: counter flow

B. Cross flow

The fluids may move in cross flow (perpendicular to each other), as shown by the finned and unfinned tubular heat exchangers of Figure. The two configurations are typically differentiated by an idealization that treats fluid motion over the tubes as unmixed or mixed. The fluid is said to be unmixed because the fins inhibit motion in a direction (y) that is transverse to the main-flow direction (x). In this case the fluid temperature varies with x and y. In contrast, for the unfinned tube bundle fluid motion, hence mixing, in the transverse direction is possible, and temperature variations are primarily in the main flow direction. Since the tube flow is unmixed, both fluids are unmixed in the finned exchanger, while one fluid is mixed and the other unmixed in the unfinned exchanger. The nature of the mixing condition can significantly influence heat exchanger performance.



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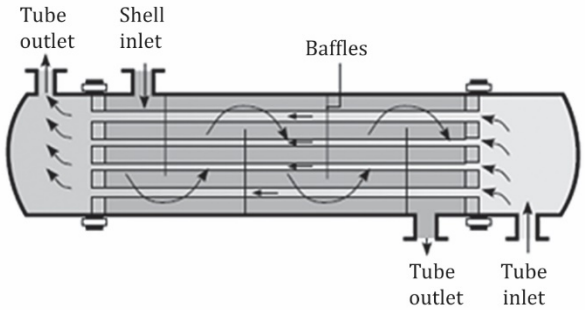
C. Shell & tube

Tubes heat exchanger contain a large number of tubes (sometimes several hundred) packed in a shell with their axes parallel to that of the shell. Heat transfer takes place as one fluid flows inside the tubes while the other fluid flows outside the tubes through the shell. Baffles are usually installed to increase the convection coefficient of the shell-side fluid by inducing turbulence and a cross-flow velocity component. In addition, the baffles physically support the tubes, reducing flow-induced tube vibration. Shell and tube heat exchanger are further classified according to the number of shell and tube passes involved. Heat exchanger in which all the tubes make one U-turn in the shell, for example, are called one shell pass and two tube pass heat exchangers. Similarly, a heat exchanger that involves two passes in the shell and four passes in the tubes is called a two shell pass and four tube pass heat exchanger. The static type regenerative heat exchanger is basically a porous mass that has a large heat storage capacity, such as a ceramic wire mesh



D. Regenerator

The hot fluid is passed through certain medium called matrix. The heat is transferred to solid matrix & accumulator. The operation is called heating period the heat stored in the matrix is subsequently transferred to the cold fluid by allowing it pass over the heated matrix.

5.2 OVERALL HEAT TRANSFER COEFFICIENT

A heat exchanger typically involves two flowing fluids separated by a solid wall. Heat is first transferred from the hot fluid to the wall by convection, through the wall by conduction, and from the wall to the cold fluid again by convection. In the analysis of heat exchangers, it is convenient to combine all the thermal resistances in the path of heat flow from the hot fluid to the cold one into a single resistance 'R_t'.

The rate of heat transfer between hot fluid and cold fluid is

$$Q = \frac{\Delta T}{\sum R_t} = UAT$$

Where, U is the overall heat transfer coefficient, whose unit is $W/m^{2}\cdot{}^{\circ}C$

For plane surface

$$UA = \frac{1}{1/A_i h_i + \delta/kA + 1/A_o h_o}$$

For $A_i = A_o = A$

$$UA = \frac{A}{1/h_i + \delta/k + 1/h_o}$$

When the wall thickness of the tube is small and the thermal conductivity of the tube material is high, as usually is the case, the thermal resistance of the tube is negligible ($R_{wall} \approx 0$).

Fins are commonly used on the gas side to enhance the product UA and thus the heat transfer on that side. When the tube is finned on one side to enhance heat transfer, the total heat transfer surface area on the fined side becomes where A_{fin} is the surface area of the fins and $A_{unfinned}$ is the area of unfinned portion of the tube surface.

For Cylindrical Surface

$$U_i A_i = \frac{1}{\left(1/A_i h_i\right) + \left[\frac{\ln\left(r_o/r_i\right)}{2KL}\right] + \left(1/A_o h_o\right)}$$

For $A_i = 2\pi r_i l, A_o = 2\pi r_o l$

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$$U_i = \frac{1}{\frac{1}{h_i} + \left(\frac{r_i}{k}\right)\ln\left(\frac{r_o}{r_i}\right) + \left(\frac{r_o}{r_i}\right)h_o}$$

$$U_o = \frac{1}{\left(\frac{r_o}{r_i}\right)\frac{1}{h_i} + \left(\frac{r_o}{k}\right)\ln\left(\frac{r_o}{r_i}\right) + \frac{1}{h_o}}$$

Fouling

The phenomena of rust formation and deposition of fluid impurities is called fouling. The performance of heat exchangers usually deteriorates with time as a result of accumulation of deposits on heat transfer surfaces. The layer of deposits represents additional resistance to heat transfer and causes the rate of heat transfer in a heat exchanger to decrease. The net effect of these accumulations on heat transfer is represented by a fouling factor R_f, which is a measure of the thermal resistance introduced by fouling.

The overall heat transfer coefficient relation is valid for clean surfaces and needs to be modified to account for the effects of fouling on both the inner and the outer surfaces of the tube.

$$R_{si} = \frac{1}{A_i.h_{si}}, R_{so} = \frac{1}{A_o.h_{so}}$$

$$UA = \frac{1}{\left(\frac{1}{A_i h_i}\right) + \left(\frac{1}{A_i h_{si}}\right) + \left[\frac{1}{2\pi k l}\ln\ln\left(\frac{r_o}{r_i}\right)\right] + \left(1/A_o h_{so}\right) + \left(1/A_o h_o\right)}$$

$$R_f = \frac{1}{U_{dirty}} - \frac{1}{U_{clean}}$$

5.3 HEAT EXCHANGER ANALYSIS

To design or to predict the performance of a heat exchanger, it is essential to relate the total heat transfer rate to quantities such as the inlet and outlet fluid temperatures, the overall heat transfer coefficient, and the total surface area for heat transfer.

1. Heat exchanger is a steady flow device
2. The heat exchanger is insulated from its surroundings, in which case the only heat exchange is between the hot and cold fluids.
3. Axial conduction along the tubes is negligible.
4. Potential and kinetic energy changes are negligible.
5. The fluid specific heats are constant.
6. The overall heat transfer coefficient is constant.

5.3.1Energy balance

Heat lost by hot fluid

$Q_h = m_h c_h (T_{h,i} - T_{h,o})$

Heat gained by cold fluid

$Q_c = m_c c_c (T_{c,i} - T_{C,i})$

In heat exchanger analysis, it is often convenient to combine the product of the mass flow rate and the specific heat of a fluid into a single quantity. This quantity is called the heat capacity rate and is defined as

$C = \dot{m}c$

Note that the only time the temperature rise of a cold fluid is equal to the temperature drop of a hot fluids is when the heat capacity rates of the two fluids are equal to each other.

The rate of heat transfer in a heat exchanger can also be expressed in an analogous manner to Newton's law of cooling as

$Q = UA\Delta T_m$

Where U is the overall heat transfer coefficient, A is the heat transfer area, and ΔT_m is an appropriate average temperature difference between the two fluids.

Heat lost by hot fluid is equal to heat gained by cold

$Q_h = Q_c = Q_{ex}$

5.4 LOG ARITHMIC TEMPERATURE DIFFERENCE (LMTD)

5.4.1 Double pipe heat exchangers

The temperature of the fluids changes in the direction of flow and consequently there occurs a change in the thermal heat causing the flow of heat. The form of T_m may be determined by applying an energy balance to differential elements in the hot and cold

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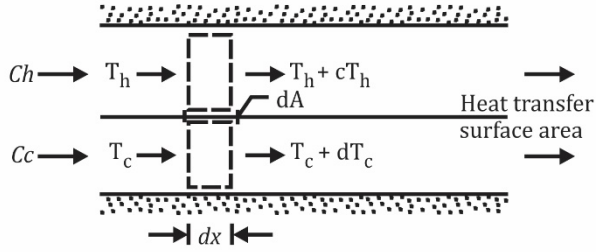
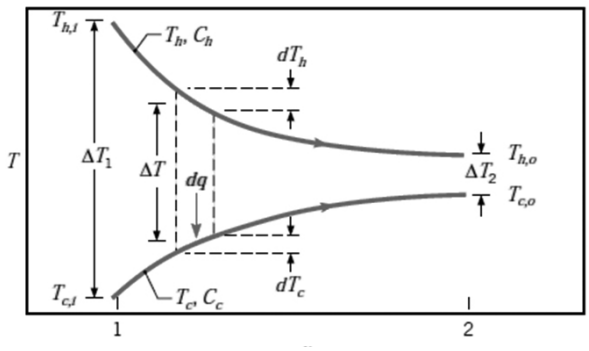
fluids. Each element is of length dx and heat transfer surface area dA,





By energy balance

$$dQ = UdA\left(T_h - T_c\right) = -\dot{m}C_h dT_h = \dot{m}C_c dT_c$$

-ve sign denotes decrease

$C_h = m_h c_h$, $C_c = m_c c_c$, Let $\theta = T_h - T_c$

or $d\theta = dT_h - dT_c$ Or

$$dT_h = \frac{dQ}{-C_h}, \quad dT_c = \frac{dQ}{C_c}$$

$$d\theta = -dQ\left[\frac{1}{C_c} + \frac{1}{C_h}\right]$$ For parallel flow heat exchange

$$d\theta = -dQ\left[\frac{1}{C_h} - \frac{1}{C_c}\right]$$ for counter flow heat exchanger

The heat transfer across the surface area dA may also be expressed as

$$dQ = UdA\left(T_h - T_c\right)$$ Or

$$dQ = UdA\,\theta = \frac{-d\theta}{\left(1/C_h \pm 1/C_c\right)}$$

By integrating the above expression

$$U\int_0^A dA = \frac{-1}{\left(\frac{1}{C_h} \pm \frac{1}{C_c}\right)}\int_{\theta_1}^{\theta_2}\frac{d\theta}{\theta}$$

+ve corresponds to parallel flow heat exchanger & -ve correspond is counters flow heat exchanger

$$\ln\frac{\theta_2}{\theta_1} = -UA\left(\frac{1}{C_h} \pm \frac{1}{C_c}\right)$$ &

$$Q = -\frac{\theta_2 - \theta_1}{\left(\frac{1}{C_h} \pm \frac{1}{C_c}\right)}$$

By combing above equations

$$Q = UA\left[\frac{\left(\theta_2 - \theta_1\right)}{\ln\ln\theta_2/\theta_1}\right]$$ Or

$$T_m = \left[\frac{\left(\theta_2 - \theta_1\right)}{\ln\ln\theta_2/\theta_1}\right]$$

For Parallel flow

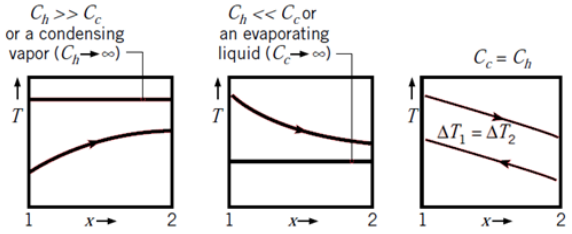
$\theta_1 = T_{h,i} - T_{c,i}, \theta_2 = T_{h,o} - T_{c,o}$

For counter flow

$\theta_1 = T_{h,i} - T_{c,o}, \theta_2 = T_{h,o} - T_{c,i}$

$\left(\text{LMTD}\right)_{\text{counter flow}} > \left(\text{LMTD}\right)_{\text{parallel flow}}$

Counter flow heat exchange can transfer more heat than a similar flow heat exchange. Conversely a counter flow exchanger needs a smaller heating surface for same rate of heat transfer.



Special heat exchangers a) Condenser, b) Evaporator, c) A counter flow heat exchanger with equal heat capacities

In a counter-flow heat exchanger, the temperature difference between the hot and cold fluids will remain constant along the heat exchanger when the heat capacity rates of the two fluids are equal (that is,ΔT=constant when C_h=C_c or $m_h C_{ph}$=$m_c C_{pc}$). Then we have ΔT_1=ΔT_2, and

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the log mean temperature difference relation above gives $\Delta T_{Lm}=\frac{0}{0}$, which is indeterminate. It can be shown by the application of L' hospital's rule that in this case we have $\Delta T_{Lm}=\Delta T_1=\Delta T_2$, as expected.

5.4.2 Multipass and Cross Flow heat Exchangers Use of a Correction Factor

For cross flow and multipass shell & tube heat exchangers

$$\Delta T_{lm}=F\left(\Delta T_{lm}\right)_{CF}$$

F is the correction Factor

$$\Delta T_{lm}=\frac{\Delta T_1-\Delta T_2}{\ln\left(\Delta T_1/\Delta T_2\right)}$$

$$\Delta T_1=\Delta T_{h,in}-\Delta T_{c,out}$$

$$\Delta T_2=\Delta T_{h,out}-\Delta T_{c,in}$$

F depends on Geometry & inlet outlet temperatures

In Literature plot of CF & P are given for various multipass heat exchangers.

$$\text{where , } P=\frac{t_2-t_1}{T_1-t_1}, R=\frac{T_1-T_2}{t_2-t_1}$$

$$t_2\to T_{c,out}, t_1\to T_{h,in}, t_1\to T_{c,in}, t_2\to T_{h,out}$$

Note: The correction Factor F = 1 for condenser / Boiler.

5.5 EFFECTIVENESS AND NUMBER OF TRANSFER UNITS (NTU).

Heat exchanger design methodology is based on known fluid inlet temperature, estimated heat transfer coefficients. Unknown parameters such outlet temperature or the surface area is to be determined. LMTD method is suitable for designing new heat exchanger whose inlet, outlet temperatures are known. If only the inlet temperatures are known, use of the LMTD method requires a cumbersome iterative procedure. It is therefore preferable to use an alternative approach termed the effectiveness–NTU (or NTU) method.

Capacity Ratio

It is defined as ratio of minimum to maximum heat capacity rate

$$\text{if } m_h c_h < m_c c_c : C=\frac{m_h c_h}{m_c c_c}$$

OR

$$\text{if } m_c c_c < m_h c_h : C=\frac{m_c c_c}{m_h c_h}$$

$$\Delta T_h=\frac{Q}{m_c h_c}, \Delta T_c=\frac{Q}{m_c c_c}$$

Heat Exchange effectiveness (ϵ)

Heat exchanger analysis is the determination of the heat transfer rate and the outlet temperatures of the hot and cold fluids for prescribed fluid mass flow rates and inlet temperatures when the type and size of the heat exchanger are specified. The heat transfer surface area A of the heat exchanger in this case is known, but the outlet temperatures are not.

The effectiveness of heat exchanger is defines as

$$\epsilon=\frac{Q}{Q_{max}}=\frac{\text{Actual heat transfer rate}}{\text{Maximum possible heat transfer rate}}$$

Actual heat transfer is determined by energy balance

$$Q_{act}=m_h c_h\left(T_{h,i}-T_{h,o}\right)=m_c c_c\left(T_{c,o}-T_{c,i}\right)$$

The heat transfer in a heat exchanger will reach its maximum value when

1) The cold fluid is heated to the inlet temperature of the hot fluid
2) The hot fluid is cooled to the inlet temperature of the cold fluid. Smaller heat capacity rate will experience a larger temperature change. Therefore, the maximum heat transfer rate in a heat exchanger will occur for fluid with lower hear capacity rate

The maximum possible heat transfer rate is given

$$Q_{max}=C_{min}\left(T_{h,i}-T_{C,i}\right)$$

$\left(T_{h,i}-T_{C,i}\right)$ is max temp difference for any heat exchange.

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NTU (Number of Transfer Units)

NTU is a measure of the size of heat exchanger. It provides indication of the size of heat exchanger. Effectiveness relations of the heat exchangers typically involve the dimensionless group UA/C_{min}. This quantity is called the number of transfer units NTU

$$\boxed{NTU = \frac{UA}{C_{min}}}$$

Effectiveness of Parallel flow heat exchanger

From LMTD analysis it can be obtained

$$\ln\left(\frac{\theta_1}{\theta_2}\right) = UA\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

$$\ln\left(\frac{T_{h,i} - T_{c,i}}{T_{h,o} - T_{c,o}}\right) = UA\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

From the definition of effectiveness

$$\epsilon = \frac{m_h c_h \left(T_{h,i} - T_{h,o}\right)}{C_{min}\left(T_{h,i} - T_{C,i}\right)} = \frac{m_c c_c \left(T_{c,o} - T_{c,i}\right)}{C_{min}\left(T_{h,i} - T_{C,i}\right)}$$

Or $$T_{h,o} = T_{h,i} - \frac{C_{min}\left(T_{h,i} - T_{C,i}\right)}{m_h c_h}\epsilon,$$

$$T_{c,o} = T_{c,i} + \frac{C_{min}}{m_c c_c}\left(T_{h,i} - T_{C,i}\right)\epsilon$$

By rearranging above equations

$$\frac{T_{h,o} - T_{c,o}}{\left(T_{h,i} - T_{C,i}\right)} = \left[1 - \epsilon C_{min}\left\{\frac{1}{C_h} + \frac{1}{C_c}\right\}\right] \quad \text{Or}$$

$$\ln\left[1 - \epsilon C_{min}\left\{\frac{1}{C_h} + \frac{1}{C_c}\right\}\right] = -UA\left\{\frac{1}{C_h} + \frac{1}{C_c}\right\}$$

The effectiveness of parallel flow heat exchanger is give by

$$\epsilon = \frac{1 - \exp\left[\frac{-UA}{C_c}\left(1 + \frac{C_c}{C_h}\right)\right]}{\left(1 + \frac{C_c}{C_h}\right)\frac{C_{min}}{C_c}}$$

In above Equation take either C_c or C_h to be C_{min} (both approaches give the same result).

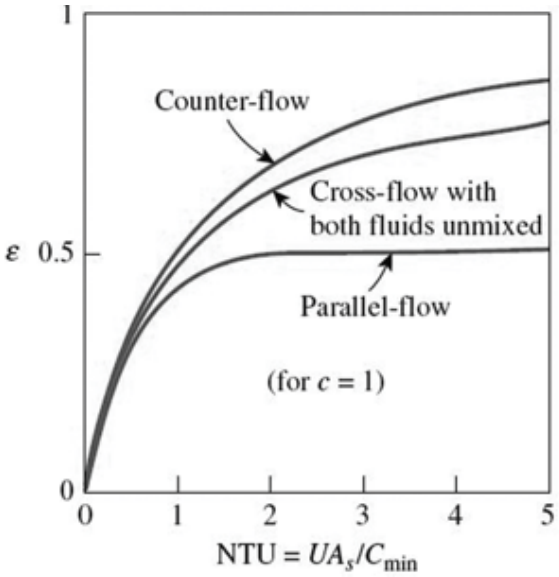
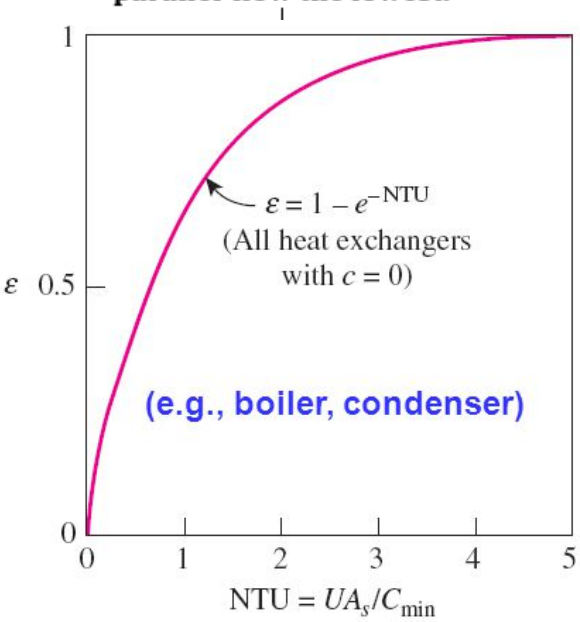


FIGURE: For a specified NTU and capacity ratio C. the counter-flow heat exchanger has the highest effectiveness and the parallel-flow the lowest.



FIGURE

The effectiveness relation reduces to $\varepsilon = \varepsilon_{max} = 1 - \exp(-NTU)$ for all heat exchangers when the capacity ratio $c = 0$.

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$$\epsilon_{\text{parallel flow}} = \frac{1-\exp\left[\frac{-UA}{C_{min}}\left(1+\frac{C_{min}}{C_{max}}\right)\right]}{\left(1+\frac{C_{min}}{C_{max}}\right)}$$

$$\epsilon_{\text{parallel flow}} = \frac{1-\exp\left[-NTU(1+C)\right]}{(1+C)}$$

Where, $C = C_{min}/C_{max}$ is the capacity ratio & $NTU = UA/C_{min}$

Effectiveness of Counter flow heat exchanger

By carrying the analysis similar to the parallel flow heat exchanger we obtain

$$\epsilon_{\text{counter flow}} = \frac{1-\exp\left[\frac{-UA}{C_{min}}\left(1-\frac{C_{min}}{C_{max}}\right)\right]}{1-\frac{C_{min}}{C_{max}}\exp\left[\frac{-UA}{C_{min}}\left(1-\frac{C_{min}}{C_{max}}\right)\right]}$$

$$\epsilon_{\text{counter flow}} = \frac{1-\exp\left[-NTU(1-C)\right]}{1-C\exp\left[-NTU(1-C)\right]}, \text{ For } C<1$$

$$\epsilon_{\text{counter flow}} = \frac{NTU}{1+NTU}, \text{ For } C=1$$

Points to remember

1. Form the figure it is evident that with increase in NTU, effectiveness increases but NTU larger than 3, is not suitable large size cannot be justified economically.
2. For same capacity ratio & NTU, $\epsilon_{\text{counter flow}} > \epsilon_{\text{parallel flow}}$
3. For given NTU, lower capacity ratio heat exchanger has higher effectiveness.
4. Effectiveness becomes maximum for C = 0 and min for C = 1.The case$C = C_{min}/C_{max} \to 0, C_{max} \to \infty$, this means that fluid is undergoing phase change.
5. For condenser or boiler c = 0, and effectiveness for all type of heat exchangers is given by
   $\epsilon = 1 - \exp(-NTU)$

Example:1

In a food processing plant, a brine solution is heated from $-12^{\circ}C$ to $-6.5^{\circ}C$ in a double pipe parallel flow heat exchanger by water entering at $35^{\circ}C$ and leaving at $20.5^{\circ}C$ at the rate of $9kg/min$. Determine the heat exchanger area for an overall heat transfer coefficient of $860W/m^2K$. For water $c_p = 4.186\times10^3 J/kgK$.

Solution:

Heat transfer from the water, $Q = mc_p\Delta t$

$= 9\times4.186\times10^3(35-20.5)$

$= 546.27\times10^3 J/min$

$= 9104.5 J/s$

Log-mean temperature difference,

$$\theta_m = \frac{\theta_1-\theta_2}{\log_e(\theta_1/\theta_2)}$$

For parallel flow arrangement:

$\theta_1 = t_{h1} - t_{c1} = 35-(-12) = 47^{\circ}C$

$\theta_2 = t_{h2} - t_{c2} = 20.5-(-6.5) = 27^{\circ}C$

The subscripts h and c represent the hot and cold fluids respectively:

$$\therefore \theta_m = \frac{47-27}{\log_e(47/27)} = 36.10^{\circ}C$$

Heat exchange, $Q = UA\theta_m$

$\therefore$ Heating surface area,

$$A = \frac{Q}{U\theta_m} = \frac{9104.5}{860\times36.10} = 0.293m^2$$

Example:2

For a double-pipe heat exchanger in which the overall heat transfer coefficient varies linearly with the temperature difference i.e., $U = a+b\theta$, determine an expression for the heat transfer using subscripts 1 and 2 to represent values at each end of the exchanger.

Solution:

For a parallel flow heat exchanger, the heat flow dQ through an elementary strip of area dA is given by:

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$$dQ = -m_h c_h dt_h = m_c c_c dt_c$$

$$= -C_h dt_h = C_c dt_c$$

$$dt_h = -\frac{dQ}{C_h} \text{ and } dt_c = \frac{dQ}{C_c}$$

$$d(t_h - t_c) = -dQ\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

$$dQ = \frac{-d(t_h - t_c)}{1/C_h + 1/C_c} = \frac{-d\theta}{1/C_h + 1/C_c} \quad .....(i)$$

Also $dQ = UdA\theta = (a + b\theta)dA\theta$ ...(ii)

Through integration of expression (i)

$$Q = \frac{\theta_1 - \theta_2}{1/C_h + 1/C_c}; \left(\frac{1}{C_h} + \frac{1}{C_c}\right) = \frac{\theta_1 - \theta_2}{Q} \text{ ...(iii)}$$

From expression (i) and (ii)

$$\left(a + b\theta\right)dA\theta = \frac{-d\theta}{1/C_h + 1/C_c}$$

$$= \frac{-d\theta}{(\theta_1 - \theta_2)/Q} = -\left(\frac{Qd\theta}{\theta_1 - \theta_2}\right)$$

Separating the variable and integrating between the inlet and outlet sections of the heat exchanger:

$$\int_{\theta_1}^{\theta_2} \frac{d\theta}{\theta(a + b\theta)} = -\left(\frac{\theta_1 - \theta_2}{Q}\right)\int_0^A dA$$

$$\frac{\theta_1 - \theta_2}{Q}A = \frac{1}{a}\log_e\left[\frac{\theta}{a + b\theta}\right]_{\theta_1}^{\theta_2}$$

$$= \frac{1}{a}\log_e\left[\frac{\theta_2}{a + b\theta_2} - \frac{\theta_1}{a + b\theta_1}\right]$$

$$= \frac{1}{a}\log_e\left[\frac{\theta_2(a + b\theta_1)}{\theta_1(a + b\theta_2)}\right] = \frac{1}{a}\log_e\left(\frac{\theta_2 U_1}{\theta_1 U_2}\right) \text{..(iv)}$$

From the identities

$U_1 = a + b\theta_1$ and $U_2 = a + b\theta_{2,}$ the constant a is found to be

$$a = \frac{U_1\theta_2 - U_2\theta_1}{\theta_2 - \theta_1}$$

Upon substitution of this value in the expression (iv)

$$-\left(\frac{\theta_1 - \theta_2}{Q}\right)A = \frac{\theta_2 - \theta_1}{U_1\theta_2 - U_2\theta_1} = \frac{1}{a}\log_e\left(\frac{\theta_2 U_1}{\theta_1 U_2}\right)$$

Therefore, the heat exchange rate is :

$$Q = \frac{U_1\theta_2 - U_2\theta_1}{\log_e(\theta_2 U_1 / \theta_1 U_2)}A$$

Example:3

A counter-flow concentric tube heat exchanger is used to cool the lubricating oil of a large industrial gas turbine engine. The oil flows through the tube at $0.19kg/s(c_p = 2.18kJ/kgK)$, and the coolant water flows in the annulus in the opposite direction at a rate of $0.15kg/s(c_p = 4.18kJ/kgK)$. The oil enters the coolant at 425 K and leaves at 345 K while the coolant enters at 285 K. How long must the tube be made to perform this duty if the heat transfer coefficient from oil to tube surface is $2250W/m^2K$ and from tube surface to water is $5650W/m^2K$? The tube has a mean diameter of 12.5 mm and its wall presents negligible resistance to heat transfer.

Solution: A heat balance over the exchanger gives:

heat lost by oil = heat gained by water

$$m_o c_o(t_{o1} - t_{o2}) = m_w c_w(t_{w2} - t_{w1})$$

The subscripts o and w refer to oil and water respectively .Inserting the appropriate values:

$$0.19 \times 2.18(425 - 345) = 0.15 \times 4.18(t_{w2} - 285)$$

∴ Outlet temperature of water;

$$t_w = 285 + \frac{0.19 \times 2.18 \times 80}{0.15 \times 4.18} = 337.85K$$

From an energy balance on the oil, the heat transfer is given by

$$Q = m_o c_o(t_{o1} - t_{o2})$$

$$= 0.19 \times 2.18(425 - 345) = 33.136kJ/s$$

$$= 33.136 \times 10^5 J/s$$

For counter current operation of the exchanger

$$\theta_1 = t_{o1} - t_{w2} = 425 - 337.85 = 87.15K$$

$$\theta_2 = t_{o2} - t_{w1} = 345 - 285 = 60K$$

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∴ log-mean temperature difference is,

$$\theta_m = \frac{\theta_1 - \theta_2}{\log_e(\theta_1/\theta_2)} = \frac{87.15-60}{\log_e(87.15/60)} = 72.79K$$

Let U be overall heat transfer coefficient between oil and water;

$$\frac{1}{U} = \frac{1}{h_0} + \frac{1}{h_w}$$

$$U = \frac{h_o h_w}{h_o + h_w} = \frac{2250\times5650}{2250+5650} = 1609.18W/m^2K$$

Heat exchanger, $Q = UA\theta_m$

∴ Heat surface area,

$$A = \frac{Q}{U\theta_m} = \frac{33.136\times10^3}{1609.18\times7279} = 0.2829m^2$$

The surface area also equals π d l where d and l represent the tube diameter and length respectively.

∴ Required length of the tube is

$$l = \frac{0.2829}{\pi\times12.5\times10^{-3}} = 7.21m$$

Example:4

Hot water having specific heat $4200J/kgK$ flows through a heat exchanger at the rate of 4 kg/min with an inlet temperature of $100°C$. A cold fluid having a specific heat $2400J/kgK$ flows in at a rate of 8 kg/min and with inlet temperature $20°C$. Make calculations for maximum possible effectiveness if the fluid flow conforms to (a) parallel flow arrangement (b) counter flow arrangement.

Solution: Thermal capacity rates of the hot and cold fluids are :

$$C_h = m_h c_h = \frac{4}{60}\times4200 = 280W/K$$

$$C_c = m_c c_c = \frac{8}{60}\times2400 = 320W/K$$

Obviously,

$C_{min} = 280W/K$ and $C_{max} = 320W/K$

Capacity ratio, $C = \frac{C_{min}}{C_{max}} = \frac{280}{320} = 0.875$

The effectiveness for a parallel flow heat exchanger is

$$\in = \frac{1-\exp\left[-NTU(1+C)\right]}{1+C}$$

Effectiveness will be maximum when $NTU = \infty$ for the given value of capacity ratio C.

∴ Maximum possible effectiveness

$$\frac{1-\exp\left[-\infty\right]}{1+C}$$

$$= \frac{1}{1+C} = \frac{1}{1+0.875} = 0.533$$

(b) The effectiveness for a counter flow heat exchanger is

$$\in = \frac{1-\exp\left[-NTU(1-C)\right]}{1-C\exp\left[-NTU(1-C)\right]}$$

Effectiveness will be maximum when $NTU = \infty$ for the given value of capacity ratio C

∴ Maximum possible effectiveness

$$= \frac{1-\exp(-\infty)}{1-C\exp(-\infty)} = \frac{1-0}{1-0} = 1$$

Example:5

A counter-flow exchanger of the surface area $8m^2$ is to be used to heat a process liquid by using a high temperature water available from another part of the plant. If the overall coefficient of heat transfer is $450W/m^2K$, find the exit temperatures of the process liquid and water stream from the data given below:

	Hot Fluid (water)	Cold Fluid (process liquid)
Inlet Temperature K	365	300
Mass Flow Rate kg/s	1.0	3.0
Specific Heat kJ/kgK	4.2	2.1

Solution:

Thermal capacity rate of the hot (water stream) and cooling (process liquid) fluid are:

$$C_h = m_c c_h = 1.0\times4.2 = 4.2kJ/sK$$

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$C_c = m_c c_c = 3.0 \times 2.1 = 6.3 kJ/sK$

Obviously,

$C_{min} = 4.2 kJ/sK$ and $C_{max} = 6.2 kJ/sK$

Capacity ratio

$C = C_{min}/C_{max} = 4.2/6.3 = 0.667$

Number of transfer units,

$$NTU = \frac{UA}{C_{min}} = \frac{450 \times 8}{4.2 \times 10^3} = 0.867$$

The effectiveness for a counter flow heat exchanger is,

$$\in = \frac{1 - \exp\left[-NTU(1-C)\right]}{1 - C\exp\left[-NTU(1-C)\right]}$$

$$= \frac{1 - \exp\left[-0.857(1-0.667)\right]}{1 - 0.667\exp\left[-0.857(1-0.667)\right]}$$

$$= \frac{1 - \exp\left[-0.285)\right]}{1 - 0.667\exp(-0.285)} = 0.495$$

Alternatively using the parameters

$C = 0.667$ and $NTU = 0.857$, we get

$\in = 0.495$

The hot fluid has the minimum thermal capacity and therefore in terms of temperature differences

$$\in = \left(\frac{t_{h1} - t_{h2}}{t_{h1} - t_{c1}}\right) \text{ or } 0.495 = \left(\frac{365 - t_{h2}}{365 - 300}\right)$$

$\therefore$ Outlet temperature of the hot fluid (water),

$t_{h2} = 365 - 0.495(365 - 300) = 332.82K$

From energy balance on the two fluids,

$m_h c_h (t_{h1} - t_{h2}) = m_c c_c (t_{c2} - t_{c1})$

or

$10 \times 4.2 \times (365 - 332.82) = 3.0 \times 2.1 (t_{c2} - 300)$

$\therefore$ Outlet temperature of the cold fluid (process liquid),

$$t_{c2} = 300 + \frac{1.0 \times 4.2 \times (365 - 332.82)}{3.0 \times 2.1} = 321.45K$$

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GATE QUESTIONS

Q.1 The overall heat transfer coefficient for a shell and tube heat exchanger for clean surfaces are U_o = 400 W/m² K. The fouling factor after one year of operation is found to be h_{do} = 2000 W/ m² K. The overall heat transfer coefficient at this time is

(A) 1200 W/m² K (B) 894 W/m² K
(C) 333 W/m² K (D) 287 W/m² K

[GATE-2001]

Q.2 If the baffle spacing in a shell and tube heat exchanger increases, then the Reynolds number of the shell side fluid

(A) Remains unchanged
(B) Increases
(C) Increases or decreases depending on number of shells passes
(D) Decreases

[GATE-2002]

Q.3 A double pipe countercurrent heat exchanger is designed to cool 3400 kg/hr of benzene flowing in the inner pipe from 80 °C to 35 °C. Water enters at 20 °C and exits at 37 °C in the annular space. The inside pipe has an inner diameter of 3.5 cm and wall thickness of 3.56 mm. The outer pipe has an inner diameter of 5.25 cm and is insulated. Neglecting the wall resistance to heat transfer from the inner pipe, and assuming the individual film heat transfer coefficient for water to be 6600 W/m² °C, Calculate :

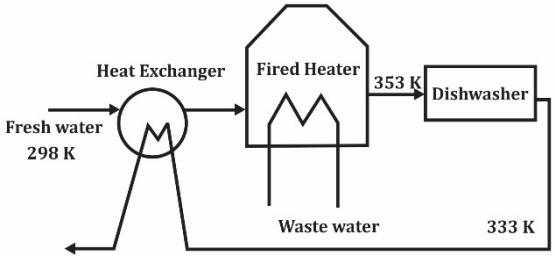
(a) The individual heat transfer coefficient for benzene flowing in the Inner Pipe.
(b) The overall heat transfer coefficient based on inside diameter of Inner Pipe.
(c) The total length required for the heat exchanger.

$$Nu = 0.023\ (Re)^{0.8}\ (Pr)^{0.3}$$

Where Nu is the Nusselt Number and Pr is the Prandtl Number.
Average properties of benzene :
Viscosity= 4×10^{-4} kg/m s,
Thermal conductivity=0.147 W/m °C
Specific heat capacity 1880 J/kg °C,
Density= 837 kg/m³.

[GATE-2002]

Q.4 A canteen requires hot water for its dish-washing. For this purpose, the canteen draws 0.4 kg/s of water at 298 K and heats it to 353 K in a fired-heater. The wastewater leaving the dishwasher is at 333 K. In order to save energy, it is proposed to recover heat from the wastewater to partially heat up the incoming water in a counter-current exchanger as shown schematically below :



Assume that there is no loss of water in the dishwasher and a minimum approach temperature of 10 K should be maintained in the exchanger.

(a) Determine the maximum temperature to which the incoming water can be heated in the exchanger.
(b) If the overall heat transfer coefficient in the exchanger is 1200 W/m² K and the specific heat capacity of water is 4186 J/kg K, compute the area of the heat exchanger.
(c) If the cost of the exchanger in 1982 was $2\times A^{0.41}$ (in lakh rupees), where A is the exchanger area in m² and the

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Marshall and Swift cost indices in 1982 and 2000 are 315 and 400 respectively, Determine the cost of the exchanger in the year 2000.

[GATE-2002]

Q.5 A process stream of dilute aqueous solution flowing at the rate of 10 kg/s is to be heated. Steam condensate at 95 °C is available for heating purpose, also at a rate of 10 kg/s. A 1-1 shell and tube heat exchanger is available. The best arrangement is
(A) counter flow with process stream on shell side
(B) counter flow with process stream on tube side
(C) parallel flow with process stream on shell side
(D) parallel flow with process stream on tube side

[GATE-2003]

Q.6 Steam is to be condensed in a shell and tube heat exchanger, 5 m long with a shell diameter of 1 m. Cooling water is to be used for removing the heat. Heat transfer coefficient for the cooling water whether on shell side or tube side is same. The best arrangement is
(A) vertical heat exchanger with steam on tube side
(B) vertical heat exchanger with steam on shell side
(C) horizontal heat exchanger with steam on tube side
(D) horizontal heat exchanger with steam on shell side

[GATE-2003]

Q.7 A fluid is flowing inside the inner tube of a double pipe heat exchanger with diameter 'D'. For a fixed mass flow rate, the tube side heat transfer coefficient for turbulent flow conditions is proportional to
(A) $D^{0.8}$ (B) $D^{-0.2}$
(C) D^{-1} (D) $D^{-1.8}$

[GATE-2003]

Q.8 Air is to be heated by condensing steam. Two heat exchangers are available
(i) a shell and tube heat exchanger, and
(ii) a finned tube heat exchanger
Tube side heat transfer area is equal in both cases.
The recommended arrangement is
(A) Finned tube heat exchanger with air inside and steam outside
(B) Finned tube heat exchanger with air outside and steam inside
(C) Shell and tube heat exchanger with air inside tubes and steam on shell side
(D) Shell and tube heat exchanger with air on shell side and steam inside tubes.

[GATE-2003]

Q.9 Hot water (0.01 m^3/min) enters the tube side of a co-current shell and tube heat exchanger at 80 °C and leaves at 50 °C. Cold oil (0.05 m^3/min) of density 800 kg/m^3 and specific heat of 2 kJ/kg-K enters at 20 °C. The log mean temperature difference in °C is approximately
(A) 32 (B) 37
(C) 45 (D) 50

[GATE-2004]

Q.10 Baffles are used in heat exchangers in order to
(A) Increase the tube side fluid's heat transfer coefficient
(B) Promote vibration in the heat exchanger
(C) Promote cross flow and turbulence in the shell side fluid
(D) Prevent shell expansion due to thermal effects.

[GATE-2005]

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Q.11 A counter-current flow double pipe heat exchanger is used to heat water flowing at 1 kg/s from 40 °C to 80 °C. Oil is used for heating and its temperature changes from 100 °C to 70 °C. The overall heat transfer coefficient is 300W/m² °C. If it is replaced by a 1-2 shell and tube heat exchanger with counter-current flow configuration with water flowing in shell and oil flowing in the tube, what is the excess area required with respect to the double pipe heat exchanger? The correction factor Ft for LMTD (Log Mean Temperature Difference) based on the above double pipe heat exchanger is 0.5. The heat transfer coefficient remains unchanged, and the same inlet and outlet conditions are maintained. Cp, water = 4180 J/kg °C Cp,oil = 2000 J/kg °C

(A) 0 m² (B) -20.15 m²
(C) 22.6 m² (D) 9.69 m²

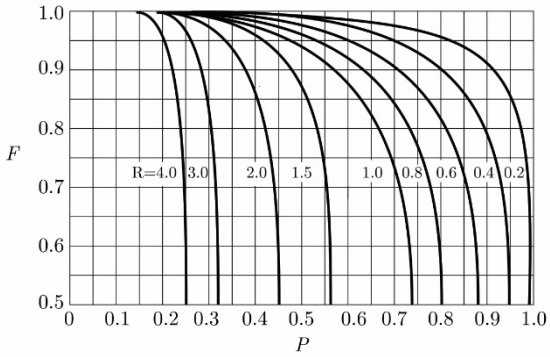
[GATE-2005]

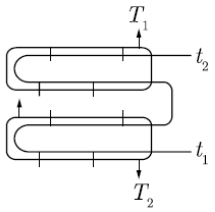
Q.12 Fluid flows in an annulus of inner diameter 0.8 m and outer diameter 1m. Heat is transferred to the fluid from inner tube surface of the annulus. What is the equivalent diameter for heat transfer in metre?

(A) 0.45 (B) 0.20
(C) 1.64 (D) 0.90

[GATE-2005]

Q.13 A process fluid has to be cooled from 22 °C to 2 °C using brine in a 2-4 shell and tube heat exchanger shown below. The brine enters at -3 °C and leaves at 7 °C . The overall heat transfer coefficient is 500W/m²-K. The design heat load is 30 kW. The brine flows on the tube side and the process fluid on the shell side. The heat transfer area in m² is





$$P=\frac{t_2-t_1}{T_1-t_1} \qquad Q=\frac{T_1-T_2}{t_2-t_1}$$

(A) 1.1 (B) 5.77
(C) 6.59 (D) 7.53

[GATE-2006]

Q.14 A hot fluid entering a well-stirred vessel is cooled by feeding cold water through a jacket around the vessel. Assume the jacket is well-mixed.
For the following data:
Mass flow-rate of the hot fluid =0.25 kg/s
Mass flow-rate of cold water = 0.4 kg/s
Specific heat of oil = 6000 J/kg–K
Specific heat of cold water = 4184 J/kg–K
The inlet and exit temperatures of the hot fluid are 150 °C and 100 °C respectively,
Inlet temperature of cold water = 20 °C
The overall heat transfer coefficient is 500 W/m²-K. The heat transfer area in m² is

(A) 1.82 (B) 2.1
(C) 3 (D)4.26

[GATE-2007]

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Q.15 The following list of options P, Q, R and S are some of the important considerations in the design of a shell and tube heat exchanger.
P : Square pitch permits the use of more tubes in a given shell diameter.
Q : The tube side clearance should not be less than one-fourth of the tube diameter.
R : Baffle spacing is not greater than the diameter of the shell or less than one-fifth of the shell diameter.
S : The pressure drop on the tube side is less than 10 psi.
Pick out the correct combination of true statements from the following
(A) P, Q and R
(B) Q, R and S
(C) R, S and P
(D) P, Q, R and S

[GATE-2007]

Q.16 A double-pipe heat exchanger is to be designed to heat 4 kg/s of a cold feed from 20 °C to 40 °C using a hot stream available at 160 °C and a flow rate of 1 kg/s.
The two streams have equal specific heat capacities and the overall heat transfer coefficient of the heat exchanger is 640 W/m^2-K. Then the ratio of heat transfer areas required for the co-current to counter-current modes of operations is
(A) 0.73 (B) 0.92
(C) 1.085 (D) 1.25

[GATE-2009]

Q.17 Which one of the following statements about baffles in a shell and tube heat exchanger is false? Baffles
(A) Act as a support to the tube bundle
(B) Reduce the pressure drop on the shell-side
(C) Alter the shell-side flow pattern
(D) Help in increasing the shell-side heat transfer coefficient

[GATE-2010]

Common Data Questions
Hot oil at 150 °C is used to pre-heat a cold fluid at 30 °C in a 1 : 1 shell and tube heat exchanger. The exit temperature of the hot oil is 110 °C. Heat capacities (product of mass flow rate and specific heat capacity) of both the streams are equal. The heat duty is 2 kW.

Q.18 Under co-current flow conditions, the overall heat transfer resistance (1/UA) is
(A) 0.4 °C /W (B) 0.04 °C /W
(C) 0.36 °C /W (D) 0.036 °C /W

[GATE-2010]

Q.19 Under counter-current flow conditions, the overall heat transfer resistance (1/UA) is
(A) 0.4 °C /W (B) 0.04 °C /W
(C) 0.36 °C /W (D) 0.036 °C /W

[GATE-2010]

Q.20 Oil at 120 °C is used to heat water at 30 °C in a 1-1 co-current shell and tube heat exchanger. The available heat exchange area is S_1. The exit temperatures of the oil and the water streams are 90 °C and 60 °C respectively. The co-current heat exchanger is replaced by a 1-1 counter-current heat exchanger having heat exchange area S_2. If the exit temperatures and the overall heat transfer coefficients are same, the ratio of S_1 to S_2 is
(A) ∞ (B) 1.1
(C) 0.91 (D) 0

[GATE-2011]

Q.21 In a counter-flow double pipe heat exchanger, oil (m = 2 kg/s, C_p = 2.1 kJ/kg.^{0}C) is cooled from 90^{0}C to 40^{0}C by water (m = 1 kg/s, C_p = 4.2 kJ/kg.^{0}C)

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which enters the inner tube at 10^0C. The radius of the inner tube is 3 cm and its length is 5 m. Neglecting the wall resistance, the overall heat transfer coefficient based on the inner radius, in kW/m^2 K, is

(A) 0.743 (B) 7.43
(C) 74.3 (D) 2475

[GATE-2012]

Q.22 In a 1-1 pass floating head type shell and tube heat exchanger, the tubes (od = 25 mm; id = 21 mm) are arranged in a square pitch. The tube pitch is 32 mm. The thermal conductivity of the shell side fluid is 0.19 W/m-K, and the Nusselt number is 200. The shell-side heat transfer coefficient (in W/m^2.K), rounded off to the nearest integer, is

(A) 1100 (B) 1400
(C) 1800 (D) 2100

[GATE-2012]

Q.23 The effectiveness of a heat exchanger in the $\varepsilon - NTU$ method is defined as

a) $\dfrac{\text{increase in temperature of the cold fluid}}{\text{decrease in temperature of the hot fluid}}$

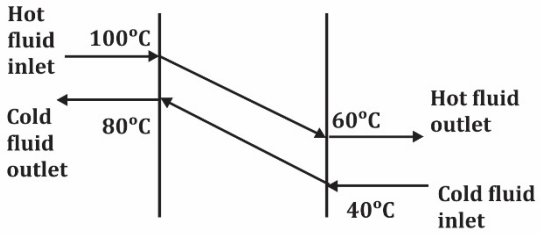
b) $\dfrac{\left(\begin{array}{l}\text{actual exit temperature}\\ \text{attained by the cold fluid}\end{array}\right)}{\left(\begin{array}{l}\text{maximum exit temperature}\\ \text{attainable by the cold fluid}\end{array}\right)}$

c) $\dfrac{\left(\begin{array}{l}\text{actual exit temperature}\\ \text{attained by the hot fluid}\end{array}\right)}{\left(\begin{array}{l}\text{maximum exit temperature}\\ \text{attainable by the hot fluid}\end{array}\right)}$

d) $\dfrac{\left(\text{actual heat transfer rate}\right)}{\left(\begin{array}{l}\text{possible heat transfer rate}\\ \text{from hot fluid to cold fluid}\end{array}\right)}$

[GATE-2013]

Q.24 In a double-pipe counter-current heat exchanger, the temperature profiles shown in the figure were observed. During operation, due to fouling inside the pipe, the heat transfer rate reduces to half of the original value. Assuming the flow rates and the physical properties of the fluids, do not change the LMTD (in $^\circ$C) in the new situation is



(A) 0 (B) 20
(C) 40 (D) Indeterminate

[GATE-2013]

Q.25 An oil with a flow rate of 1000 kg/h is to be cooled using water in a double-pipe counter-flow heat exchanger from a temperature of 70°C to 40°C. Water enters the exchanger at 25°C and leaves at 40°C. The specific heats of oil and water are 2 kJkg^{-1}K^{-1} and 4.2 kJkg^{-1}K^{-1}, respectively. The overall heat transfer coefficient is 0.2 kWm^{-2}K^{-1}. The minimum heat exchanger area (in m^2) required for this operation is

(A) 0.69 (B) 3.85
(C) 10.56 (D) 56.32

[GATE-2014]

Q.26 In a 1-1 pass shell and tube heat exchanger, steam is condensing in the shell side at a temperature (T_s) of 135°C and the cold fluid is heated from a temperature (T_1) of 20°C to a

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temperature of 90°C. The energy balance equation for this heat exchanger is

$$\ln\frac{T_s - T_1}{T_s - T_2} = \frac{UA}{mC_p}$$

Where U is the overall heat transfer coefficient, A is the heat transfer area, m is the mass flow rate of cold fluid and C_p is the specific heat. Tube side fluid is in a turbulent flow and the heat transfer coefficient can be estimated from the following equation:

$$Nu = 0.023(\mathrm{Re})^{0.8}(\mathrm{Pr})^{1/3}$$

Where Nu is the Nusselt number, Re is the Reynolds number and Pr is the Prandtl number. The condensing heat transfer coefficient in the shell side is significantly higher than the tube side heat transfer coefficient. The resistance of the wall to the heat transfer is negligible. If only the mass flow rate of the cold fluid is doubled, what is the outlet temperature (in °C) of the cold fluid at steady state?

(A) 80.2 (B) 84.2

(C) 87.4 (D) 88.6

[GATE-2016]

Q.27 In a heat exchanger, the inner diameter of a tube is 25 mm and its outer diameter is 30 mm. The overall heat transfer coefficient based on the inner area is 360 W/m^2.°C. Then, the overall heat transfer coefficient based on the outer area, rounded to the nearest integer, is ___ W/m^2.°C.

[GATE-2017]

Q.28 A hot liquid is to be cooled in a 1-1 shell and tube heat exchanger from 80°C to 50°C. Cooling water enters the tube side at 30°C, and exits at 45 °C. The properties of the liquids are constant. Also, the overall heat transfer coefficient is same for counter- current and co-current modes. The percentage saving in the heat transfer area for counter-current option with respect to the concurrent option is _____ (rounded off to third decimal place).

[GATE-2018]

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ANSWER KEYS

1	2	3(a)	3(b)	3(c)	4(a)	4(b)	4(c)	5	6
(C)	(D)	(1428)	(1210)	(22.6)	(323)	(3.48)	(4.23)	(B)	(B)
7	8	9	10	11	12	13	14	15	16
(D)	(B)	(A)	(C)	(C)	(D)	(D)	(A)	(B)	(C)
17	18	19	20	21	22	23	24	25	26
(B)	(D)	(B)	(B)	(B)	(B)	(D)	(C)	(B)	(B)
27	28								
(300)	(27.08)								

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EXPLANATIONS

Q.1 (C)

Given: $U_0 = 400\frac{W}{m^2k}$,

Fouling factor, $h_{d_0} = 2000\frac{W}{m^2k}$

$$\frac{1}{U_{dirty}} = \frac{1}{h_0} + \frac{1}{U_{clean}}$$

$$= \frac{1}{2000} + \frac{1}{400}$$

$$= \frac{2400}{(2000)(400)}$$

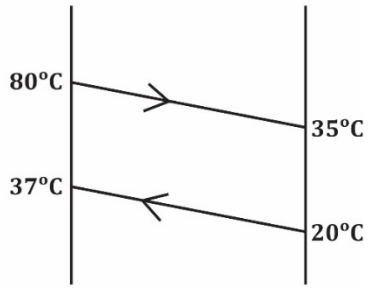
$$\frac{1}{U_{dirty}} = \frac{3}{1000}$$

$$U_{dirty} = 333\frac{W}{m^2K}$$

Q.2 (D)

Decreases

Q.3



Given

$Nu = 0.023\ Re^{0.8}\ Pr^{0.3}$

di = 0.035 m

do = 0.03856 m

Benzene properties:

$C_p = 1880$ J/kg °C

$\rho = 837\ \frac{kg}{m^3}$

$\mu = 4 \times 10^{-4}\ \frac{kg}{m\ s}$

$k = 0.147\ \frac{W}{m\ ^oC}$

(a) (1428)

$$Re = \frac{d\,G}{\mu} = \frac{0.035 \times \frac{3400}{3600\left(\frac{\pi}{4}(0.035)^2\right)}}{4 \times 10^{-4}} = 88416.13$$

$$Pr = \frac{\mu\, C_p}{k} = \frac{(1880)\left(4 \times 10^{-4}\right)}{0.147} = 5.11$$

$$So, Nu = \frac{h_i D}{k} = (0.023)(88416.13)^{0.8}(5.11)^{0.3}$$

$$h_i = \frac{340 \times 0.147}{0.035}$$

$$h_i = 1428\frac{J}{m^2\ ^oC\ s}$$

(b) (1210)

$$\frac{1}{U_i d_i} = \frac{1}{h_i d_i} + \frac{1}{h_0 d_0}$$

$$\frac{1}{U_i} = \frac{1}{h_i} + \frac{d_i}{h_0 d_0}$$

$$\frac{1}{U_i} = \frac{1}{12425} + \frac{1}{6600}\frac{(35)}{3.5 + 2(0.356)}$$

$$U_i = 1210\ \frac{W}{m^2\ ^oC}$$

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(c) (22.6)

$$m\,C_p\,\Delta T = U_i A_i\,\Delta T_{LMTD}$$

$$\left(\frac{3400}{3600}\times 1880\times 45\right) = \left(1210\times \pi D_i L\right)\left[\frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)}\right]$$

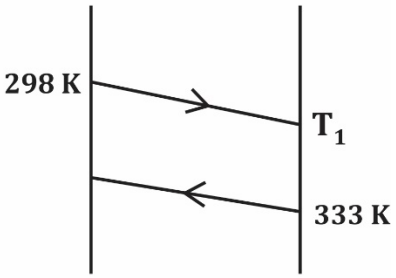
$$\frac{79,900}{(1210)\pi(0.035)} = L\left[\frac{43-15}{\ln\left(\frac{43}{15}\right)}\right]$$

$$600.84 = L(26.58)$$

$$L = 22.6\,m$$

Q.4

(a) (323)



$$T_1 - 333 = 10\,K$$

$$T = 323\ K$$

(b) (3.48)

$$U = 1200\ \frac{W}{m^2\,^oC},\ C_p = 4186\ \frac{J}{kg\,K}$$

$$m\,C_p\Delta T = U\,A\,\Delta T_{LMTD}$$

$$(0.4)(4186)(323-298) = (1200)A(10)$$

$$A = 3.48\,m^2$$

(C) (4.23)

		Cost index
[1982]	cost $= 2\times A^{0.41}$	315
[2000]	cost $= C_{2000}$	400

$$\frac{C_{2000}}{C_{1982}} = \frac{400}{315}$$

$$C_{2000} = \frac{400}{315}\times 2\times(3.48)^{0.41}$$

$$C_{2000} = 4.23\ \text{lakh}$$

Q.5 (B)

Counter flow with process stream on tube side.

Q.6 (B)

Vertical Heat Exchange with steam on shell side.

Q.7 (D)

From Dittus–Boelter Equation

$$Nu = 0.023\,(Re)^{0.8}\,(Pr)^n$$

$$\begin{pmatrix} n = 0.4, \text{ for fuild heated} \\ n = 0.3, \text{ for fluid cooled} \end{pmatrix}$$

$$\frac{hD}{k} \propto \left(\frac{\rho \bar{V} D}{\mu}\right)^{0.8}$$

$$hD \propto \left(\bar{V}D\right)^{0.8}$$

$$h.D \propto \left[\frac{\dot{m}}{\frac{\pi}{4}D^2}D\right]^{0.8}$$

$$h.D \propto \frac{1}{D^{0.8}}$$

$$h \propto D^{-1.8}$$

Q.8 (B)

Finned tube Heat exchange with air outside and steam inside.

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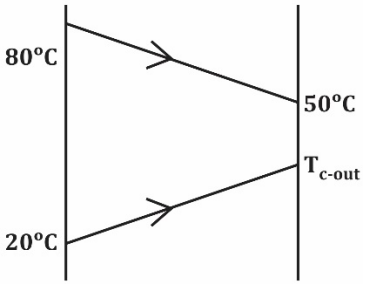
Q.9 (A)



$$V_{Hot} = 0.01\frac{m^3}{min}$$

$$V_{cold} = 0.05\ \frac{m^3}{min}$$

$$T_{h,in} = 80\ ^oC$$

$$T_{h,out} = 50\ ^oC$$

$$T_{c,in} = 20\ ^oC$$

$$m_{hot} \times C_{p\text{-}hot} \times \Delta T_{hot} = m_{cold} \times C_{p\text{-}cold} \times \Delta T_{cold}$$

$$\left[(0.01)(1000)\right] = \left[(0.05)(800)\right]$$
$$\left[(4.2)(80-50)\right] = \left[(2)(T_{c,out}-20)\right]$$

$$T_{c,out} = 35.75\ ^oC$$

$$\Delta T_{LMTD} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)}$$

$$\Delta T_1 = 80 - 20 = 60\ ^oC$$

$$\Delta T_2 = 50 - 30.75 = 14.25\ ^oC$$

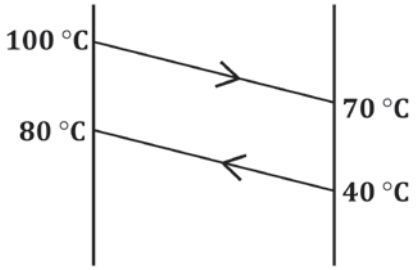
$$\Delta T_{LMTD} = \frac{60 - 14.25}{\ln\left(\frac{60}{14.25}\right)}$$

$$\Delta T_{LMTD} = 31.82\ ^oC$$
$$\approx 32\ ^oC$$

Q.10 (C)

Promote cross flow and turbulence in the shell side fluid

Q.11 (C)



$$T_{H\text{-}in} = 400\ ^oC$$

$$T_{H\text{-}out} = 70\ ^oC$$

$$T_{C\text{-}in} = 40\ ^oC$$

$$T_{C\text{-}out} = 80\ ^oC$$

$$\Delta T_1 = 20\ ^oC$$

$$\Delta T_2 = 30\ ^oC$$

$$\Delta T_{LMTD} = \frac{\Delta T_2 - \Delta T_1}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)} = \frac{30-20}{\ln\left(\frac{30}{20}\right)} = 24.66\ ^oC$$

$$Q_{(DPHE)} = UA_1\left[\Delta T_{LMTD}\right]$$

$$Q_{(STHE)} = UA_2F_t\left(\Delta T_{LMTD}\right)$$

So, $A_2 = 2A_1$

$$\text{Excess Area Required} = A_2 - A_1$$
$$= 2A_1 - A_1$$
$$= A_1$$

$$m_w c_{p\text{-}w} \Delta T_w = UA_1 \Delta T_{LMTD}$$

$$(1)\ (4180)(80-40) = (300)\,A_1\,(24.66)$$

$$A_1 = 22.60\,m^2$$

Q.12 (D)

$$\text{Hydralic Radius}\left(D_H\right) = \frac{\text{C/S Area of channel}}{\text{wetted parameter of channel}}$$

$$= \frac{\frac{\pi}{4}\left(D_0^2 - D_i^2\right)}{\pi\left(D_0 + D_i\right)} = \frac{1}{4}\left(D_0 - D_i\right)$$

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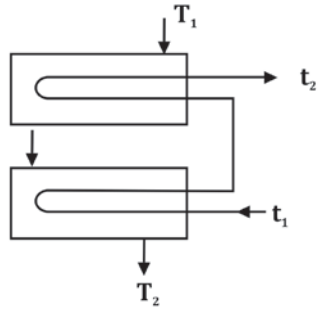
$$D_{equivalent} = 4\left(D_H\right)$$
$$= 4\left(\frac{1}{4}\right)\left(D_0 - D_i\right)$$
$$= \left(1 - 0.8\right) = 0.2\,m$$

Q.13 (D)



$T_1 = 22\ ^oC$

$T_2 = 2\ ^oC$

$t_1 = -3\ ^oC$

$t_2 = 7\ ^oC$

$$U = 500\ \frac{W}{m^2K}$$

$Q = 30,000$ watt

$$P = \frac{t_2 - t_1}{T_1 - t_1} = \frac{7 - (-3)}{22 - (-3)} = 0.4$$

$$R = \frac{T_1 - T_2}{t_2 - t_1} = \frac{22 - 2}{7 - (-1)} = \frac{20}{10} = 2$$

So, from graph, $F_t = 0.87$

$$\Delta T_{LMTD} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)} = \frac{(22-7)-(2-(-3))}{\ln\left[\frac{22-7}{2-(-3)}\right]}$$

$\Delta T_{LMTD} = 9.102\ ^oC$

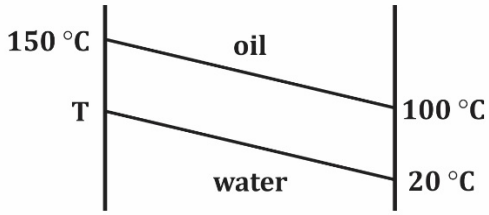
$\Delta T_{LMTD}^{\ *} = F_t \times \Delta T_{LMTD}$

$= (0.87)\,(9.102)$

$\Delta T_{LMTD} = 7.92\ ^oC$

$$Q = U\,A\,\Delta T_{LMTD}$$
$$30,000 = (500)\,A\,(7.92)$$
$$A = 7.53\ m^2$$

Q.14 (A)



Given : oil data

$$m_{oil} = 0.25\ \frac{kg}{s},\ T_i = 150\ ^oC$$

$$C_{p\text{-}oil} = 6000\ \frac{J}{kg.k},\ T_o = 100\ ^oC$$

water data:

$$\mu_{water} = 0.4\ \frac{kg}{s},\ t_i = 20\ ^oC$$

$$C_{pw} = 4184\ \frac{J}{kg\ K},$$

$$U = 500\frac{W}{m^2\ K}$$

$m_w\ C_{pw}\ \Delta T_w = m_o\ C_{po}\ \Delta T_o$

$(0.4)\,(4184)\,(T - 20) = (0.25)\,(6000)(150 - 100)$

$$T - 20 = \frac{0.25\,(6000)(50)}{(0.4)(4184)}$$

$T = 64.81\ ^oC$

$\Delta T_1 = (150 - 64.81) = 85.18\ ^oC$

$\Delta T_2 = 100 - 20 = 80\ ^oC$

$$\Delta T_{LMTD} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)} = \frac{85.18 - 80}{\ln\left(\frac{85.18}{80}\right)} = 82.56\ ^oC$$

$M_w C_{p_w}\ \Delta T_w = UA\ \Delta T_{LMTD}$

$(0.4)\,(4184)\,(64.81 - 20) = 500\ A\,(82.56)$

$A = 1.816\ m^2$

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Q.15 (B)

Q.16 (C)

$T_{h\text{-}in} = 160\ ^{o}C$

$T_{c\text{-}in} = 20\ ^{o}C$

$T_{c\text{-}out} = 40\ ^{o}C$

$m_h = 1\ kg/s$

$m_c = 4\ kg/s$

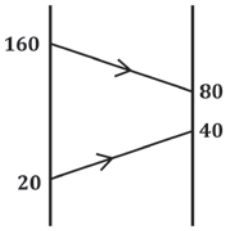
Energy Balance

$m_h . C_{p\text{-}h}\ \Delta T_h = m_c\ C_{p\text{-}c}\ \Delta T_c$

$(1)(160 - T_{h\text{-}out}) = 4\ (40 - 20)$

$T_{h\text{-}out} = 80\ ^{o}C$

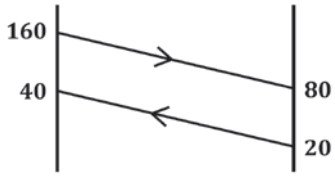
Co-Current Flow:



$$\Delta T_{LMTD\text{-}1}\,(\text{Co-current}) = \frac{140 - 40}{\ln \frac{140}{40}}$$

$\Delta\ T_{LMTD\text{-}1} = 79.82\ ^{o}C$

Counter-Current Flow:



$$\Delta T_{LMTD\text{-}2}\ (\text{Counter Current}) = \frac{120 - 60}{\ln \frac{120}{60}}$$

$$= 86.56\ ^{o}C$$

$Q = UA\ \Delta T_{LMTD}$

$$A \propto \frac{1}{\Delta T_{LMTD}}$$

Now by Reciprocity rule

$$\text{So, } \frac{A_{\text{counter current}}}{A_{\text{co-current}}} = \frac{\Delta T_{LMTD\text{-}2}}{\Delta T_{LMTD\text{-}1}} = \frac{86.56}{79.82} = 1.084$$

Q.17 (B)

Reduce pressure drop on the shell-side.

Q.18 (D)

$T_{h-in} = 150\ ^{o}C$

$T_{c-in} = 30\ ^{o}C$

$T_{h-out} = 110\ ^{o}C$

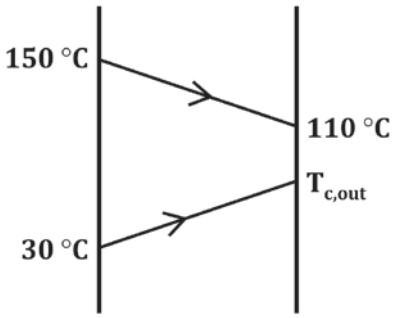
$(m\ C_p)_h = (m\ C_p)_c$

$Q = 2\ kW = 2000\ W$

Co-current Flow

$$(mC_p)_h\,(150 - 110) = (mC_p)_c\,(T_{c\text{-}out} - 30)$$

$T_{c-out} = 70\ ^{o}C$



$$\text{So, } \Delta T_{LMTD} = \frac{(150 - 30) - (110 - 70)}{\ln\left(\frac{150 - 30}{110 - 70}\right)}$$

$$= \frac{120 - 40}{\ln\left(\frac{120}{40}\right)} = 79.81$$

$Q = UA\ \Delta T_{LMTD}$

$$\frac{1}{UA} = \frac{79.82}{2000} = 0.036\ \frac{^{o}C}{W}$$

Q.19 (B)

Counter Current Flow:

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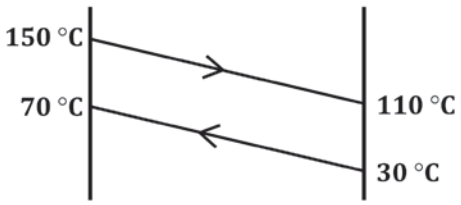


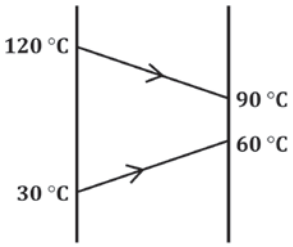
$$Q = UA\,\Delta T_{LMTD}$$

$$\frac{1}{UA} = \frac{80}{2000}$$

$$\frac{1}{UA} = 0.04\ \frac{^{o}C}{W}$$

Q.20 (B)

Case-I : Co-current Flow

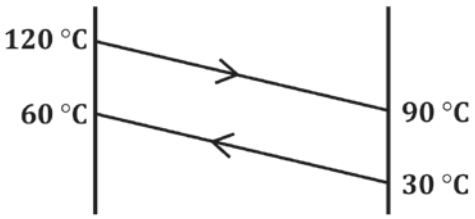


$$\Delta T_{LMTD\text{-}1} = \frac{(120-30)-(90-60)}{\ln\left(\frac{120-30}{90-60}\right)}$$

$$= \frac{90-30}{\ln\left(\frac{90}{30}\right)}$$

$$= 54.61$$

Case-II : Counter Current flow



$$\Delta T_{LMTD\text{-}2} = \frac{(120-90)+(60-30)}{2}$$

$$= 60\ ^{o}C$$

$$\frac{Q_1}{Q_2} = \frac{U\ S_1\ \Delta T_{LMTD\text{-}1}}{U\ S_2\ \Delta T_{LMTD\text{-}2}}$$

$$\frac{60}{54.61} = \frac{S_1}{S_2}$$

$$\frac{S_1}{S_2} = 1.098 = 1.1$$

Q.21 (B)

Given:

$$m_h = 2\frac{kg}{s},\ C_{p,h} = 2100\ \frac{J}{kg\ ^{\circ}C}$$

$$m_c = 1\frac{kg}{s},\ C_{p,c} = 4200\ \frac{J}{kg\ ^{\circ}C}$$

$$m_c C_{p,c} \Delta T_c = m_h C_{p,h} \Delta T_h$$

$$(1)(4200)(T-10) = (2)(2100)(90-40)$$

$$(T-10) = 50$$

$$T = 60\ ^{\circ}C$$

$$Q = U_i A_i \Delta T$$

$$(2)(2100)(90-40) = U_i\,(\pi \times 0.06 \times 5) \times 30$$

$$U_i = 7.427\ \frac{kW}{m^2}$$

Q.22 (B)

Given: O.D. $= 25\,mm = 0.025\,m$

I.D. $= 21\,mm = 0.021\,m$

Pitch $= 32\,mm = 0.032\,m$

$$k_{fluid} = 0.19\ \frac{W}{m\text{-}K}$$

$$Nu = 200$$

Equivalent diameter for square pitch:

$$D_e = \frac{P_t - \frac{\pi}{4}d_0^2}{\frac{\pi}{4}d_0} = \frac{(32)^2 - \frac{\pi}{4}(25)^2}{\frac{\pi}{4}(25)}$$

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$\therefore D_{eq.} = 27.17\,mm = 0.0271\,m$

$$Nu = \frac{hD_{eq.}}{k}$$

$$200 = \frac{h(0.0271)}{0.19}$$

$$h = 1400\frac{W}{m^2\,K}$$

Q.23 (D)

Q.24 (C)

$$Q_2 = \frac{Q_1}{2}[\text{Given}]$$

$$\frac{Q_1}{Q_2} = \frac{q_h}{\left(\frac{q_h}{2}\right)} = \frac{m_h c_h (100-60)}{m_h c_h (100-T_h)}$$

$$2 = \frac{(40)}{(100\text{-}T_h)}$$

$$\therefore T_h = 80\,°C$$

$$\frac{q_c}{q_c/2} = \frac{m_c C_c (80\text{-}40)}{m_c C_c (T_c\text{-}40)}$$

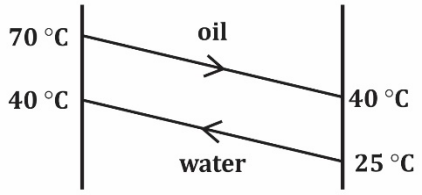
$$2 = \frac{40}{(T_c\text{-}40)}$$

$$\therefore T_c = 60\,°C$$

So, the new arrangement becomes:

$$\Delta T_{LMTD} = 40\,°C$$

Q.25 (B)



Given:

$$m_{oil} = 1000\frac{kg}{h} = \frac{1000}{3600}\frac{kg}{s}$$

$$C_{p,oil} = 2\frac{kJ}{kg\,°C} = 2000\frac{J}{kg\,°C}$$

$$C_{p,water} = 4.2\frac{kJ}{kg\,°C}$$

$$U = 0.2\frac{kW}{m^2K} = 200\frac{W}{m^2K}$$

$$\Delta T_{LMTD} = \frac{(T_{h,in} - T_{c,out}) - (T_{h,out} - T_{c,in})}{\ln\left(\frac{T_{h,in} - T_{c,out}}{T_{h,out} - T_{c,in}}\right)}$$

$$= \frac{(70-40)-(40-25)}{\ln\left(\frac{70-40}{40-25}\right)}$$

$$\Delta T_{LMTD} = 21.64\,K$$

$$Q = mC_p\Delta T = UA\Delta T_{LMTD}$$

$$\left(\frac{1000}{3600}\right)(2000)(70-40) = (200)(A)(21.64)$$

$$A = 3.85\,m^2$$

Q.26 (B)

Given: $T_s = 135\,°C,\ T_1 = 20\,°C,\ T_2 = 90\,°C$

$m_1 = m,\ m_2 = 2m$

$$\ln\frac{(T_s - T_1)}{(T_s - T_2)} = \frac{UA}{mC_p}$$

$$Nu = 0.023Re^{0.8}Pr^{1/3}$$

$$\text{Now, } \frac{1}{U} = \frac{1}{h_t} + \frac{1}{h_s}$$

$$h_s >>> h_t$$

$$\frac{1}{U} = \frac{1}{h_t} \Rightarrow U = h_t$$

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$$Nu = \frac{hD}{k} = 0.023 Re^{0.8} Pr^{1/3}$$

$$h \propto m^{0.8}$$

$$\text{So,} \frac{\ln\left(\frac{T_s - T_1}{T_s - T_2}\right)}{\ln\left(\frac{T_s - T_1}{T_s - T_2'}\right)} = \frac{\frac{UA}{m_1 C_p}}{\frac{UA}{m_2 C_p}}$$

$$\frac{\ln\left(\frac{T_s - T_1}{T_s - T_2}\right)}{\ln\left(\frac{T_s - T_1}{T_s - T_2'}\right)} = \frac{\frac{m_1^{0.8}}{m_1}}{\frac{m_2^{0.8}}{m_2}}$$

$$\frac{\ln\left(\frac{135 - 20}{135 - 90}\right)}{\ln\left(\frac{135 - 20}{135 - T_2'}\right)} = \left(\frac{m_2}{m_1}\right)^{0.2}$$

$$\frac{0.9382}{\ln\left(\frac{115}{135 - T_2'}\right)} = (2)^{0.2}$$

$$0.81 = \ln\left(\frac{115}{135 - T_2'}\right)$$

$$e^{0.81} = \frac{115}{135 - T_2'} = 2.26$$

$$\therefore T_2' = 84.18\,°C$$

Q.27 (300)

Given: $d_i = 25\,mm = 0.025\,m$

$d_o = 30\,mm = 0.030\,m$

$$U_i = 360 \frac{W}{m^2\,°C}$$

$$Q = U_i A_i \Delta T = U_o A_o \Delta T$$

Overall HTC based on inside dia.:

$$U_i = \frac{1}{\left[\frac{1}{h_i} + \frac{(r_o - r_i)A_i}{kA_{lm}} + \frac{A_i}{A_o h_o}\right]}$$

Overall HTC based on outer dia.:

$$U_o = \frac{1}{\left[\frac{A_o}{A_i h_i} + \frac{(r_o - r_i)A_o}{kA_{lm}} + \frac{1}{h_o}\right]}$$

$$U_i A_i = U_o A_o$$

$$U_o = \frac{A_i}{A_o}(U_i) = \frac{\pi d_i L}{\pi d_o L}(U_i)$$

$$U_o = \left(\frac{0.025}{0.030}\right) \times 360$$

$$U_o = 300 \frac{W}{m^2\,°C}$$

Q.28 (27.08)

$T_{h-in} = 80\ ^oC$

$T_{h-out} = 50\ ^oC$

$T_{c-in} = 30\ ^oC$

$T_{c-out} = 45\ ^oC$

CASE-1: Co-Current flow

$$\Delta T_{LMTD}(\text{Co-current}) = \frac{(T_{h-in} - T_{c-in}) - (T_{h-out} - T_{c-out})}{\ln\left(\frac{T_{h-in} - T_{c-in}}{T_{h-out} - T_{c-out}}\right)}$$

$$= \frac{(80 - 30) - (50 - 5)}{\ln\frac{(80 - 30)}{(50 - 5)}}$$

$$= \frac{50 - 5}{\ln(10)} = 19.54\ °C$$

CASE-2: Counter-Current flow

$$\Delta T_{LMTD}(\text{Counter-current}) = \frac{(80 - 45) - (50 - 30)}{\ln\frac{(80 - 45)}{(50 - 30)}}$$

$$= \frac{35 - 20}{\ln\left(\frac{35}{20}\right)} = 26.80\ °C$$

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Heat transfer is given by

$$Q=U\ A\ \triangle T_{LMTD}$$

$$\text{So, } A \propto \frac{1}{\triangle T_{LMTD}}$$

$$\frac{A_1\,(\text{Co}-\text{current})}{A_2\,(\text{Counter}-\text{current})}=\frac{\triangle T_{LMTD}\,(\text{Counter}-\text{current})}{\triangle T_{LMTD}\,(\text{Co}-\text{current})}$$

$$=\frac{26.80}{19.54}$$

$$\frac{A_2}{A_1}=0.729$$

$$\%\text{Saving of Area}=\frac{A_1-A_2}{A_1}$$

$$=1-\frac{A_2}{A_1}=1-0.729$$

$$=27.08\ \%$$

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ASSIGNMENT QUESTIONS

Q.1 Which of the following law of thermodynamics is responsible for heat transfer?
a) Zeroth law of thermodynamics
b) First law of thermodynamics
c) Second law of thermodynamics
d) Third law of the thermodynamics

Q.2 Heat is mainly transferred by conduction, convection and radiation in
a) insulated pipes carrying hot water
b) refrigerator freezer coil
c) boiler furnace
d) condensation of steam in a condenser

Q.3 Arrangement of silver, air, aluminum and lead in order of increasing thermal conductivity at room temperature
a) Air, Aluminum, Silver, Lead
b) Air, Aluminum, Lead, Silver
c) Lead, Air, Aluminum, Silver
d) Air, Lead, Aluminum, Silver

Q.4 Consider the following materials:
1. Carbon 2. Mica
3. Bakelite 4. Fiber glass
Which of these materials are good conductors of heat, but bad conductors of electricity?
a) 1 only b) 2 only
c) 2 and 3 d) 3 and 4

Q.5 Which one of the following has a highest thermal conductivity?
a) Boiling water b) Steam
c) Solid ice d) Rain water

Q.6 On a summer day, a scooter rider feels more comfortable while on the move more than while at a stop light because
a) an object in motion captures less solar radiation
b) air is transparent to radiation and hence it is cooler than the body
c) more heat is lost by convection and radiation while in motion
d) air has a low specific heat and hence it is cooler

Q.7 As the thickness of insulation around a heated cable gradually increases from zero, heat transfer from the conductor
a) goes on decreasing monotonically
b) goes on increasing monotonically
c) first increases and then decreases
d) first decreases and then increases

Q.8 Usually fins are provided to increase the rate of heat transfer but fin also acts as insulation which one of the following non dimensional number decides this factor
a) Eckert number b) Biot number
c) Fourier number d) Peclet number

Q.9 Fins are used to increase the heat transfer rate from surface by
a) increasing the temperature difference
b) increasing the effective surface area
c) increasing the convective heat transfer coefficient
d) none of the above

Q.10 Up to critical radius of insulation
a) added insulation increases heat loss
b) added insulation decreases heat loss

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c) convection heat loss is less than conduction heat loss
d) heat flux decrease

Q.11 A critical radius of insulated pipe leads to
a) minimum heat transfer
b) no effect on heat transfer
c) no heat transfer at all
d) maximum heat transfer

Q.12 The non-dimensional parameter known as Stanton number (St) is used in
a) forced convection heat transfer in flow over flat plate
b) condensation heat transfer with laminar film layer
c) natural convection heat transfer over flat plate
d) unsteady heat transfer from bodies in which internal temperature gradient cannot be neglected

Q.13 In convection heat transfer one of the significant dimensionless number is
a) Schmidt number
b) Nusselt number
c) Poisson number
d) Froude number

Q.14 Fraction of radiative energy leaving one surface that strikes the other surface is called
a) radiative flux
b) emissive power of the first surface
c) view factor
d) reradiating flux

Q.15 What is the basic equation of thermal radiation from which all other equations of radiation can be derived
a) Stefan-Boltzman equation
b) Plank's equation
c) Wein's equation
d) Rayleigh Jeans formula

Q.16 In a lumped parameter model, the temperature variation with time is
a) Cubic b) Linear
c) Exponential d) Sinusoidal

Q.17 When metallic surfaces are oxidized the emissivity
a) decreases
b) increases
c) remains unaltered
d) unpredictable

Q.18 A heat exchanger with heat transfer surface area A and overall heat transfer coefficient U handles two fluids of heat capacities C_{max} and C_{min}. The parameter NTU (Number of transfer units) used in the analysis of heat exchanger is defined as
a) $\frac{AC_{min}}{U}$ b) $\frac{U}{AC_{min}}$
c) AUC_{min} d)

Q.19 What is the equivalent thermal conductivity of a composite wall made of two layers of different material of thermal conductivity K_1 and K_2 and having thickness δ_1 and δ_2 respectively
a) $\frac{2(K_1+K_2)}{\delta_1+\delta_2}$ b) $\frac{K_1+K_2}{K_1K_2}\frac{\delta_1\delta_2}{\delta_1+\delta_2}$
c) $\frac{K_1K_2(\delta_1+\delta_2)}{K_1\delta_2+K_2\delta_1}$ d) $\frac{K_1}{\delta_1}+\frac{K_2}{\delta_2}$

Q.20 An aluminum sheet of 10 mm thickness is brought into contact with an iron sheet of 20 mm thickness. The outer surface of aluminum is kept at 100°C. whereas the outer surface of iron is maintained at 0°C. If the ratio of thermal conductivity of aluminum and iron is 3:1, then the interface temperature in "C is
a) 40 b) 60
c) 75.7 d) 85.7

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Q.21 A metal wall has an area of 5 m^2, thickness 10 cm and a thermal conductivity 200 W/mK what is the value of thermal resistance of the wall in K/W

a) 10^{-4} b) 10^{-3}

c) 2×10^{3} d) 10^{4}

Q.22 Which one of the following differential equation represents the unsteady state heat transfer

a) $\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}=\frac{1}{\alpha}\frac{\partial T}{\partial r}$

b) $\frac{\partial T}{\partial x}+\frac{\partial T}{\partial y}+\frac{\partial T}{\partial Z}=0$

c) $\frac{\partial T}{\partial x}+\frac{\partial T}{\partial y}+\frac{\partial T}{\partial z}+\frac{q_G}{K}=0$

d) $\frac{\partial T}{\partial x}+\frac{\partial T}{\partial y}=0$

Q.23 A steel plate of thermal conductivity 50 W/m-K and thickness 10 cm passes a heat flux by conduction of 25 kW/m^2. If the temperature of the hot surface of the plate is 100°C then what is the temperature of the cooler side of the plate

a) 30°C b) 40°C

c) 50°C d) 60°C

Q.24 A composite wall of a furnace has 3 layers of equal thickness having thermal conductivities in the ratio of 1:2:4. What will be the temperature drop ratio across the three respective layers?

a) 1:2:4 b) 4:2:1

c) 1:1:1 d) log 4:1og 2:1og 1

Q.25 Which one of the following has maximum thermal conductivity?

a) Silver b) Copper

c) Gold d) Diamond

Q.26 A metallic rod of uniform diameter and length L connects two heat sources each at 500°C. The atmospheric temperature is 30°C. The temperature gradient $\frac{dT}{dL}$ at the centre of the bar will be

a) $\frac{500}{L/2}$ b) $-\frac{500}{L/2}$

c) $-\frac{470}{L/2}$ d) zero

Q.27 A hollow sphere has inner and outer surface areas of $2m^2$ and 8 m^2 respectively. For a given temperature difference across the surfaces, the heat flow is to be calculated considering the material of the sphere as a plane wall of the same thickness. What is the equivalent mean area normal to the direction of heat flow?

a) $6m^2$ b) $5m^2$

c) 4 m^2 d) None of these

Q.28 A composite wall having three layers of thickness 0.3 m, 0.2 m and 0.1 m and of thermal conductivities 0.6, 0.4 and 0.1 W/mK, respectively, is having surface area 1 m^2. If the inner and outer temperatures of the composite wall are 1840 K and 340 K, respectively, what is the rate of heat transfer?

a) 150W b) 1500W

c) 75 W d) 750 W

Q.29 A man sits on the floor by a fire burning at some distance at same level. The mode of heat transfer mainly responsible for the man receiving heat is

a) conduction b) convection

c) radiation d) advection

Q.30 For a laminar flow of air (Pr = 0.71) over a flat plate at zero incidence, the thickness of the velocity boundary layer δ and the thermal boundary layer δ_1 are such that

a) $\delta_1 > \delta$ b) $\delta_1 = \delta$

c) $\delta_1 < \delta$ d) $0.5 \le \frac{\delta_1}{\delta} \le 1.5$

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Q.31 Heat is lost from a 100 mm diameter steam pipe placed horizontally in ambient at 30°C. If the Nusselt number is 25 and thermal conductivity of air is 0.03 W/mK then the heat transfer coefficient will be

a) 7.5 W/m²K b) 16.2 W/m²K
c) 25.2 W/m²K d) 30 W/m²K

Q.32 Match List-I (Dimensionless quantity) with List-II (Application) and select the correct answer using the codes given below the lists:

List-I

A. Stanton number
B. Grashof number
C. Peclet number
D. Schmidt number

List-II

1. Natural convection
2. Mass transfer
3. Forced convection
4. Forced convection for small Prandtl number

Codes:

	A	B	C	D
a)	2	1	4	3
b)	3	1	4	2
c)	2	4	1	3
d)	3	4	1	2

Q.33 Nusselt number for fully developed turbulent flow in a pipe is given by $Nu = c(Re^a Pr^b)$. The values of a and b are

a) a= 0.5 and b = 0.33 for heating and cooling both
b) a = 0.5 and b = 0.4 for heating and b = 0.3 for cooling
c) a = 0.8 and b = 0.4 for heating and b = 0.3 for cooling
d) a = 0.8 and b = 0.3 for heating and b = 0.4 for cooling

Q.34 The wavelength of the radiation emitted by a body depends upon

a) The nature of its surface
b) The area of its surface
c) The temperature of its surface
d) All the above factors.

Q.35 For two infinite parallel Gray planes with emissivity $\in_1$ and $\in_2$ respectively, the expression for emissivity factor?

a) 1 b) $\frac{\in_1 \in_2}{\in_1 + \in_2 - \in_1 \in_2}$

c) $\in_1 + \in_2$ d) $\frac{1}{\in_1} + \frac{1}{\in_2}$

Q.36 If the temperature of a solid surface changes from 27°C to 627°C. It's emissive power will increase in the ratio of

a) 3 b) 9
c) 27 d) 81

Q.37 The radiative heat transfer rate per unit area (W/m²) between two plane parallel gray surface (emissivity = 0.9) maintained at 400 K and 300 K is (Stefan-Boltzman constant = 5.67×10^{-8} W/m²K⁴)

a) 1020 b) 464
c) 812 d) 567

Q.38 Two long parallel surfaces each of emissivity 0.7 are maintained at different temperatures and accordingly have radiation heat exchange between them. It is desirable to reduce 75% of this radiation heat transfer by inserting thin parallel shields of equal emissivity on both sides. The number of shields should be

a) one b) two
c) three d) four

Q.39 For conduction through a spherical wall with constant thermal conductivity and with inner side temperature greater than outer wall temperature, (one dimensional heat transfer), what is the type of temperature distribution?

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a) Linear b) Parabolic
c) Hyperbolic d) None of the above

Q.40 Match **List-I** (surface with orientations) with **List-II** (equivalent emissivity) and select the correct answer using the codes given below:

List-I

A. Infinite parallel planes
B. Body 1 completely enclosed by body 2 but body 1 is very small
C. Radiation exchange between two small Grey bodies
D. Two concentric cylinders with large lengths

List-II

1. $\in_1$

2. $\dfrac{1}{\frac{1}{\epsilon_1}+\frac{1}{\epsilon_2}-1}$

3. $\dfrac{1}{\frac{1}{\epsilon_1}+\frac{A_1}{A_2}\left(\frac{1}{\epsilon_2}-1\right)}$

4. $\in_1 \in_2$

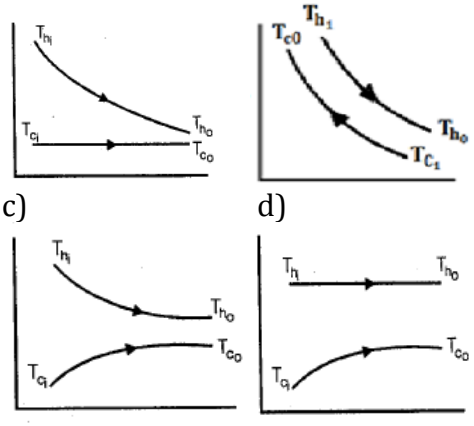
Codes:

	A	B	C	D
a)	3	1	4	2
b)	2	4	1	3
c)	2	1	4	3
d)	3	4	1	2

Q.41 Hot air enters a single pass counter flow heat exchanger at 700 K and leaves at
400 K the cooling water enters at 300 K and leaves at 600 K. The LMTD for the heat exchanger is
a) 100 K b) 156.5 K
c) 144.3 K d) 200 K

Q.42 Which one of the following graphs, showing temperature distributions of different heat exchangers, pertains steam condenser?

a) b)



c) d)

Q.43 Fins are made as thin as possible to:
a) Reduce the total weight
b) Accommodate more number of fins
c) Increase the width for the same profile area
d) Improve flow of coolant around the fins

Q.44 Which one of the following heat exchanger gives parallel straight line pattern of temperature distribution for both cold and hot fluid
a) Parallel-flow with unequal heat capacity
b) Counter-flow with equal heat capacity
c) Parallel flow with equal heat capacity
d) Counter flow with unequal heat capacity

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ANSWER KEY:

1	2	3	4	5	6	7	8	9	10	11	12	13	14
(c)	(c)	(d)	(a)	(c)	(c)	(c)	(b)	(b)	(a)	(d)	(a)	(b)	(c)
15	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**
(b)	(c)	(c)	(d)	(c)	(d)	(a)	(a)	(c)	(b)	(d)	(d)	(c)	(d)
29	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**
(c)	(a)	(a)	(b)	(c)	(c)	(b)	(d)	(c)	(c)	(c)	(c)	(a)	(d)
43	**44**												
(b)	(b)												

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EXPLANATIONS

Q.1 **(c)**

Q.2 **(c)**
For radiation the magnitude of temperature difference should be large enough. Convection & conduction is also predominant in boiler furnace.

Q.3 **(d)**
Air, K = 0.024 W/mK
Lead, K = 35 W/mK
Aluminum, K = 234 W/mK
Silver, K = 420 W/mK.

Q.4 **(a)**
Mica, Bakelite and Fiber glass are insulator material and carbon in the form of diamond is a good conductor of heat but bad conductor of electricity

Q.5 **(c)**
K_{water}= 0.556 W/m^{0}C
K_{ice} = 2.22 W/m^{0}C
K_{steam} = 0.0206 W/m^{0}C

Q.6 **(c)**
More heat is lost by forced convection as the value of heat transfer coefficient is large because it depends on fluid properties, velocity of flow and surface condition.

Q.7 **(c)**
As the radius of insulation around a heated cable gradually increases from zero to critical radius, heat transfer increases and when the radius of insulation exceeds form the critical radius of insulation heat transfer decreases.

Q.8 **(b)**

Q.9 **(b)**

Since area is crucial in any mode of heat transfer hence by increasing effective surface area heat transfer rate can be enhanced.

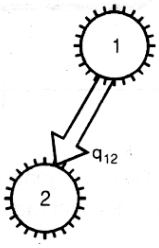
Q.10 **(a)**
Up to critical thickness heat transfer increases and after critical thickness there will be reduction in heat transfer, when more insulation is provided.

Q.11 **(d)**

Q.12 **(a)**

Q.13 **(b)**

Q.14 **(c)**
View factor represent the fraction of the energy emitted by one surface and received by the other.



$$F_{12} = \frac{q_{12}}{q_1}$$

Where
q_1 = Total energy emitted by body 1
q_{12} = Energy emitted by (1) and received by (2)

Q.15 **(b)**

Q.16 **(c)**

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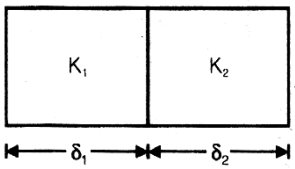
$$\frac{T-T_\infty}{T_i-T_\infty}=e^{\frac{-hA}{\rho Vc}t}$$

Hence temperature variation will be exponential.

Q.17 (c)

Q.18 (d)

Q.19 (c)



$R_{eq} = R_1 + R_2$

$$\frac{\delta_1+\delta_2}{k_{eq}A}=\frac{\delta_1}{k_1A}+\frac{\delta_2}{k_2A}$$

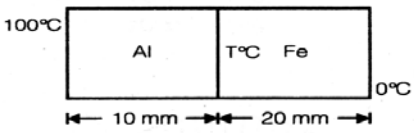
$$\frac{\delta_1+\delta_2}{k_{eq}}=\frac{\delta_1}{k_1}+\frac{\delta_2}{k_2}$$

$$\frac{\delta_1+\delta_2}{k_{eq}}=\frac{\delta_1k_2+k_1\delta_2}{k_1k_2}$$

$$k_{eq}=\frac{k_1k_2(\delta_1+\delta_2)}{k_1\delta_2+k_2\delta_1}$$

Q.20 (d)

$$\frac{K_{Al}}{K_{fe}}=3$$



$$Q_{Al}=\frac{K_{Al}A(100-T)}{10}$$

$$Q_{Fe}=\frac{K_{Fe}\times A(T-0)}{20}$$

$$\frac{K_{Al}(100-T)}{10}=\frac{K_{Fe}(T)}{20}$$

$$\frac{3K_{Fe}(100-T)}{10}=\frac{K_{Fe}(T)}{20}$$

$600 - 6T = T$

$600 = 7T$

$$\Rightarrow T=\frac{600}{7}=85.7^{\circ}C$$

Q.21 (a)

$$\text{Thermal resistance}=\frac{L}{KA}$$

$$=\frac{0.1}{5\times200}=10^{-4}k/W$$

Q.22 (a)

Q.23 (c)

$$\frac{Q}{A}=\frac{K(T_1-T_2)}{L}$$

$$25\times10^3=\frac{50\times(100-T_2)}{0.1}$$

$$T_2=50^{\circ}C$$

Q.24 (b)

$$\text{Since } \Delta T\propto\frac{1}{K}$$

Q.25 (d)

Q.26 (d)

Q.27 (c)

For sphere

$A_{gm} = \sqrt{A_1A_2}$

$= \sqrt{2 \times 8} = 4\ m^2$

Q.28 (d)

$$q=\frac{T_1-T_2}{R_{total}}$$

$$=\frac{(1840-340)}{\frac{0.3}{0.6\times1}+\frac{0.2}{0.4\times1}+\frac{0.1}{0.1\times1}}=750\ W$$

Q.29 (c)

Q.30 (a)

$$(Pr)^{\frac{1}{3}}=\frac{\delta}{\delta_1}$$

Since $Pr < 1$

$\therefore \delta_1 > \delta$

Q.31 (a)

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$$Nu = \frac{hD}{K}$$

$$25 = \frac{h \times 0.1}{0.03}$$

$$h = \frac{0.03 \times 25}{0.1} = 7.5\ W/m^2K$$

Q.32 (b)

Q.33 (c)

Full developed Turbulent flow in a pipe.

$Nu = 0.023(Re^{0.8}Pr^n)$

Dittus –Boelter equation

n = 0.4 for heating of fluids

n = 0.3 for cooling of fluids

Q.34 (c)

Q.35 (b)

For two parallel Grey surface

$$Q = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$

Q.36 (d)

Since emissive power = $f(T^4)$

$$= \frac{(900)^4}{(300)^4} = 81$$

Q.37 (c)

$$Q = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$

$\epsilon_1 = \epsilon_2 = 0.9$

$\sigma = 5.67 \times 10^{-8}\ W/m^2K^4$

$$Q = \frac{5.67 \times 10^{-8}(400^4 - 300^4)}{\frac{1}{0.9} + \frac{1}{0.9} - 1}$$

$Q = 812\ W/m^2$

Q.38 (c)

$$q_{withshield} = \left(\frac{1}{N+1}\right) q_{without\ shield}$$

where N = No. of radiation shields

$$0.25\ q = \left(\frac{1}{N+1}\right) q$$

N + 1 = 4

N = 3

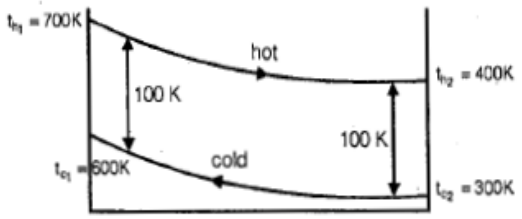
Q.39 (c)

Temp distribution would be

$$\frac{t - t_1}{t_2 - t_1} = \frac{\frac{1}{r} - \frac{1}{r_1}}{\frac{1}{r_2} - \frac{1}{r_1}}$$

Q.40 (c)

Q.41 (a)



Hence LMTD = 100K

Q.42 (d)

Q.43 (b)

Q.44 (b)

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